10-10-2022

Optimal synthesis of crank-rocker mechanisms with optimum transmission angle for desired stroke and time-ratio using genetic programming

Bahman Ahmadi
University of Kurdistan

Behnam Ahmadi
Michigan Technological University, bahmadi@mtu.edu

Follow this and additional works at: https://digitalcommons.mtu.edu/michigantech-p
Part of the Mechanical Engineering Commons

Research Article

Advances in Mechanical Engineering

Advances in Mechanical Engineering
2022, Vol. 14(10) 1–10

DOI: 10.1177/16878132221131291
journals.sagepub.com/home/ade
SAGE

Optimal synthesis of crank-rocker mechanisms with optimum transmission angle for desired stroke and time-ratio using genetic programming

Bahman Ahmadi[1] and Behnam Ahmadi[2]

Abstract
Dimensional synthesis of crank-rocker mechanisms applied to provide some desired values of stroke and time ratio, is of utmost importance for designing an efficient mechanism. In the synthesis and manufacturing of crank-rocker mechanisms, the designers are further challenged by other design criteria, such as quality of motion. In this study, a novel approach based on genetic programming (GP) is proposed for dimensional synthesis of planar crank-rocker mechanisms with optimum transmission angle over the desired stroke and time-ratio. An analytical approach is elaborated which leads to an interesting relationship of length of the coupler and rocker links. It is, therefore, advised that by adopting equal lengths for coupler link and rocker link, one can guarantee the optimality of the transmission angle's deviation (*TA*), ensuring the Grashof condition. Consequently, through an inverse modeling approach, GP method is utilized to construct some explicitly mathematical formulas to represent all the sizes and dimensions of the crank-rocker mechanism based on the any desired values of both stroke and time-ratio. In this way, an input-output data set consisting of all the dimensions and the sizes of the links as the input variables and both the stroke and time-ratio as output variables is first constructed using the pertinent mathematical equations. Indeed, such approach of inverse modeling using GP simplifies greatly the synthesis of the crack-rocker mechanisms for any desired values of both stroke and time-ratio. The proposed approach has been applied for kinematic synthesis of a crank-rocker mechanism. A comparison between the obtained results of this work and the analytic method, clearly illustrates the efficiency of the proposed approach.

Keywords
Mechanism synthesis, crank-rocker, genetic programming, optimization, inverse modeling

Date received: 17 May 2022; accepted: 16 September 2022

Handling Editor: Chenhui Liang

Introduction

Mechanisms as basic components of machines are widely used in a widening area of engineering and scientific domains. The synthesis of a mechanism is the study of procedures of combining parametric members whose combination provides some desired behaviors.[1] Dimensional synthesis, as a kind of inverse problem, is concerned with evaluation of geometrical sizes of

[1]Department of Mechanical Engineering, Faculty of Engineering, University of Kurdistan, Sanandaj, Kurdistan, Iran
[2]Department of Mechanical Engineering–Engineering Mechanics, Michigan Technological University, Houghton, MI, USA

Corresponding author:
Bahman Ahmadi, Department of Mechanical engineering, Faculty of Engineering, University of Kurdistan, Pasdaran Boulevard, Sanandaj, Kurdistan 66177-15175, Iran.
Email: b.ahmadi@uok.ac.ir

mechanisms. Dimensional syntheses of crank-rocker mechanisms to provide some desired values of stroke and time-ratio, have been sort of challenging problems in the case of planar mechanisms. However, in the synthesis and manufacturing of crank-rocker mechanisms, the designers are challenged by some other design criteria, for example, quality of motion. Addressing this issue is very important to design an efficient mechanism.[2] The quality of motion is measured by transmission angle's deviation from ideal value of 900.[3]

Generally, synthesis of the crank-rocker mechanisms has been addressed during the last decades. In earlier studies, Joe Brodell and Soni[4] introduced a design chart obtained from an analytical approach to synthesize the crank-rocker mechanism with good quality of motion and unit time-ratio. Balaji Rao and Lakshminarayana[5] presented the optimal synthesis of RSSR crank-rocker mechanisms for different combinations of oscillation angle, time-ratio, and axis angle. The authors in[6] suggested a synthesis procedure which satisfies a prescribed stroke and time-ratio wherein the required definite objective function is optimized. Soylu,[7] investigated a synthesis procedure for the planar mechanism in which the maximum deviation of transmission angle is constrained to be less than a certain desired limit. The graphical and analytical approaches associated with design charts considering some performance criteria, such as stroke, transmission angle, and time-ratio was addressed in Lun and Leu.[8] Balli and Chand[3] investigated the transmission angle's influence on the different parameters of a mechanism, for example, friction, mechanical advantage, pressure angle, transmission force, input crank angle, tolerance, and the performance sensitivity. The authors also studied the various mechanism's defects, such as branching, order, circuit, and poor transmission angle in Balli and Chand.[9] Eren and Aydemir[10] introduced a kinematic synthesis scheme for the synthesis of four-bar sley drive linkages. The authors considered the transmission angle, the stroke, and the time-ratio as the parameters affecting the sley trajectory. Hassaan et al.[11] used Powell's optimization method for optimal design of crank-rocker linkages for a given stroke and time-ratio. Khader[12] proposed the design nomograms for directly designing the crank-rocker mechanism links' ratios with a definite determined transmission angle limit. Rufino and Ferreira[13] described a variable stroke mechanism and presented a kinematic model to determine the requirements for synthesis of the mechanism. El-Shakeryet al.[14] suggested both analytical and graphical methodologies to optimally determine links' lengths of crank-rocker four-bar linkage to obtain a targeted design with definite transmission angle. Yildiz[15] addressed the structural design optimization of a four-bar linkage for a trunk lid of a sedan type passenger car, incorporating three different population-based optimization techniques. The authors in Refs.[2,16,17] used game theory to present a framework for multi-objective optimal synthesis of four-bar mechanisms for path-generating applications. It should be mentioned that the performance criteria stroke and time-ratio were not taken into account by authors. However, many research works have been devoted to the optimal synthesis of path-generating four-bar mechanism using suitable optimization schemes and computational intelligence techniques.[2,17–24]

Modeling and identification of data driven systems have always been of a great interest for research workers and have become a new topic of research in a broad range area in engineering science.[25–31] Generally, an exactly mathematical relationship between input–output data is needed to model as well as to predict the data-driven system, precisely. However, obtaining such mathematical modeling is so cumbersome and even is not possible in poorly identified systems. Recently, some deep learning algorithms as a part of a broader category of machine learning schemes in conjunction with evolutionary algorithms have drawn considerable attention in computation of imprecisely complex environments. In evolutionary algorithms (EAs), genetic programming is defined as an extension to genetic algorithms (GAs) in which the structures undergoing adaptation are not strings but are hierarchical computer programs of dynamically varying shape and size. Recent research works that set the stage for current GP research topics and applications is various, and includes topology optimization of structures,[32,33] optimal modeling of complex systems,[34] controller design,[35] image processing,[36] object detection,[37] data mining,[38] control of robots,[39] and so on.

The available kinematic design methods in mechanism literature calculate the link lengths of a crank rocker mechanism according to minimum transmission angle criteria as a design constraint. On the other hand, as the occurrence of failure of Grashof condition is catastrophic in a four-bar mechanism, guaranteeing Grashof constraint feasibility is of the most importance at both the synthesis stage and the manufacturing process. As far as the authors know, there is no design approach in the literature for a crank-rocker mechanism to determine the link lengths of the mechanism analytically, taking into account the deviation of transmission angle from ideal value as an objective function and guaranteeing the Grashof condition, simultaneously. Moreover, there is still a clear gap in development and implementing meta-heuristic approaches to generate the structural descriptions of mechanisms.

This study tries to overcome these shortcomings by incorporating GP meta-modeling and inverse modeling approaches to solve the optimization problems arising in kinematic synthesis of crank-rocker mechanisms. The objective of the optimization problem is defined as

to design a mechanism that has a pre-determined value for stroke and time ratio and simultaneously guarantees the optimality of the motion quality and Grashof condition as well . In this regard, the lengths of the mechanism links are considered as design variables. It is, analytically, advised that keeping equal the length of rocker link and connecting rod link would automatically satisfy the optimality of the transmission angle's deviation. Then, for each mechanism, the stroke and the time ratio are geometrically obtained as a function of the lengths of the mechanism links. In order to apply GP method for the inverse modeling purpose, some input-output data are constructed based on stroke and time ratio as functions of sizes and lengths of the crank-rocker mechanism. Finally, with the use of the GP, the optimal mathematical model for each design variables are obtained as the functions of both stroke and time ratio.

It is shown that the GP model can successfully find almost simple and easy to use mathematical relations of mechanism's links needed to synthesize the mechanism with the best quality of motion for any desired values of both stroke and time ratio simultaneously. In fact, the proposed mathematical forms presented in this study, provide the capability to synthesize the length of mechanism's links for any desired value of stroke and time ratio within the estimated ranges. Moreover, such novel application of the proposed approach of the optimum synthesis of crank-rocker mechanisms leads to a very interesting relationship of length of mechanism's link. It is advised that keeping equal the length of rocker link and connecting rod link would automatically satisfy the optimality of the transmission angle's deviation. The proposed approach has been applied for kinematic synthesis of a four-bar crank-rocker mechanism. A comparison between the obtained results of this work and the analytical method, clearly illustrates the efficiency of the proposed method.

Methodology

In this section, a general algorithm for the optimally synthesis of mechanisms has been presented by using genetic programming method. The proposed algorithm has been summarized in Figure 1. The objective of the optimization problem is defined as to design a mechanism that has some desired performance criteria. Generally, the lengths of the mechanism links are considered as design variables, that is, output variables. For each mechanism, the performance criteria, that is, input variables are geometrically obtained as a function of the lengths of the mechanism links. Then, in order to apply genetic programming method for the inverse modeling purpose, some input-output data are constructed based on the performance criteria as functions

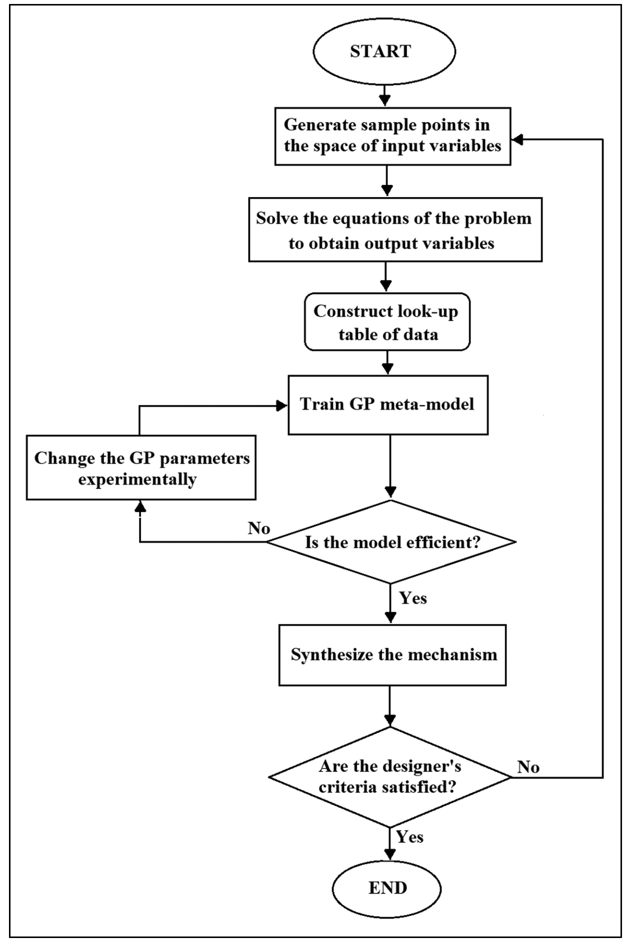

Figure 1. Schematic algorithm of the proposed approach for synthesis of mechanisms.

of sizes and lengths of the mechanism. Finally, with the use of the GP the optimal mathematical model for each design variable are obtained as the functions of performance criteria. The following case study illustrate how the proposed approach depicted in Figure 1 can be used to improve the overall design of a four-bar crank-rocker mechanism for any desired stroke and time-ratio while optimizing transmission angle' deviation.

Crank-rocker mechanism: A case study

The four-bar mechanism is the simplest and the most popular linkages, which consists of three movable links and a fixed link. A crank-rocker is a type of four-bar mechanism which is widely used for converting continuous rotary motion to oscillatory motion with a quick return feature. A four-bar linkage is called a crank-rocker when the shortest link is connected to the ground link. As illustrated in Figure 2, r_1 is the reference bar, r_2 is the input link with rotatory motion which is called crank, r_3 is a connecting rod link, and r_4 is the output link which is called rocker and oscillates between two limiting angles.

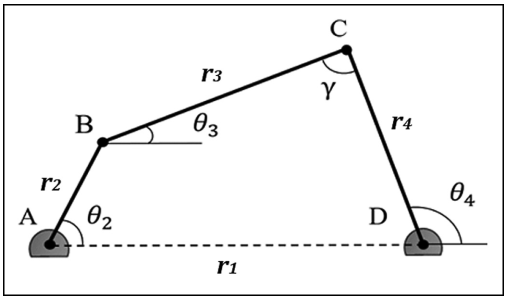

Figure 2. Schematic of a four-bar mechanism.

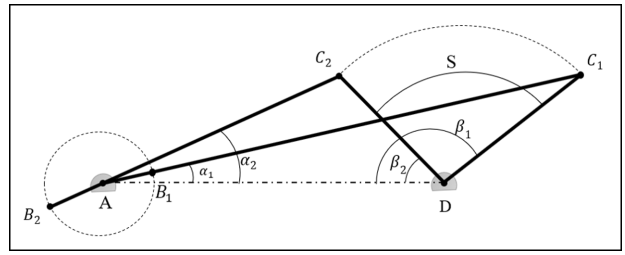

Figure 3. Schematic of crank-rocker mechanism at its limiting situation.

Angles θ_3, θ_4, and γ represent the interior joint angles and obtained as follows for a certain crank angle (θ_2) as follows

$$BD = \sqrt{{r_1}^2 + {r_2}^2 - 2r_1r_2\ cos\theta_2} \tag{1}$$

$$\gamma = \cos^{-1}\left(\frac{{r_3}^2 + {r_4}^2 - BD^2}{2r_3r_4}\right), \tag{2}$$

$$\theta_3 = 2\ \tan^{-1}\left(\frac{-r_2\ sin\theta_2 + r_4\ sin\gamma}{r_1 + r_3 - r_2\cos\theta_2 - r_4\cos\gamma}\right), \tag{3}$$

$$\theta_4 = 2\ \tan^{-1}\left(\frac{r_2\ sin\theta_2 - r_3\ sin\gamma}{r_4 - r_1 + r_2\cos\theta_2 - r_3\cos\gamma}\right). \tag{4}$$

In this study, the stroke, time-ratio, and transmission angle's deviation (TA) are considered as performance criteria in the synthesis of mechanism. The stroke of a crank-rocker mechanism is the angle range that rocker oscillates and time ratio is a measure of its quick return feature. Figure 3 shows a planar four-bar crank-rocker mechanism at its limiting situations.

According to Figure 3 and by using cosine law, the stroke and time-ratio of the mechanism are kinematically computed as a function of four parameters, that is, the length of the mechanism's links. The parameters β_1, β_2, α_1, and α_2 are defined as follows

$$\beta_1 = \cos^{-1}\left(\frac{{r_1}^2 + {r_4}^2 - (r_3 + r_2)^2}{2r_1r_4}\right) \tag{5a}$$

$$\beta_2 = \cos^{-1}\left(\frac{{r_1}^2 + {r_4}^2 - (r_3 - r_2)^2}{2r_1r_4}\right) \tag{5b}$$

$$\alpha_1 = \cos^{-1}\left(\frac{{r_1}^2 + (r_3 + r_2)^2 - {r_4}^2}{2r_1(r_3 + r_2)}\right) \tag{5c}$$

$$\alpha_2 = \cos^{-1}\left(\frac{{r_1}^2 + (r_3 - r_2)^2 - {r_4}^2}{2r_1(r_3 - r_2)}\right) \tag{5d}$$

The stroke and time ratio are then calculated using equations (6) and (7), respectively.

$$S = \beta_1 - \beta_2, \tag{6}$$

$$TR = \frac{(2\pi - \beta)}{\beta}, \tag{7}$$

in which $\beta = \pi - \alpha$, and $\alpha = \alpha_1 - \alpha_2$.

The transmission angle' deviation is formulated as

$$TA = (\gamma_{max} - 90^o)^2 + (\gamma_{min} - 90^o)^2 \tag{8}$$

According to equation (8), TA is to minimize the deviation of the transmission angle from 90°. Figure 4 illustrates the minimum and maximum values of transmission angles which can be obtained by equation (9)

$$\gamma_{min} = \cos^{-1}\left(\frac{{r_3}^2 + {r_4}^2 - (r_1 - r_2)^2}{2r_3r_4}\right), \tag{9a}$$

$$\gamma_{max} = \cos^{-1}\left(\frac{{r_3}^2 + {r_4}^2 - (r_1 + r_2)^2}{2r_3r_4}\right). \tag{9b}$$

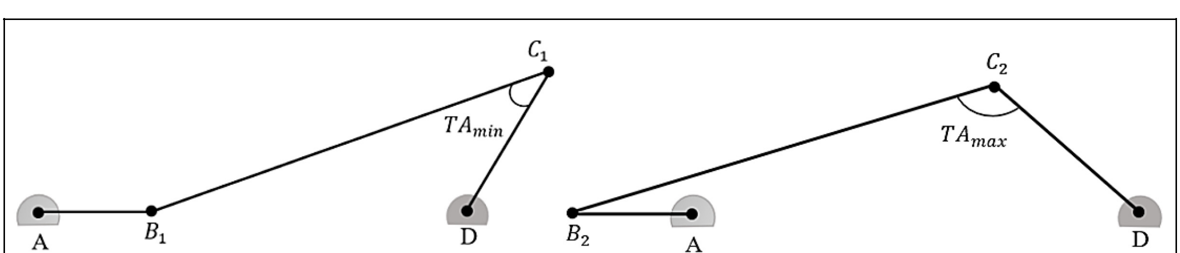

Figure 4. The minimum and maximum values of transmission angles.

where, γ refers to transmission angle of mechanism. In order to allow for the complete turn of the input link, the Grashof condition should be satisfied as an inequality constraint during the synthesis process. Grashof condition for a four-bar crank-rocker is defined as

$$r_{min} + r_{max} < r_a + r_b \,, \tag{10}$$

where, r_{min} and r_{max} are the lengths of minimum and maximum links and r_a and r_b are the lengths of two other links.

Optimization of transmission angle

In the synthesis and manufacturing of mechanisms, design engineers usually are faced quality of motion. Addressing this important issue is essential to synthesize an accurate, efficient, and reliable mechanism that functions satisfactorily during its period of service. The transmission angle of mechanism employed as a measure of the quality of force transmission of the mechanism. For instance, it helps to decide the "Best" among a family of possible mechanisms for most effective force transmission.

In this section, an analytical approach adopted from Ahmadi et al.[24] is given to obtain an explicitly mathematical equation which guarantees the optimality of the transmission angle's deviation and also ensures the Grashof condition.

In this way, equation (8) can be rewritten using equation (9) as follows,

$$\begin{aligned} TA = &\left[\cos^{-1}\left[\frac{(r_1+r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right] - \frac{\pi}{2}\right]^2 \\ &+ \left[\cos^{-1}\left[\frac{(r_1-r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right] - \frac{\pi}{2}\right]^2 \end{aligned} \tag{11}$$

In order to obtain a relationship between r_3 and r_4 which leads to optimality of TA, one can set the gradient vector of equation (11) to zero with respect to r_3 and r_4, that is,

$$\begin{aligned} \frac{\partial TA}{\partial r_3} = &\frac{\left(r_3^2 - r_4^2 - (r_1+r_2)^2\right)\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{(r_1+r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]\right]}{\sqrt{1 - \left[\frac{(r_1+r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]^2}} \\ &+ \frac{\left(r_3^2 - r_4^2 - (r_1-r_2)^2\right)\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{(r_1-r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]\right]}{\sqrt{1 - \left[\frac{(r_1-r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]^2}} = 0 \,, \end{aligned} \tag{12a}$$

$$\begin{aligned} \frac{\partial TA}{\partial r_4} = &\frac{\left(r_4^2 - r_3^2 - (r_1+r_2)^2\right)\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{(r_1+r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]\right]}{\sqrt{1 - \left[\frac{(r_1+r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]^2}} \\ &+ \frac{\left(r_4^2 - r_3^2 - (r_1-r_2)^2\right)\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{(r_1-r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]\right]}{\sqrt{1 - \left[\frac{(r_1-r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]^2}} = 0 \,. \end{aligned} \tag{12b}$$

By taking substitution variables α and β as

$$\alpha = \frac{\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{(r_1+r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]\right]}{\sqrt{1 - \left[\frac{(r_1+r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]^2}} \,, \tag{13a}$$

$$\beta = \frac{\left[\frac{\pi}{2} - \cos^{-1}\left[\frac{(r_1-r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]\right]}{\sqrt{1 - \left[\frac{(r_1-r_2)^2 - r_3^2 - r_4^2}{2r_3r_4}\right]^2}} \,, \tag{13b}$$

The necessary conditions of equation (12) is transferred to equation (14) using some algebraic manipulation as

$$\alpha\,(r_1+r_2)^2 + \beta\,(r_1-r_2)^2 = 0 \,, \tag{14a}$$

$$(\alpha+\beta)\left(r_4^2 - r_3^2\right) = 0 \,. \tag{14b}$$

Evidently, both α and β in equation (13) cannot be equal to zero at the same time. Moreover, the sum of $(\alpha + \beta)$ in equation (14b) cannot be zero. By this way, the set of solution is obtained as an feasible solution of RRS as $r_3 = r_4$.

In order to obtain real solution, the radicand terms in equation (13) should be positive values. Readily, by considering $r_3 = r_4$ in equation (13) one can conclude that

$$r_3 = r_4 > \frac{r_1 + r_2}{2} \tag{15a}$$

$$r_3 = r_4 > \frac{|r_1 - r_2|}{2} \tag{15b}$$

in which, $|.|$ denotes the absolute value. The equation (15) simply represent Grashof condition. Moreover, by using $r_3 = r_4$, the equation (5) can be restated as follows

$$\beta_1 = \cos^{-1}\left(\frac{{r_1}^2 - {r_2}^2 - 2r_2r_3}{2r_1r_3}\right) \tag{16a}$$

$$\beta_2 = \cos^{-1}\left(\frac{{r_1}^2 - {r_2}^2 + 2r_2r_3}{2r_1r_3}\right) \tag{16b}$$

Table 1. Input-output data pairs for the crank-rocker four-bar mechanism.

Index	**S** (°)	**TR**	r_1 (cm)	r_2 (cm)	r_3 (cm)
1	20	1.08	45.29	4.25	28.03
2	21.5	1.15	45.71	4.54	27.05
3	22.3	1.12	45.32	4.75	27.54
4	23	1.06	44.67	4.98	28.43
5	24	1.17	45.54	5.10	26.88
6	25.1	1.09	44.65	5.45	28.11
7	26	1.2	45.55	5.55	26.54
8	27.4	1.05	43.95	6.05	28.74
9	28	1.11	44.44	6.12	27.96
10	29.2	1.07	43.90	6.47	28.56
11	30	1.18	44.84	6.53	27.04
12	30.7	1.1	43.98	6.80	28.22
13	31.5	1.16	44.45	6.93	27.40
14	32	1.06	43.43	7.19	28.80
15	33	1.13	43.98	7.34	27.91
16	34.2	1.19	44.41	7.57	27.10
17	35	1.08	43.24	7.92	28.67
18	36.5	1.14	43.65	8.22	27.93
19	37.8	1.17	43.79	8.51	27.57
20	38	1.05	42.58	8.76	29.14
21	39.3	1.15	43.43	8.94	27.92
22	40	1.07	42.54	9.26	28.99
23	40	1.2	43.86	9.05	27.26
24	41	1.1	42.73	9.47	28.67
25	42	1.16	43.24	9.64	27.91
26	43.6	1.09	42.35	10.19	28.89
27	44	1.18	43.24	10.14	27.73
28	45	1.11	42.41	10.54	28.71
29	46.9	1.16	42.76	10.97	28.14
30	48.5	1.13	42.29	11.48	28.61
31	50	1.17	42.60	11.82	28.16
32	51	1.09	41.62	12.27	29.18
33	52.5	1.11	41.72	12.66	29.01
34	54	1.15	42.06	13.00	28.59
35	55	1.14	41.87	13.32	28.76

$$\alpha_1 = \cos^{-1}\left(\frac{{r_1}^2 + {r_2}^2 + 2r_2r_3}{2r_1(r_2 + r_3)}\right) \tag{16c}$$

$$\alpha_2 = \cos^{-1}\left(\frac{{r_1}^2 + {r_2}^2 - 2r_2r_3}{2r_1(r_3 - r_2)}\right) \tag{16d}$$

Such analytical approach indicates that keeping equal the length of r_3 and r_4 would automatically guarantees the optimality of the motion quality and also ensures the entire turn of crank link.

Results and discussion

As shown in Table 1, the sample points include two inputs (S, TR) and three independent outputs ($r_1, r_2,\ r_3$) which are considered as the input-output data pairs for GP algorithm. Subsequently, GP takes S and TR as input variables to generate mathematical functions and to fit as accurately as possible for each link's length of r_1, r_2 and r_3. It should be mentioned that the variations of two input parameters are set to $20^{\circ} \leq S \leq 55^{\circ}$ and $1.05 \leq TR \leq 1.2$. The input samples are depicted in Figure 5. Moreover, the limits of link lengths are considered as:

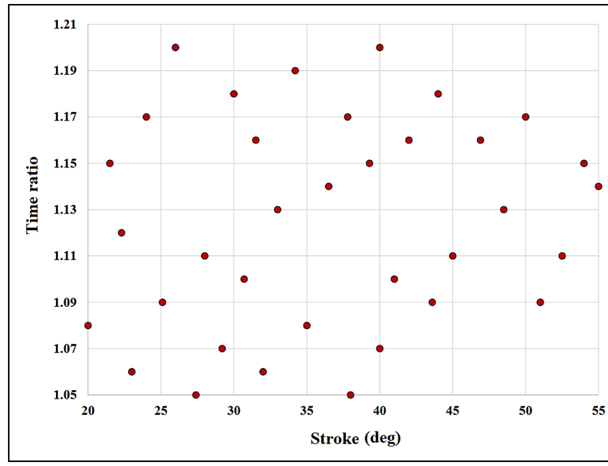

Figure 5. The input samples.

$$1 < r_1,\quad r_2,\quad r_3 < 50 \tag{17}$$

The main advantage of GP for the modeling process is its ability to produce models that build an understandable structure, that is, a formula or equation relating input and output variables, which might shed engineering insight into the processes involved.[40] Thus, for mechanism synthesis instances, GP may offer advantages over other techniques since GP can self-modify, through the genetic loop, a population of function trees in order to finally generate an optimal and physically interpretable model. Specifically, a comparative analysis of GP and multi-layer perceptron artificial neural network (MLP-ANN) for meta-modeling of the sample points is provided in Table 2. The results show that across all three systems GP provided greater training and prediction capability. The close proximity of training and prediction performance of GP implies relatively greater resistance to overfitting. The analysis of computational effort by GP and MLP-ANN reveals that GP requires a greater amount of computing time. However, this increased cost of fitting a GP metamodel may not be significant when the data is computationally expensive.[41] Furthermore, GP produces explicit functions in contrast with the models generated by other non-parametric approaches, such as MLP-ANN.

In order to succeed the inverse modeling procedure, the GP algorithm is deployed as a surrogate model. The GP algorithm utilizes computer programs as individuals of its population and models them with a tree structure.[33] More complete details on genetic programming and its terminology is presented in Refs.[30,33,34]

Table 2. Comparison of the proposed GP and MLP-ANN method.

Method	Accuracy (R^2)			Computation cost (s)
	r_1	r_2	r_3	
GP	0.97	0.97	0.95	
MLP-ANN	0.88	0.89	0.83	

Table 3. The parameters of GP.

Population size	100
Generation	200
Maximum depth	7
Probability of crossover	0.9
Probability of mutation	0.15
Fitness selection method	Binary tournament selection
Stopping criteria	Maximum generation
Function set	{plus, multiple, minus, divide, power, sin(.), cos(.)}
Terminal set	$\{r_1,\ r_2,\ r_3,\ \text{rand}\}$

In the optimization process, the inverse modeling error is evolutionarily minimized as the fitness function which is expressed as follows

$$F = \sum_{i=1}^{k} |rn_{mi} - rn_{di}|, \tag{18}$$

in which, r_m represents the length obtain from GP's models and r_d stands for the desired length of links. To construct the GP-type meta-model, 35 input–output data sets (Table 1) have been generated by equations (6) and (7) under above-mentioned constraints. Table 3 lists the determined parameters to control the genetic programming performance in this study. It should be mentioned that, the control parameters are adjusted experimentally.

According to Table 3, the initial population in GP execution consists of 100 individuals generated by ramped half and half method with the maximum depth of trees as 7. In the last generation, there are 100 mathematical models with different levels of accuracy and complexity for each normalized length. The best model related to the fitness function considered as the final solution to the problem.

The corresponding mathematical expressions of such GP- type models for optimum $r_1, r_2,\ r_3$ are given by equation (19). In order to examine the efficiency of GP, the output of such non-linear meta-models is shown in Figures 6 to 8 for r_1, r_2, and r_3, respectively. As it can be seen, the suggested GP models impressively provide high correlation coefficients. These mathematical models enable designers to synthesize a crank-rocker mechanism for any desired pre-determined value of stroke and time ratio.

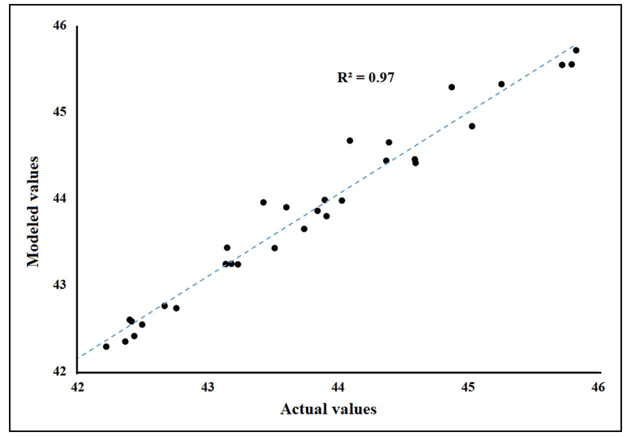

Figure 6. Scatter diagram of actually values versus estimated values for design variables r_1.

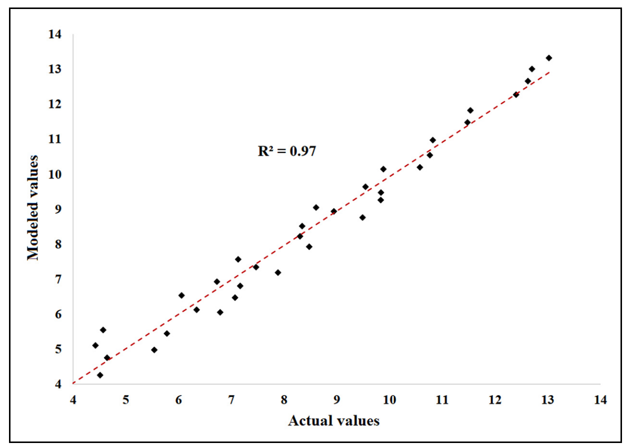

Figure 7. Scatter diagram of actually values versus estimated values for design variables r_2.

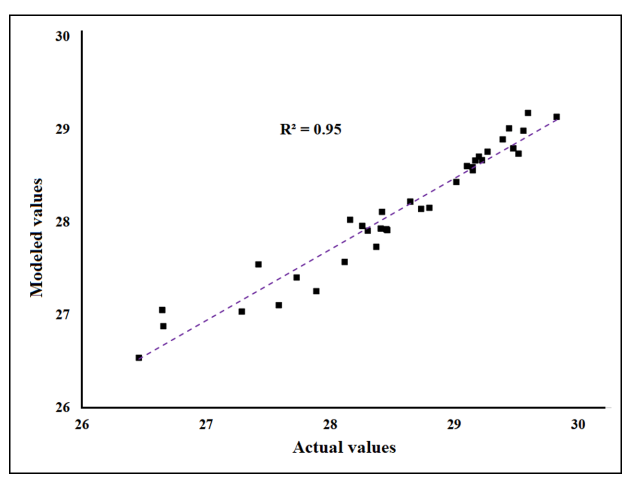

Figure 8. Scatter diagram of actually values versus estimated values for design variables r_3.

Table 4. Comparative results for crank-rocker four-bar mechanism.

Parameter	GP model	Analytic method
r_1 (cm)	43.43	41.06
r_2 (cm)	7.59	10
r_3 (cm)	28.80	25.01
r_4 (cm)	28.80	36.47
S (°)	31.13	32
TR	1.07	1.06
Error S (%)	2.71	–
Error TR (%)	0.94	–
γ_{min} (°)	77	57
γ_{max} (°)	124	112
TA (deg^2)	1373	1520

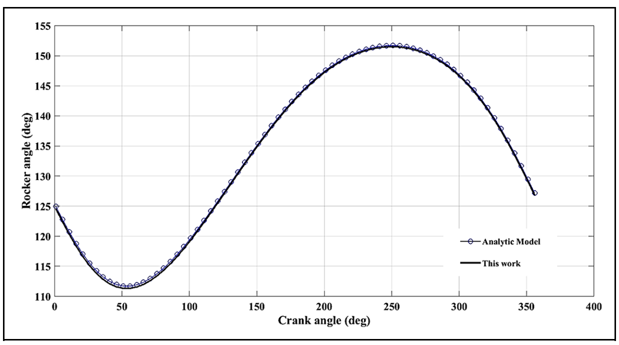

Figure 9. Rocker displacement.

$$r_1 = 9.03\left(TR - \frac{\sin(S \times TR)}{TR}\right) + 38.61 \quad (19a)$$

$$r_2 = 3\,S\left(S + \frac{1}{TR} + 2.79\right) \quad (19b)$$

$$r_3 = r_4 = 2\,(S - TR) - sin\left(S + \frac{9.15}{TR} + 7.84\right) + \frac{9.23}{TR} + 20.12 \quad (19c)$$

The proposed method is now applied effectively to the optimal synthesis of a four-bar mechanism. It is desired to synthesize a crank-rocker four-bar mechanism to provide a stroke of $S = 32°$ (0.558 rad) and TR = 1.06 with the best possible value of quality of motion. Table 4 provides the performance comparison of the synthesized mechanism using the obtained equation (19) of this paper for the given stroke and time ratio with that of analytic method. It should be mentioned that, in analytic method, the optimal values of r_1, r_2, r_3, and r_4 are directly calculated by equations (6) and (7) for any desired values of S and TR. Figure 9 depicts the rocker displacement for mechanism obtained by this work and the analytic method.

The ideal value of transmission angle is 90° in order to provide the best possible quality of force transmission at the joint. That is why most mechanism designers

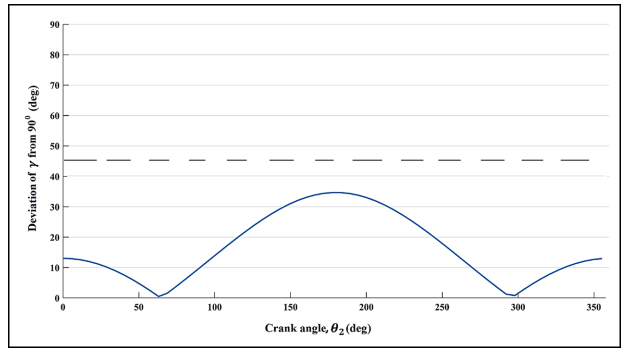

Figure 10. Deviation of transmission angle from 90°.

try to keep the minimum transmission angle' deviation above a specified value to promote smooth running and good force transmission. Generally, to obtain a good quality of motion, it is recommended in the literature, for example, that the maximum deviation of the transmission angle from 90° should be less than 45°.[1] The constraint limits on the minimum and maximum values of transmission angles are considered as

$$45^\circ \leqslant \gamma_{min} \leqslant 90^\circ, \quad (20a)$$

$$90^\circ \leqslant \gamma_{max} \leqslant 135^\circ. \quad (20b)$$

It should be noted that, for a Grashof four-bar mechanism the transmission angle can generally vary from 0° to 90°. In the other hand, for a non-Grashof linkage the transmission angle will be 0° in the toggle positions which occur when the output rocker and the coupler are colinear.[1] However, the equation (15) obtained from the proposed approach would automatically guarantees the optimality of the motion quality and also ensures the Grashof condition, that is, the transmission angle will not be 0° or 180°. The values of TA over the entire range of motion for the mechanism is also shown in Figure 10. It is evident from Figure 10 that the synthesized mechanism keeps the transmission angle within the recommended range while simultaneously improve the overall design of four-bar mechanism to met the desired stroke and time-ratio.

More importantly, it should be mentioned now that the genetically obtained synthesis equations of this paper can now be readily employed for the design of any crank-rocker mechanism used in real-world practical applications such as the sley drive of weaving machines,[10] the mixer mechanism,[42] the shaper linkage,[43] windshield wiper mechanism,[44] the continuously variable transmission (CVT) system,[45] etc. For instance, the application of the proposed approach for the design of four-bar linkage sley drive mechanisms is elaborated. The minimum transmission angle' deviation, the stroke, and the time ratio are the parameters affecting the sley motion curve. The optimum values of these parameters should be used by taking into account the desired sley motion on the one

hand and dynamics of the mechanism on the other hand. Once the required sley motion curve is determined from the shed geometry of a weaving machine, the kinematic design approach introduced in the proposed methodology can be used to optimize a four-bar sley drive mechanism with the best quality of motion.

Conclusion

In this paper, by using genetic programming, a unified approach for optimal syntheses of four-bar crank-rocker mechanisms was proposed so that any desired values of both stroke and time-ratio are achieved wherein the optimality of the transmission angle deviation as well as Grashof condition is guaranteed analytically. It was proved that keeping equal the length of connecting rod link and rocker link would guarantee the optimality of the transmission angle deviation and also ensures the Grashof condition. In this way, the length of three links, namely, reference bar, crank, and connecting rod was considered as design variables. A complete set of input-output data was obtained by considering the pertinent mechanism's geometry relations and constraints. It is shown that GP can successfully find almost simple and easy to use mathematical relations of design variables needed to synthesize the mechanism for desired values of both stroke and time-ratio simultaneously. In fact, the proposed mathematical forms presented in this paper, provide the capability to synthesize the length of mechanism's links for any desired value of stroke and time-ratio with optimum transmission angle. Further considerations are suggested to conduct as future works:

- In the case of this study, the dynamic considerations have not been taken into account. However, the future works may consider several dynamic performance criteria such as maximum values of the ground bearing forces during one input crank revolution, maximum required value of driving torque, maximum shaking force/moment, etc.
- The present study addressed the problem of dimensional synthesis of planar linkages in a deterministic environment, that is, the dimensional tolerance and joint clearance were not considered. However, in real world applications, the dimensional tolerance and joint clearance are investable in the stages of manufacturing and assembling. Certainly, any experimental attempt is required to hire some robust design approaches and uncertainty analysis which can be done as future research.
- Further possible future research avenues are to introduce advances from the GP literature to further improve the generalization ability and also to investigate the capability of the other methods of artificial intelligence in metamodeling of mechanism synthesis applications.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD

Bahman Ahmadi https://orcid.org/0000-0002-3109-6223

References

1. Norton RL. *Design of machinery: an introduction to the synthesis and analysis of mechanisms and machines*. Boston: McGraw-Hill Higher Education, 2004.
2. Ahmadi B, Ahmadi B, Chegini SN, et al. Multi-objective reliability-based optimal synthesis of path generating four-bar mechanisms: a cooperative game theoretic approach. *Proc IMechE, Part C: J Mechanical Engineering Science* 2022; 236: 2298–2311.
3. Balli SS and Chand S. Transmission angle in mechanisms (triangle in mech). *Mech Mach Theory* 2002; 37: 175–195.
4. Joe Brodell R and Soni AH. Design of the crank-rocker mechanism with unit time ratio. *J Mech* 1970; 5: 1–4.
5. Balaji Rao LV and Lakshminarayana K. Optimal designs of the RSSR crank-rocker mechanism—I. General time ratio. *Mech Mach Theory* 1984; 19: 431–441.
6. Cleghorn WL and Fenton RG. Optimum synthesis of an angular function generating mechanism with prescribed time ratio and rocker angular swing amplitude. *Mech Mach Theory* 1984; 19: 319–324.
7. Soylu R. Analytical synthesis of mechanisms—Part 1 transmission angle synthesis. *Mech Mach Theory* 1993; 28: 825–833.
8. Lun M and Leu Y. Design of crank-rocker mechanisms with optimum transmission angle over working stroke. *Mech Mach Theory* 1996; 31: 501–511.
9. Balli SS and Chand S. Defects in link mechanisms and solution rectification. *Mech Mach Theory* 2002; 37: 851–876.
10. Eren R and Aydemir A. An approach to kinematic design of four-bar sley drive mechanisms in weaving. *J Text Inst* 2004; 95: 193–205.
11. Hassaan GA, Al-Gamil M and Lashin M. Optimal kinematic synthesis of a 4-bar planar crank-rocker mechanism for a specific stroke and time ratio. *Int J Mech Prod Eng Res Dev* 2013; 3: 87–98.
12. Khader KM. Nomograms for synthesizing crank rocker mechanism with a desired optimum range of transmission angle. *Int J Min Metall Mech Eng* 2015; 3: 155–160.
13. Rufino CH and Ferreira JV. Kinematics of a variable stroke and compression ratio mechanism of an internal combustion engine. *J Braz Soc Mech Sci Eng* 2018; 40: 1–13.

14. El-Shakery S, Ramadan R and Khader K. Analytical and graphical optimal synthesis of crank-rocker four bar mechanisms for achieving targeted transmission angle deviations. *Jordan J Mech Ind Eng* 2020; 14: 303–313.
15. Yildiz A. Parametric synthesis of two different trunk lid mechanisms for sedan vehicles using population-based optimisation algorithms. *Mech Mach Theory* 2021; 156: 104130.
16. Ahmadi B, Nariman-zadeh N, Jamali A, et al. Strategic design of mechanical systems in the non-cooperative environment using the game theory. *Modares Mech Eng* 2016; 16: 317–326.
17. Ahmadi B, Nariman-zadeh N and Jamali A. Path synthesis of four-bar mechanisms using synergy of polynomial neural network and Stackelberg game theory. *Eng Optim* 2017; 49: 932–947.
18. Matekar SB and Gogate GR. Optimum synthesis of path generating four-bar mechanisms using differential evolution and a modified error function. *Mech Mach Theory* 2012; 52: 158–179.
19. Khorshidi M, Soheilypour M, Peyro M, et al. Optimal design of four-bar mechanisms using a hybrid multi-objective GA with adaptive local search. *Mech Mach Theory* 2011; 46: 1453–1465.
20. Singh R, Chaudhary H and Singh AK. Defect-free optimal synthesis of crank-rocker linkage using nature-inspired optimization algorithms. *Mech Mach Theory* 2017; 116: 105–122.
21. Mishra R, Kanti Naskar T and Acharya S. Optimum Design of elastic and flexible linkages for motion and path generation. *Mater Today Proc* 2018; 5: 4629–4636.
22. Nariman-Zadeh N, Felezi M, Jamali A, et al. Pareto optimal synthesis of four-bar mechanisms for path generation. *Mech Mach Theory* 2009; 44: 180–191.
23. Ghotbi E. Multi-objective optimization of mechanism design using a bi-level game theoretic formulation. *Concurr Eng* 2016; 24: 266–274.
24. Ahmadi B, Nariman-zadeh N and Jamali A. A Stackelberg game theoretic multi-objective synthesis of four-bar mechanisms. *Struct Multidiscipl Optim* 2019; 60: 699–710.
25. Gölcü M, Sekmen Y, Erduranlı P, et al. Artificial neural-network based modeling of variable valve-timing in a spark-ignition engine. *Appl Energy* 2005; 81: 187–197.
26. Nezamivand Chegini S, Haghdoust Manjili MJ, Ahmadi B, et al. New bearing slight degradation detection approach based on the periodicity intensity factor and signal processing methods. *Measurement* 2021; 170: 108696.
27. Parsa M, Carranza EJM and Ahmadi B. Deep GMDH neural networks for predictive mapping of mineral prospectivity in terrains hosting few but large mineral deposits. *Nat Resour Res* 2022; 31: 37–50.
28. Sfidari E, Amini A, Kadkhodaie A, et al. Electrofacies clustering and a hybrid intelligent based method for porosity and permeability prediction in the South Pars Gas Field, Persian Gulf. *Geopersia* 2012; 2: 11–23.
29. Jamali A, Ahmadi B, Ghamati M, et al. Reliability-based optimal controller design for systems with probabilistic uncertain parameters using fuzzy limit state function. *J Vib Control* 2015; 21: 1413–1429.
30. Sette S and Boullart L. Genetic programming: principles and applications. *Eng Appl Artif Intell* 2001; 14: 727–736.
31. Nezamivand Chegini S, Amini P, Ahmadi B, et al. Intelligent bearing fault diagnosis using swarm decomposition method and new hybrid particle swarm optimization algorithm. *Soft Comput* 2022; 26: 1475–1497.
32. Assimi H, Jamali A and Nariman-zadeh N. Sizing and topology optimization of truss structures using genetic programming. *Swarm Evol Comput* 2017; 37: 90–103.
33. Khayyam H, Jamali A, Assimi H, et al. Genetic programming approaches in design and optimization of mechanical engineering applications. In: Jazar R and Dai L (eds) *Nonlinear approaches in engineering applications*. Cham: Springer, 2020, pp. 367–402.
34. Jamali A, Khaleghi E, Gholaminezhad I, et al. Modelling and prediction of complex non-linear processes by using Pareto multi-objective genetic programming. *Int J Syst Sci* 2016; 47: 1675–1688.
35. Mohammadi A, Nariman-Zadeh N and Jamali A. The archived-based genetic programming for optimal design of linear/non-linear controllers. *Trans Inst Meas Contr* 2020; 42: 1475–1491.
36. Quintana MI, Poli R and Claridge E. Morphological algorithm design for binary images using genetic programming. *Genet Program Evolvable Mach* 2006; 7: 81–102.
37. Krawiec K and Lijewski P. Genetic graph programming for object detection. In: *International conference on artificial intelligence and soft computing*, Zakopane, Poland, 25–29 June 2006, pp. 804–813. Springer.
38. Curry R, Lichodzijewski P and Heywood MI. Scaling genetic programming to large datasets using hierarchical dynamic subset selection. *IEEE Trans Syst Man Cybern B* 2007; 37: 1065–1073.
39. Murata T and Okada D. Using genetic network programming to get comprehensible control rules for real robots. In: *2006 IEEE international conference on evolutionary computation*, CEC 2006, part of WCCI2006, Vancouver, BC, Canada, 16–21 July 2006, pp. 1983–1988. IEEE.
40. Muttil N and Chau KW. Neural network and genetic programming for modelling coastal algal blooms. *Int J Environ Pollut* 2006; 28: 223.
41. Can B and Heavey C. A comparison of genetic programming and artificial neural networks in metamodeling of discrete-event simulation models. *Comput Oper Res* 2012; 39: 424–436.
42. Tsunazawa Y, Soma N and Sakai M. DEM study on identification of mixing mechanisms in a pot blender. *Adv Powder Technol* 2022; 33: 103337.
43. Fomin A and Kiselev S. Structural and kinematic analysis of a shaper linkage with four-bar assur group. In: *International conference on industrial engineering*, Moscow, Russia, May 2018, pp. 1411–1419. Springer.
44. Khan S, Jamal A, Ali S, et al. Dynamic modeling and analysis of a four-bar mechanism for automobile applications. In: *2020 international conference on electrical, communication, and computer engineering (ICECCE)*, Istanbul, Turkey, 12–13 June 2020, pp. 1–6. IEEE.
45. Yildiz A, Kopmaz O and Cetin ST. Dynamic modeling and analysis of a four-bar mechanism coupled with a CVT for obtaining variable input speeds. *J Mech Sci Technol* 2015; 29: 1001–1006.