2023

DYNAMIC MODE DECOMPOSITION APPROACH FOR ESTIMATING THE SHAPE OF A CABLE

Yash Manik Chavan
Michigan Technological University, ychavan@mtu.edu

DYNAMIC MODE DECOMPOSITION APPROACH FOR ESTIMATING THE SHAPE OF A CABLE

By

Yash Manik Chavan

A REPORT

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

In Mechanical Engineering

MICHIGAN TECHNOLOGICAL UNIVERSITY

2023

This report has been approved in partial fulfillment of the requirements for the Degree of MASTER OF SCIENCE in Mechanical Engineering.

Department of Mechanical Engineering-Engineering Mechanics

Report Advisor:	*Vijaya Venkata Narasimha Sriram Malladi*.
Committee Member:	*Jung Yun Bae.*
Committee Member:	*Myoungkuk Park.*
Department Chair:	*Wayne Weaver*.

Table of Contents

List of Figures

List of Tables

Acknowledgements

Authors Chavan and Dr. Malladi acknowledge the support of the seed grant from the Great Lakes Research Center (GLRC), Michigan Tech for conducting this research.

Abstract

This study investigates the dynamic behavior of a flexible cable with heterogeneous stiffness using a data-driven approach. The study aims to develop accurate models describing intricate structures with rigid or flexible components. To achieve this, reflective markers were attached to the cable at equal spacing, and the motion was manually excited and captured using an 8-camera setup and OptiTrack's Motive software.

The cable displacement data at the marker locations were used as initial conditions for various Dynamic Mode Decomposition (DMD) models. The performance of the data-driven cable model is compared against the performance of the DMD modeling approach, fitting the dynamics of single- and multi-degree of freedom systems with added white noise. In this work, authors have considered using time delays and Wavelets-based DMD.

The study found that the Wavelet-based DMD (WDMD) model was the most accurate method for reconstructing the response of the cable in the test cases. The researchers suggest that this data-driven approach can be applied to predict the dynamic behavior of non-linear systems, with potential applications in civil engineering, aerospace, and robotics. Overall, this study presents a promising approach to developing accurate models of complex structures with rigid or flexible components. The findings of this study can be valuable for designing structures that can withstand dynamic loads and vibrations.

1 Introduction

This research aims to examine how a flexible cable with varying stiffness behaves using a data-driven modelling approach. The study aims to create accurate models that can describe the behavior of complex structures of both rigid and flexible elements accurately. To accomplish this, reflective markers were attached evenly along the cable on top of 3D printed masses, and the system was manually excited and recorded using an 8-camera configuration and OptiTrack's Motive software. The data obtained is then utilized to develop a data-driven model that can predict the dynamics of the flexible structure. The findings from this study can be valuable in designing structures that can withstand vibrations and dynamic loads.

This study primarily focuses on tethers used to maneuver robots or drones either under water, ground or in the air. In Tethered Underwater Robot (TUR) applications, DMD can be used to prevent the entanglement of the robot's tether, which is essential for the success of the mission. TURs are used in various applications, including underwater mapping, scientific data collection, search, and recovery, and inspecting and repairing underwater infrastructure. In multi-tethered robot missions, the risk of tether entanglement becomes even more significant as the robots move independently, increasing the likelihood of their tethers getting entangled. To address these challenges, it is necessary to know position and orientation. Frequent data availability can aid in such situations as developing models based on first principles though possible but isn’t always a viable option. Hence comes the need for data driven approaches.

2 Literature Review

Researchers are developing novel techniques to predict the dynamic behavior on how cables behave while avoiding potential entanglement when used without proper care. One of the methods utilizes visual-based approaches to monitor the cable's orientation and position on a real-time basis. In this case, the cameras and models for image processing track the movement of the cable and inform the controller of any deviations. Another approach involves use of swarming algorithm to manage the robots' movements and ensure the cable's orientation and position is maintained. The robots are programmed to move in a synchronized manner, thereby lowering the chances of entanglement and enhancing the overall outcome by making use of optimized paths. For such systems to work precise and timely information on the cable's shape is crucial and such systems are useful in real time analysis and hence there is a lack of predictability.

Some of the other approaches make use of acoustic localization or optical fibers. However, in both the cases the primary limitations lie in not having capabilities in predicting the shape of the cable. Hence it is necessary to develop a model(s) that will make use of sensing technology to simulate the cable shape based on which trajectories will be planned to maneuver the tethered robots.

Dynamic Mode Decomposition (DMD) is an approach based on dimension reduction method that splits or breaks the data into its spatial-temporal coherent modes. This involves data acquired at uniform time intervals converted into the form of a snapshot matrix. So, for a total of ***m*** snapshots taken at time steps $\boldsymbol{t_k}$ represents a dynamic system which is then expressed into snapshot matrix where X1 is a snapshot matrix containing first m-1 snapshots, X2 is the time shifted matrix containing last m-1 snapshots. The aim here is to find a best fit linear matrix ***A*** such that,

$$[\mathrm{X}_2]_{m\times n} = [A]_{m\times m}[\mathrm{X}_1]_{m\times n}$$

where, $A = X_2\, X_1^{\dagger}$, †: refers to pseudo-inverse.

The presented study focuses on the applicability of the DMD modeling technique in a particular application. The effectiveness of utilizing DMD algorithm along with data obtained from image processing techniques is being used to enhance the safety of Tethered Underwater Robot (TUR) systems by avoiding entanglements and future damages. Eventually, the aim is to develop control strategies based on DMD approach that avoids potential entanglements.

In conclusion, employing DMD based models is a promising strategy for investigating the dynamic behavior of flexible structures and has a wide range of applications across various fields. Incorporating DMD with other methods such as Image processing tools and swarming algorithms can enhance the efficiency and safety of Tethered Underwater Robot (TUR) systems, allowing researchers to investigate and understand the ecosystem of the ocean and underwater environment.

2.1 Overview of DMD

The DMD algorithm is designed to handle high-dimensional data matrices efficiently by utilizing a rank reduction approach. Singular Value Decomposition (SVD), which is the most employed method for rank reduction, involves breaking down the input data matrix X_1 into its singular values and vectors.

$$[X_1]_{m\times n} = [U]_{m\times m}[\Sigma]_{m\times n}\,[V]^T{}_{n\times n}$$

The SVD of X_1 is then converted into a lower-rank approximation of rank r, which represents the most significant modes.

$$[X_1] \approx [U_r]_{m\times r}[\Sigma_r]_{r\times r}\,[V_r]^T{}_{r\times n}$$

Using the pseudo-inverse of X_1, a reduced-order matrix $\tilde{A}$ of size r * r is computed.

$$\left[\tilde{A}\right]_{r\times r} = [U_r]^T{}_{r\times m}[X_2]_{m\times n}[V_r]_{n\times r}\,[\Sigma_r^{-1}]_{r\times r}$$
$$= [U_r]^T{}_{r\times m}[A]_{m\times m}[U_r]_{m\times r}$$

Eigen value decomposition of $\tilde{A}$ provides the dominant modes and related frequencies.

$[W,\ D] = eig(\tilde{A})$

Using eigen vectors W DMD modes (ϕ) are then found as

$$\phi = [X_2]_{m\times n}[V_r]_{n\times r}\,[\Sigma_r^{-1}]_{r\times r}[W]_{r\times r}$$

Continuous and discrete time eigenvalues are then obtained using eigen vector as follows-

$$\lambda_d = diag(D)$$

$$\omega_c = \frac{\log(\lambda_d)}{\delta t}$$

Using DMD modes, mode amplitude is determined as –

$$b = \phi \times x1^{\dagger}$$

$$x1 = [X_1]_{m\times 1}$$

And finally, response is recreated as –

$$d = b\, e^{\omega t}$$

$$X_{dmd} = \phi\, d$$

where b is the projection coefficients. These coefficients can be used to predict future behavior [1].

The Dynamic Mode Decomposition (DMD) algorithm since its introduction has been improved and modified to better analyze the dynamics of complex systems and not limited to linear systems. This report highlights some of these modifications, such as FFT DMD and Multi-resolution DMD (mrDMD). FFT DMD is one such approach that uses a short-time Fourier transform (STFT) to the system's dynamic response, which is then fed back to generalized DMD algorithm as input that further helps identify the dominant modes and patterns. mrDMD identifies coherent modes from time series data by varying the time scales and increasing the number of samples. The other variants of

DMD, such as Extended DMD, Koopman DMD, input-output DMD, and DMD with control are discussed. These modifications and enhancements have improved and widened the applicability of the DMD algorithm and made it a powerful tool for analyzing complex systems.

The Koopman DMD approach involves transforming the original system into a higher-dimensional space using the Koopman operator thereby making it able to capture previously ignored non linearities. The Koopman operator describes the evolution of observable functions over time. This higher dimension linear operator is then used to study the nonlinear dynamics of a system. By lifting the system into this higher-dimensional space, analyzing the system using DMD and then projecting back observed modes and frequencies onto original low dimension system provides the reduced order system that captures non-linearities involved.

To perform the Koopman DMD, observable functions such as monomials or Fourier modes are typically used. The Koopman DMD method has been successfully implemented in fields such as fluid dynamics, control systems, and neuroscience to name a few. Its ability to capture nonlinear dynamics using a linear operator makes it a powerful tool for analyzing complex systems and predicting their behavior.

2.2 WDMD

In this report we also discuss the wavelet-based dynamic mode decomposition (WDMD) algorithm [10]. The WDMD algorithm utilizes the wavelet transform coefficients of the measurements by expanding the dimensions of the original system. The wavelet transform is useful where Fourier transform is not suitable. Wavelet transform decomposes a signal into wavelets that are time-shifted, unlike Fourier transform, which breaks down a signal into sine waves of varying frequencies. The wavelet transform is performed by selecting varying time periods to capture the frequencies of interest at certain loss of resolution. Readers can refer to [11] for more information on wavelet transform.

The first step in the WDMD algorithm involves applying the Maximum Overlap Discrete Wavelet Transform (MODWT) to the response signal. This transform partitions the signal's energy across detail and scaling coefficients. The scaling coefficients are then partitioned into detail and scaling coefficients depending on the desired level of decomposition and considering the net energy content. Wavelets, such as Haar, Daubechies, and Symlets, with different levels can be used for this purpose. All the details and leftover scaling coefficients when combined represent the original signal. The ability of wavelet transforms to recreate the original linear as well as non-linear response is particularly useful in DMD analysis of complex systems. The wavelet coefficients thus obtained are then utilized in the generalized DMD algorithm to extract the state-space model representation.

3 Experimentation

The dynamic behavior of flexible structures such as cables is complex and highly nonlinear. Therefore, it is difficult to develop accurate mathematical models that best estimate such structures' behavior. Data-driven methods such as dynamic mode decomposition (DMD) approaches can be effectively used in modeling such systems. Before applying DMD to experimental data it is necessary to understand its effectiveness on known dynamic systems, such as single-degree of freedom (SDOF) and multi- degree of freedom (MDOF) systems. In this section, the approach to test the effectiveness of DMD algorithms on linear systems is discussed. The white noise is introduced to simulate real-world scenarios. The optimum noise level that does not significantly impact the accuracy of the models is observed by varying signal to noise ratios (SNR). The findings of are presented and implications of these findings for the application of DMD to experimental data of flexible structures, such as cables, are discussed.

3.1 SDOF

The first system considered is the single degree of freedom (SDOF) system with a mass of 1 Kg, stiffness of 98696 N/m, and damping coefficient of 6.28 Ns/m. The generalized DMD algorithm was initially chosen for the analysis. To meet the requirements of truncation, the data matrix is modified to include the future state in the same data matrix. The DMD algorithm predicts the response perfectly with almost no error. It is worth noting that for small deflections, the SDOF system behaves as a linear dynamical system, making the exact DMD algorithm an appropriate choice. The relative fitting error of the model is then determined using the Frobenius norm.

Figure 1 shows the relative fitting error of the SDOF system as the signal-to-noise ratio (SNR) varies. The model order is determined based on all relative singular values higher than the tolerance level of 10^-2. As seen, the relative error stabilizes after considering a delay of 5 snapshots. Further reduction in the tolerance level may lower the fitting error but could lead to overfitting the noise hence an undesired outcome. For an SNR greater than or equal to 30, the model order was found to be 3. Increasing number of input snapshots improves the pole estimates while maintaining a final fitted model size of 3. Figure 2 shows the distribution of all DMD poles for all cases. The figure indicates that the DMD algorithm tends to overfit the white noise for higher SNRs. For SNR= 30 and the size of the input data $X \in R^{3*998}$ results in a model of order three i.e., $A \in R^{3*3}$. The poles of the discrete system are found to be at 0.80±i0.58 and -0.68. Figure 3 shows prediction at SNR = 30 and for the mentioned model size.

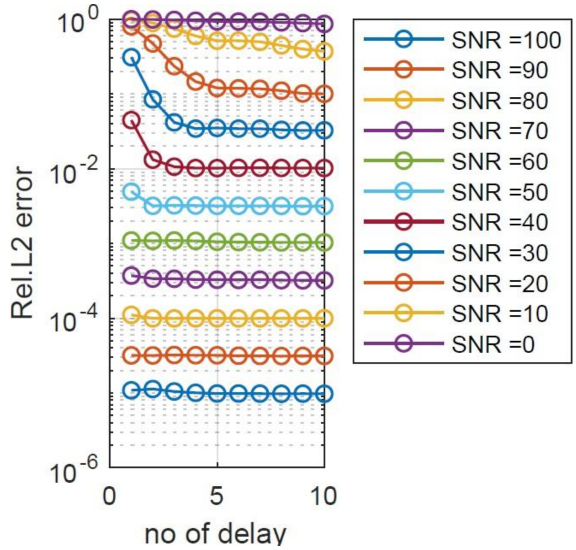

Figure 1 The relative fitting error of the normal DMD model with varying SNR

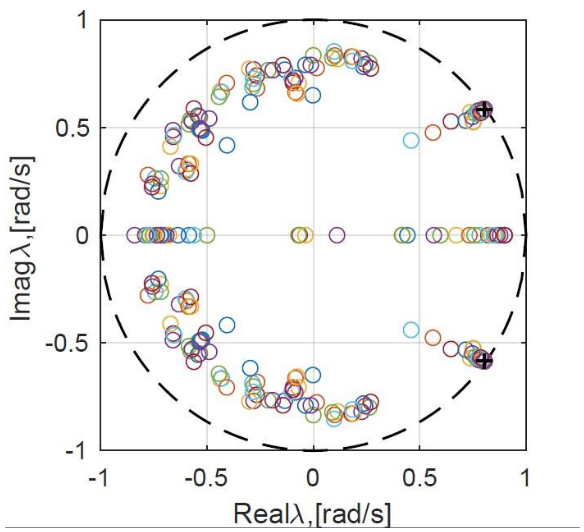

Figure 2 Location of fitted poles for all SDOF DMD models

The results of the SDOF case prove the significance of SNR in fitting the data. The study suggests that a delay of 5 snapshots and a tolerance level of 10^-2 can produce accurate fitting without overfitting the noise. Also, by using more input snapshots we can greatly improve the pole estimates and have a constant model size. The traditional DMD is proven effective for SDOF systems and hence no other DMDs are taken into consideration.

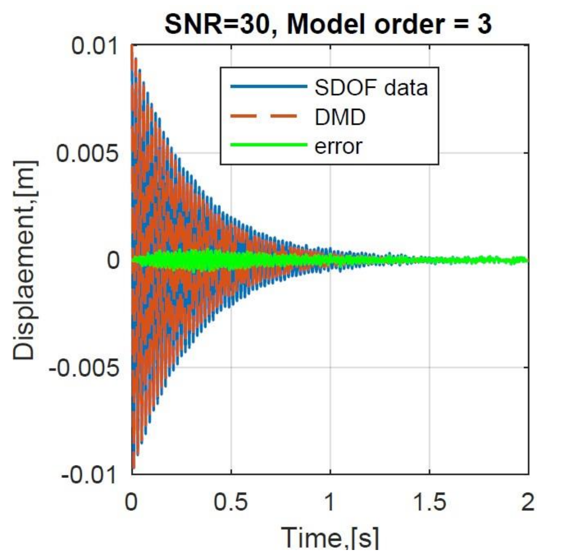

Figure 3 Time response of the input noisy data and the DMD model estimate based on initial conditions.

3.2 MDOF

A 5-degree-of-freedom (DOF) system was analyzed using Dynamic Mode Decomposition (DMD) technique during the study. This system was chosen by the author as the system has intermediate complexity between a simple SDOF system and a more complex 9-mass rope setup. The system masses, springs stiffnesses, and damping values are chosen to be 1 Kg, 300 N/m, and 6.28 Ns/m respectively. Similar to the previous SDOF system gaussian white noise was added numerically to the time signals, simulating realistic conditions. The results with and without white noise were then compared to analyze the impact of noise on the accuracy of the extracted modes.

DMD and WDMD analysis is performed on both the noisy and noiseless data, and the corresponding errors for the noisy data are presented in Figure 4. A tolerance level of 10^-9 was chosen for this study to ensure accurate results unlike 10^-2 which was used in SDOF system. This also shows that careful selection of tolerance level is needed. The impact of signal-to-noise ratio (SNR) on the DMD analysis was analyzed, and it was observed that with increase in white noise an L_2 fitting error was increased. Figure 4 shows relative L_2 errors for the DMD and Wavelet DMD (WDMD) analysis considering varying SNR.

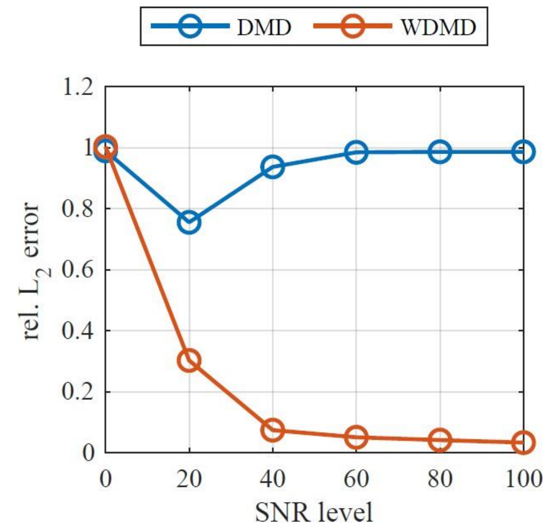

Figure 4 Compare relative L2 error of fitting original signal (SNR varied from 0 to 100) using DMD and WDMD (dB2, level 9)

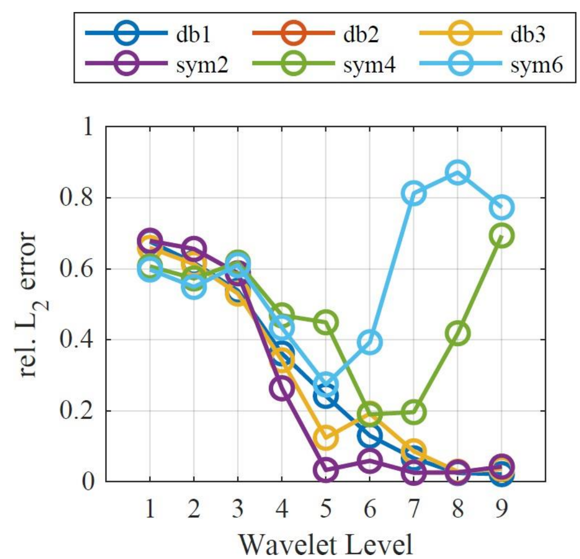

Figure 5 Compare relative L2 error of fitting original signal (SNR =80) using WDMD. The wavelet type and levels are varied.

It is also observed that the relative total fitting error of the model decreases with an increase in wavelet level, with db2 wavelet at level 9 being the best predictor for our system as shown in Figure 5. However, it is noticed that Symlets wavelets of higher levels do not necessarily perform better than the same wavelets at lower levels, except for sym4 and sym6. Overall, findings suggest that a wavelet transform-based approach can improve the accuracy of DMD algorithm for analyzing nonlinear MDOF systems.

To summarize, a DMD algorithm alone does not capture the nonlinear dynamics of the MDOF system accurately. Wavelet transform-based approach, that's WDMD approach using different wavelet such as Daubechies and Symlets wavelets of various levels (1 to 9) is found to be useful as shown previously. Figure 6 shows the predictions of WDMD and DMD algorithms for the response of all DOFs are compared and found that WDMD performed better for the given SNR.

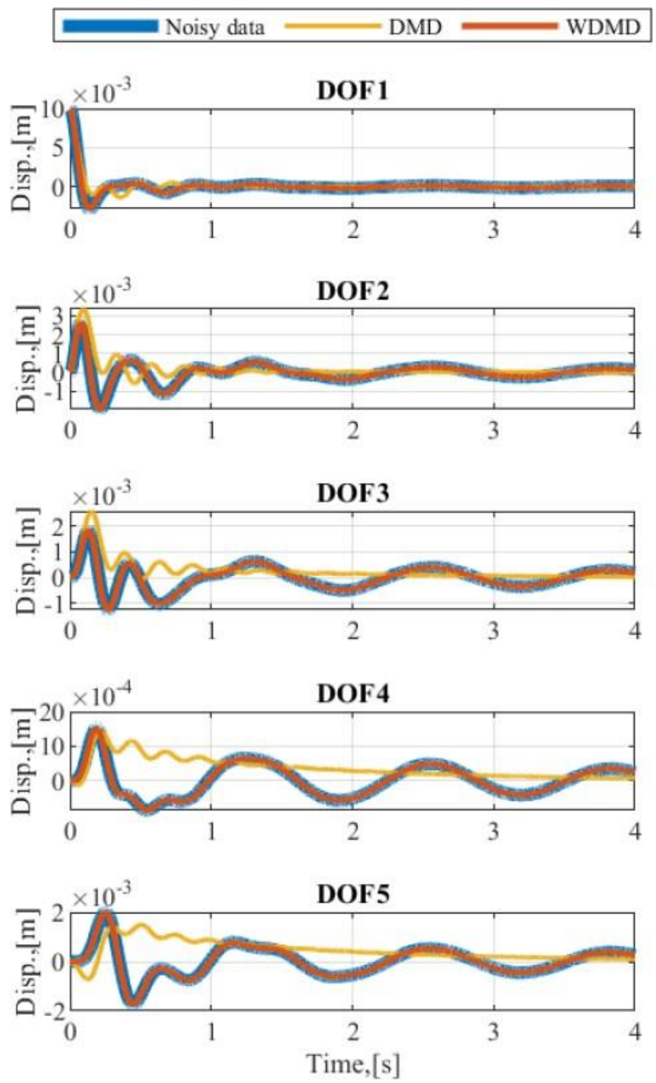

Figure 6 Compare time responses of original signal (SNR =80), DMD and WDMD (dB1, level 9)

3.3 Cable

The behavior of flexible nylon cable with plastic blocks of mass attached at regular intervals was studied under excitation where the cable was fixed at one end and free to move at the other end. Cameras and image processing techniques were used to track the motion of the cable in three dimensions and to enable the tracking, a set of reflective markers were placed on top of each plastic block. The experimental setup is shown in Figure 7.

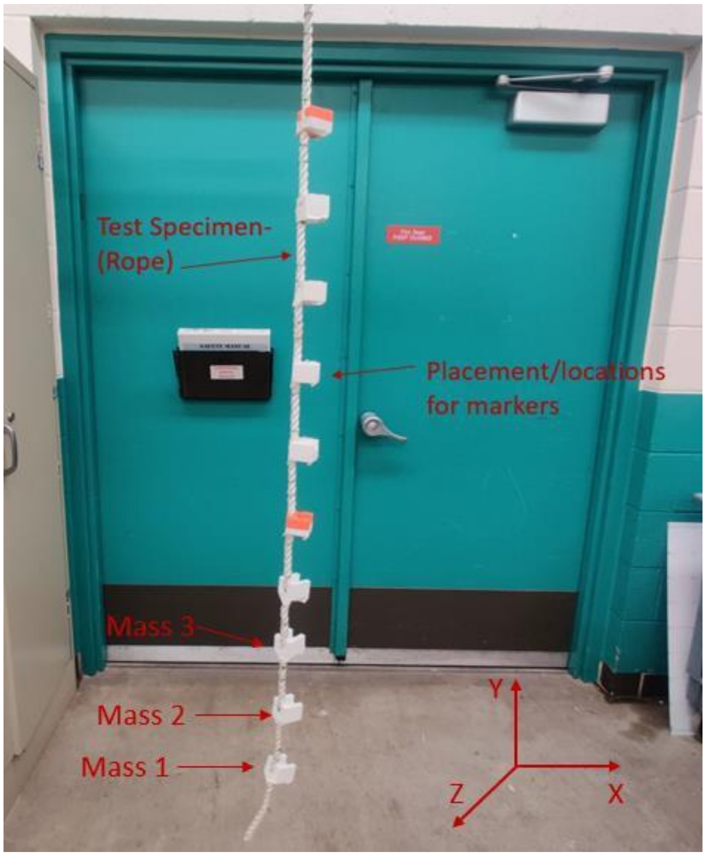

Figure 7 Experimental cable setup for DMD analysis

The cable was manually excited by releasing the free end from a random location. An 8-camera system was used to record the motion of the cable and to ensure the accuracy of the measurements the camera was calibrated. Optitrack's Motive software was used for processing the data. The sampling frequency was set to 30 FPS, and a total of 606 samples were obtained over a period of 20.17 seconds. Further a second set of data was collected with the same sampling frequency but with 1100 samples. Out of these two data sets one will be used to analyze DMD and WDMD fitting, and the other will be used to validate the model.

The length of the cable used in the experiment was around 1.2 meters and the plastic masses were attached at intervals of 10 centimeters. The origin of the Motive software was located below the first plastic mass. To obtain significant motion along with maintained stability the swing angle of the cable was limited to 20 degrees. The location of the reflective markers was crucial as that leads to acquiring the accurate marker location for the analysis techniques like DMD. The scope of this experiment was to study the shape of the cable in air and future scope of experiment is to investigate behavior of the cable in underwater environment where the environmental factors like water currents and buoyancy will be taken into consideration.

The collected data is used to create data driven models that can predict the dynamic behavior of the cable. For analyzing the collected data Wavelet-based Dynamic Mode Decomposition (WDMD) approach was used and its effectiveness to capture noisy signal was assessed. The performance of the WDMD was compared against conventional DMD algorithm. The same wavelet set as in previous study on multi-degree-of-freedom (MDOF) systems is used. Both the sets of cable experiment data were utilized in DMD and WDMD. The training data set was used to test the effectiveness of WDMD and test data set was used for validation and testing. Similar to previous 5 DOF system a tolerance level of 10^-9 was chosen for the model size. Tolerance represents the number of singular values retained during the DMD process and is dependent on truncation value r. As WDMD was used to analyze all reflective markers, there were total of 27 degrees-of-freedom in the system. Due to the systems' inherent noise no additional noise was taken into consideration.

The test was conducted on the first data set using six different waves with varying levels to evaluate the effectiveness of the WDMD method. However, it is observed that the change in type of wavelet showed no consistent change in error with increasing or decreasing level. That is no combination of wavelet and level will give consistent outcome and hence choice of an accurate wavelet at an appropriate level is crucial. As shown in figure 8, the total relative fitting error using a db1 level 9 wavelet was observed to be the least.

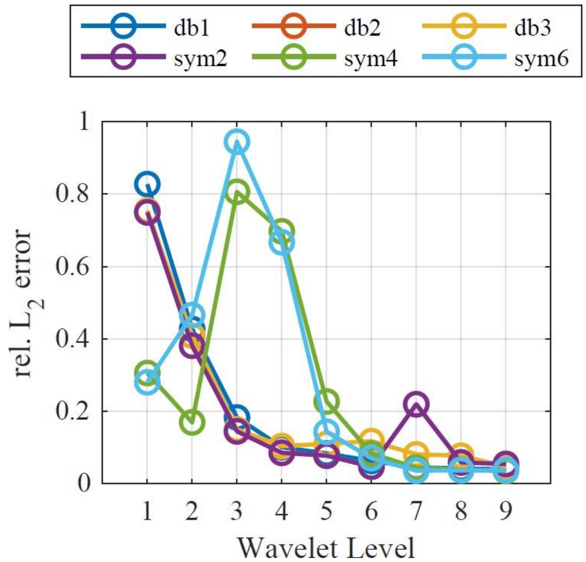

Figure 8 Comparison on relative L2 error of fitting Experimental cable data using WDMD. The wavelet type and levels are varied.

The compared experimental X, Y, and Z displacements of the markers with generalized DMD and WDMD models for the first marker placed on mass 1 using a db1 level 9 wavelet is as shown in figure 9. The figure shows that the WDMD model was able to accurately capture the response signal with a low fitting error of 0.04.

Six wavelets with their levels varied from 1 to 9 were used to evaluate the performance of the Wavelet-based Dynamic Mode Decomposition (WDMD) method on the first data set, that is training data set.

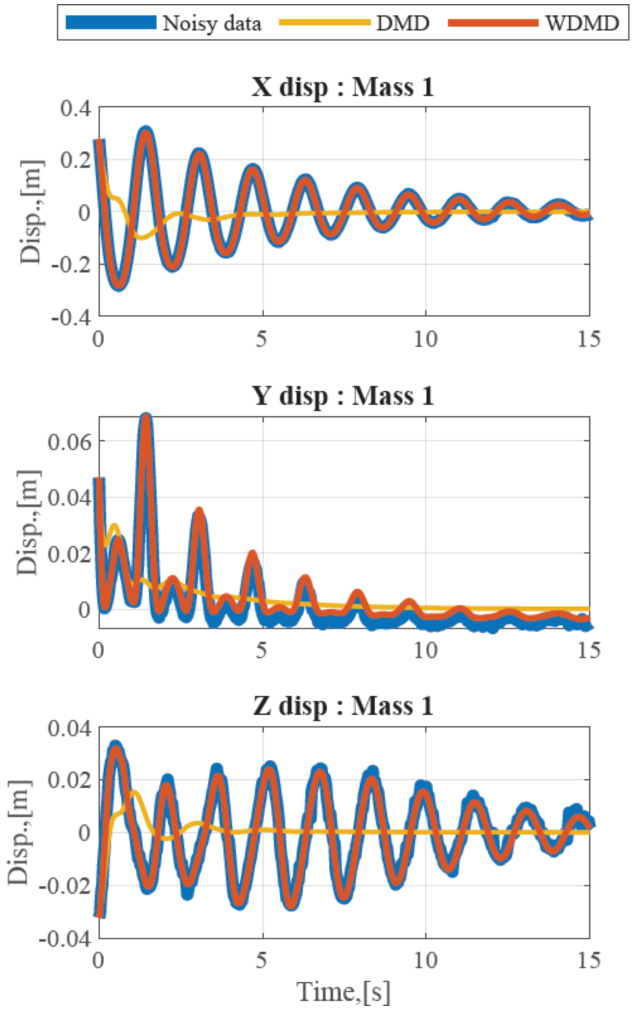

Figure 9 Compare time responses of Experimental cable data, DMD and WDMD (dB1, level 9) for Mass 1

Table 1 shows the relative fitting errors for each level. It was observed from the results that an accurate choice of wavelet at a suitable level is crucial to predict the true dynamical nature. From the table it is evident that the db1 level 9 predicts the most accurate response to our multi-degree-of-freedom (MDOF) system and has the least total relative fitting error. Using WDMD estimates, the 2D (X-Y) plot of the cable displacement is obtained which describes the cable motion as shown in figure 12(a). Overlaid results of experimental data on the same plot shows that WDMD accurately estimates the dynamic behavior of cable.

Using both DMD and WDMD models the relative errors of all markers were obtained and compared, refer figure 10. From the figure it is evident that the relative error due to WDMD was significantly lower than that of DMD. Also, the change in relative error for given or selected degrees of freedom is observed to be inconsistent, that is for same set of markers the trend in relative error in X-Y-Z is inconsistent.

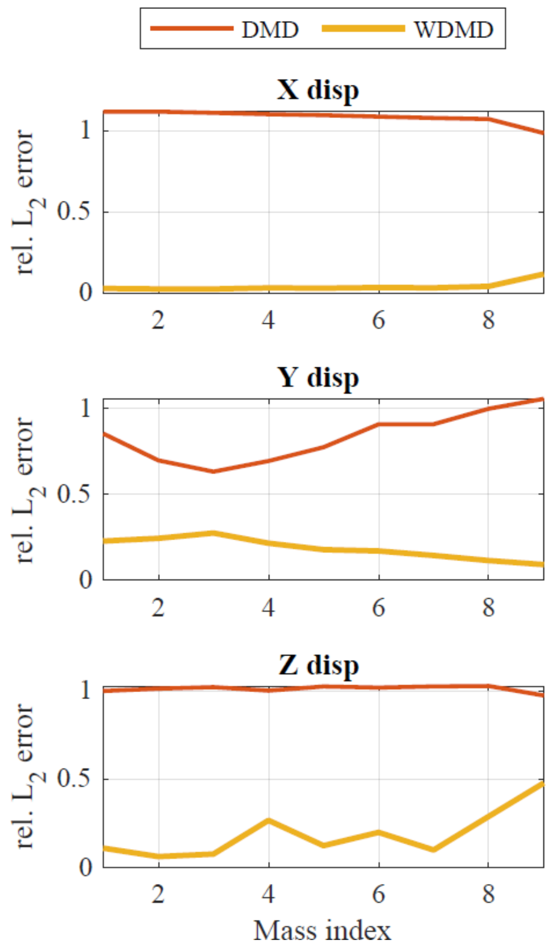

Figure 10 Compare relative L2 error of fitting Experimental cable data using WDMD for each DOF

To validate the WDMD model the same WDMD model was then tested using the test dataset. The results obtained using both data sets are found consistent with test data coordinates. The predicted time response is shown in figure 11 and figure 12(b) shows the X-Y coordinates as estimated by WDMD overlaid with test results. Unlike db1 level 9 wavelet which was found to predict the response accurately and based on which the model was built, on test data however db1 level 6 wavelet provided the accurate estimation which further concludes that the choice of wavelet and level is crucial in predicting the dynamic behavior of the cable. For the current test case, the fitting error of 0.257 was observed, which is higher than what was observed using training data set which is expected. In future runs the more robust data can further lower the fitting error.

Using the validation model the non-linear dynamical behavior of cables at discrete locations is accurately estimated as shown by the estimated X, Y, and Z displacements of the 8th marker from the test dataset. The model estimates in X direction are far better than estimates in Y and Z direction which again are expected to be as based on results from figure 10. Hence overall, it can be stated that WDMD is able to predict the non-linear dynamics of flexible structures such as cable with masses included at discrete locations.

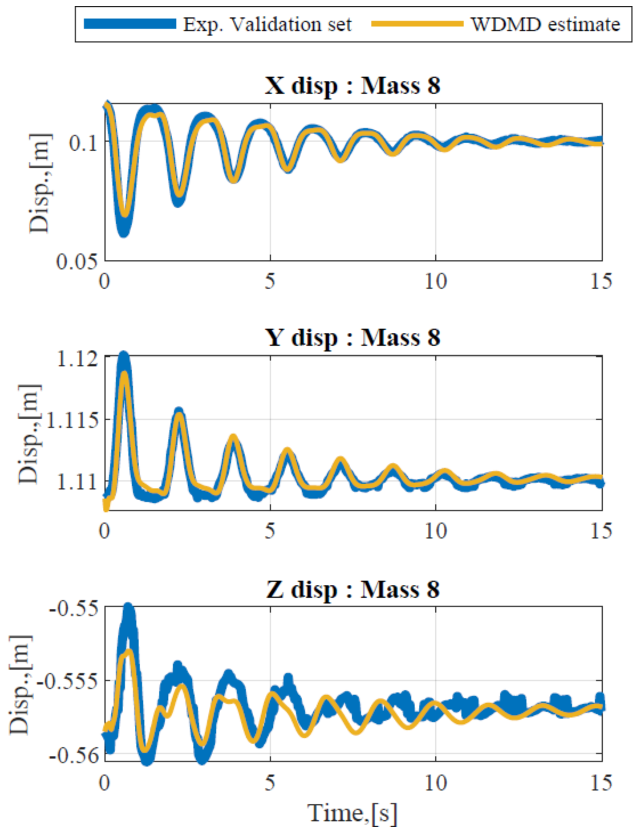

Figure 11 Validate the model by comparing time responses of Experimental cable data and WDMD (dB1, level 6) for Mass 8

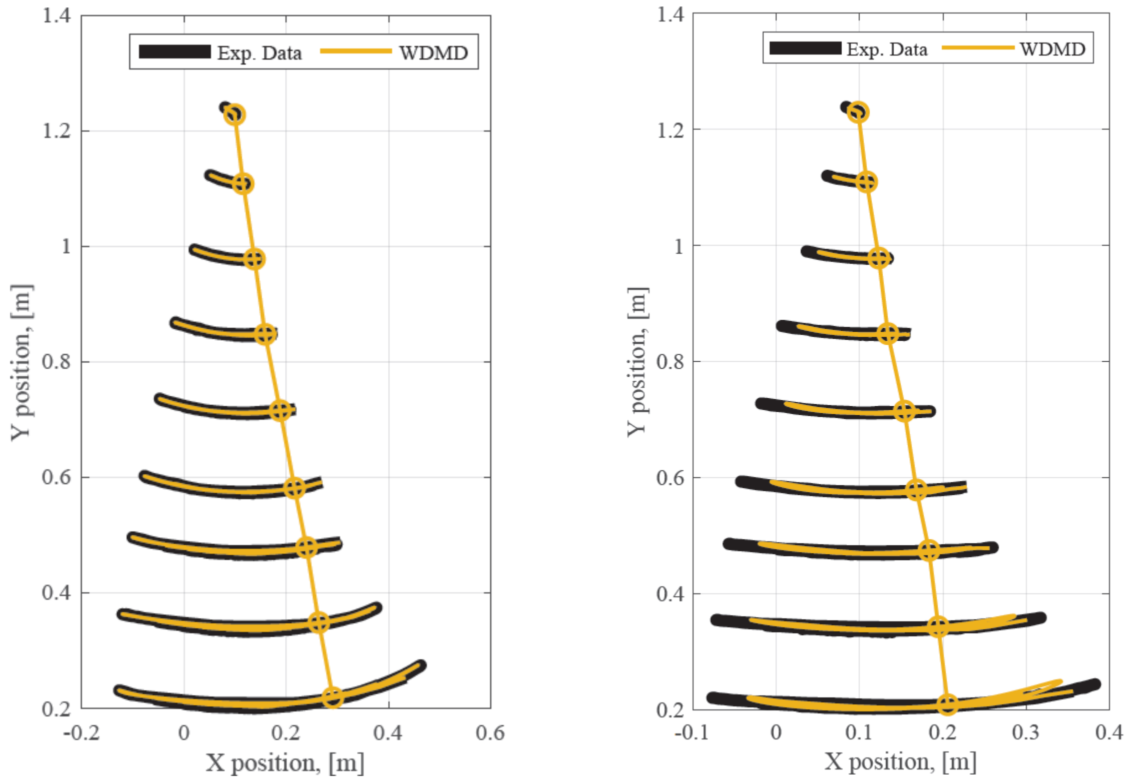

Figure 12 (a)Comparison of the 2D position of the cable using experimental estimation (yellow line) and WDMD estimation (yellow line). (b) Validation of WDMD model using experimental observations from test data (black line) and WDMD estimation (yellow line)

4 Conclusion

From the study it is concluded that WDMD algorithm is the most effective algorithm for predicting the dynamic behavior of flexible structures such as cables with varying stiffness. Compared to the results obtained from DMD and DMD with delays technique it is observed that the WDMD gives more accurate prediction of shape of cable with very lower relative error. DMD with delay can perform well when SNR is 60 and above but it can potentially overfit the noisy data.

The level of noise in the data affects the accuracy of the DMD model and thus some steps need to be performed to remove the noise in the data and to improve the accuracy. As compared to DMD, WDMD can handle the noise better and leads to accurate predictions. The accuracy of the WDMD model depends on the choice of the wavelet, level, and choice of tolerance level. Thus, for proper estimation of the dynamic behavior the wavelet and level need to be selected properly.

Thus, Combining the use of image processing tools and data driven models to capture and predict the shape of the cable at different locations shows that the dynamic behavior of flexible structures was be analyzed in real-world applications. For improving the safety and efficiency of TUR systems and other applications involving flexible structures the above method can be effectively used.

5 Reference List

[1] S. L. Brunton and J. N. Kutz, Data-Driven Science and Engineering: Machine learning, Dynamical Systems, and Control. Cambridge University Press, 2019.

[2] P. J. Schmid, “Dynamic mode decomposition of numerical and experimental data,” Journal of fluid mechanics, vol. 656, pp. 5–28, 2010.

[3] S. R. Vadyala, S. N. Betgeri, J. C. Matthews, and E. Matthews, “A review of physics-based machine learning in civil engineering,” Results in Engineering, vol. 13, p. 100316, 2022.

[4] S. Das, S. Dutta, C. Putcha, S. Majumdar, and D. Adak, “A data-driven physics-informed method for prognosis of infrastructure systems: Theory and application to crack prediction,” ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, vol. 6, no. 2, p. 04020013, 2020.

[5] T. Pagliaroli, F. Gambioli, F. Saltari, and J. Cooper, “Proper orthogonal decomposition, dynamic mode decomposition, wavelet and cross wavelet analysis of a sloshing flow,” Journal of Fluids and Structures, vol. 112, p. 103603, 2022.

[6] H. Liu, J.-H. Sun, C.-Y. Xu, X. Lu, Z. Sun, and D.- R. Zheng, “Numerical simulation and dynamic mode decomposition analysis of the flow past wings with spanwise waviness,” Engineering Applications of Computational Fluid Mechanics, vol. 16, no. 1, pp. 1849–1865, 2022.

[7] B. Thamo, D. Hanley, K. Dhaliwal, and M. Khadem, “Data-driven steering of concentric tube robots in unknown environments via dynamic mode decomposition,” IEEE Robotics and Automation Letters, 2022.

[8] V. V. S. Malladi, M. Albakri, P. Musgrave, and P. A. Tarazaga, “Investigation of propulsive characteristics due to traveling waves in continuous finite media,” in Bioinspiration, Biomimetics, and Bioreplication 2017, vol. 10162, pp. 112–119, SPIE, 2017.

[9] J. N. Kutz, S. L. Brunton, B. W. Brunton, and J. L. Proctor, Dynamic mode decomposition: data-driven modeling of complex systems. SIAM, 2016.

[10] M. Krishnan, S. Gugercin, and P. A. Tarazaga, “A waveletbased dynamic mode decomposition for modeling mechanical systems from partial observations,” Mechanical Systems and Signal Processing, vol. 187, p. 109919, 2023.

[11] S. Mallat, A Wavelet Tour of Signal Processing. Elsevier,

A Appendix A

Table 1 Variation of L2 relative error with a change in wavelet

Approach/L2 error	Total	X dir	Y dir	Z dire
DMD	1.106	1.111	0.799	1.007
WDMD ‘db1(level1)	0.827	0.828	0.601	0.870
WDMD ‘db1(level2)	0.427	0.424	0.471	0.548
WDMD ‘db1(level3)	0.183	0.175	0.459	0.275
WDMD ‘db1(level4)	0.101	0.09	0.416	0.181
WDMD ‘db1(level5)	0.083	0.069	0.426	0.172
WDMD ‘db1(level6)	0.063	0.048	0.359	0.154
WDMD ‘db1(level7)	0.045	0.034	0.232	0.145
WDMD ‘db1(level8)	0.042	0.027	0.256	0.142
WDMD ‘db1(level9)	0.040	0.027	0.226	0.138
WDMD ‘db2(level9)	0.054	0.051	0.095	0.125
WDMD ‘db3(level9)	0.047	0.043	0.1	0.126
WDMD ‘sym2(level9)	0.054	0.051	0.095	0.125
WDMD ‘sym4(level9)	0.035	0.030	0.083	0.122
WDMD ‘sym6(level9)	0.038	0.033	0.088	0.129