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DEVELOPMENT OF THE COIL VOLUME METHOD FOR TIME-DOMAIN
SIMULATION OF INTERNAL FAULTS IN TRANSFORMERS

By

Elizaveta O. Egorova

A DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

In Electrical Engineering

MICHIGAN TECHNOLOGICAL UNIVERSITY

2019

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This dissertation has been approved in partial fulfillment of the requirements for the Degree of DOCTOR OF PHILOSOPHY in Electrical Engineering.

Department of Electrical and Computer Engineering

Dissertation Co-Advisor: *Professor Bruce A. Mork*

Dissertation Co-Advisor: *Professor Jeffrey B. Burl*

Committee Member: *Dr. Sumit Paudyal*

Committee Member: *Dr. Laura E. Brown*

Interim Chair: *Dr. Glen E. Archer*

To all, who helped me to grow

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Preface

The Kron reduction calculations have been carried out by a graduate student, Hemanth Kumar Vemprala, a Ph.D. candidate at the ECE Department at Michigan Technological University.

The Kron reduction work has been submitted to the author of this dissertation in the form of tables. All the descriptive work has been performed by the author of this dissertation. The full sets of tables for the Kron reduction calculations are located in Appendices I.2 and I.4. The most important results and their analysis are located in Subsections 4.7.2.1.1 and 4.7.2.2.1.

Acknowledgements

I would like to thank:

- John Lukowski at MTU for his interest in discussing with me various topics on this project.
- Francisco Penaloza from the ATP forum for his quick and detailed replies on diverse ATP topics.
- Hemanth Kumar Vemprala, a Ph.D. candidate at MTU, for his help in Kron reduction calculations.

I would like to thank the sponsors of this project.

Abstract

A power transformer is a very important and expensive asset for a utility. As for any other power equipment, the transformer is subject to internal faults. The extent of transformer damage depends on the fault duration. Therefore, the protective relays should provide for sensitivity, selectivity, and speed of operation to minimize the effects of damaging conditions. High sensitivity of the relay allows it to identify internal faults quickly. However, the relay should not jeopardize the normal operation of the transformer. To determine desirable relay settings, a transformer model for simulation of internal faults is required.

Transformer models, shown in the literature, require detailed design data for simulation of internal faults. Such information is rarely available outside the transformer manufacturer. This dissertation proposes development and implementation of a model for simulation of internal faults in three-phase three-winding autotransformers. The model is based on typical test report data and limited core window design information.

The first objective for developing the model is to identify the components of the transformer leakage field and their interaction during internal faults.

The second objective is to develop the Cylindrical and Segmented approaches for segmentation of a faulted coil. The main attraction of the Cylindrical approach is that the sub-coils of the faulted coil form individual coils of the same height as the healthy coils and the faulted transformer remains an N -coil transformer. Therefore, calculation of binary reactances is carried out by applying the well-known energy method and formulation of the leakage admittance matrix is performed by applying Dommel's method. This allows an easy implementation of the internal fault model in the ATP software.

The third objective is to incorporate the magnetic core circuit in the transformer model. The role of the core circuit is to provide the interphase coupling and increase the transformer zero-sequence inductance compared to the air core. The interphase coupling allows simulation of internal faults during unbalanced operation and core saturation. The core circuit and its attachment method are adopted from the Hybrid model of ATPDraw.

The research showed that the Cylindrical approach is not as reliable as expected. However, it is a good start to acquiring sufficient experience before shifting the focus towards the Segmented approach. The author does not recommend continuing development of the Cylindrical approach for internal faults on the series and delta coils. The suggestion is to proceed with development and implementation of the Segmented approach using the guidance of this dissertation. The other important finding is the necessity to re-derive the adopted core circuit in the faulted phase. The adopted core circuit corresponds to a healthy three-legged transformer and, as per the results, is not suitable for simulation of internal faults.

1 INTRODUCTION

This chapter introduces a reader to the research field, defines the current problems, and briefly describes the ideas developed in this dissertation to mitigate these current problems.

1.1 Introduction to the Research Field

Power transformers are among the most important assets in electrical power systems in terms of their functions and cost. Most existing large power transformers were installed in the 1950's and 1970's when addition of new capacities was immense due to large system growth. The average age of installed large power transformers in the USA is about 40 years old, and 70% of them are 25 years old or older. Electrical equipment of such age is usually considered to be approaching the end of its useful life [1]. This fact leads to the conclusion that protection methodologies should be reevaluated to provide better sensitivity and faster relay operation for internal faults. This will allow reducing the fault duration, the extent of equipment damage, repair costs, environmental clean-up, and down time.

In [2] it is shown that about 70% to 80% of total failures originate from short-circuits between winding turns due to winding insulation degradation or wear. Several reasons for insulation degradation include strong electric fields in transformers that can develop during lightning strikes, induced forces caused by through faults, magnetizing inrush currents, etc. [3]. Also, [3] points out that failures are also caused by overheating due to prolonged overload, failure of the cooling mechanism, mechanical stresses, and moisture. An internal fault results in high radial and compressive forces and, thus, conductor insulation may not sustain the mechanical stresses caused by a fault. The faults cause a loosening of winding clamping pressure thus reducing the transformer's ability to withstand future short circuit forces.

Detection of winding inter-turn faults is critical since their effects are not easily identified using the terminal voltages and currents when a lower percentage of the faulted turns is involved. Usually, undetected turn-to-turn faults progress into phase-to-ground or phase-to-phase faults.

In the worst case, an incipient internal fault, if unnoticed, can lead to unrepairable damage of a transformer and a loss of customer service. A transformer replacement can lead to a substantial financial burden of \$1-7.5M (estimation dated of 2011) depending on voltage and MVA rating [1]. It also involves a waiting period from the order to the delivery date of about 5 to 12 months for domestic manufacturers and 6 to 16 months for manufacturers outside of the USA. However, if demand is high, the lead time increases up to 24 months. Plus, other collateral expenses include the clean-up costs due to a transformer failure.

For the same severity and duration of an internal fault, a damage to an older transformer is more extensive since older transformers have insulation degraded over the service life. In order to minimize transformer damage, protection sensitivity to internal faults should be increased.

The high sensitivity of the relay settings allows the relays to identify internal faults quickly, but it should not jeopardize the normal operation of a transformer, e.g., during high current imbalances but still under normal operation. A relay with highly sensitive settings might declare an internal fault and cause a false trip. To determine the desired sensitivity of relay settings, a tool is required. Transformer computer models which allow simulation of internal faults can be utilized to test the relay sensitivity before implementation of the protection scheme.

1.2 Project Description

This dissertation develops a transformer model which allows simulation of internal faults and uses minimum required data on a transformer: a typical test report and some of the core window design information (widths of the core legs, widths of the windings, air gaps, average height of the windings, and the number of turns in each winding). In this dissertation, the developed model is named the coil volume internal fault model because estimation of the binary reactances of a faulted transformer utilizes the cross-sectional area of the coils.

The three major objectives of this work include:

- understanding how the transformer leakage field can be represented during internal faults (leakage field pattern representation),
- finding a mathematical representation of this pattern, and
- developing the full model for simulation of internal faults by incorporating the core model.

The first objective is to identify the components of the transformer leakage field and their interaction during internal faults.

The second objective is to develop the Cylindrical and Segmented approaches for segmentation of a faulted coil. The main attraction of the Cylindrical approach is that the vertical segmentation of the faulted coil forms two (T2G faults) or three (T2T faults) sub-coils of the faulted coil which represent the individual coils of the same height as the healthy coils. In this case, a faulted transformer remains an N -coil transformer, yet with different N in the faulted phase. Therefore, calculation of the binary reactances can be carried out by applying the well-known energy method and formulation of the leakage admittance matrix can be performed by applying Dommel's method which is suitable for transformers with any number of windings. This method has been proven to give reliable results. Besides, two ATPDraw built-in transformer components, XFMR and BCTRAN, use Dommel's method for formulation of the leakage admittance matrix. Such a sequence allows an easy implementation of the coil volume internal fault model, based on the Cylindrical approach, in ATP software.

The Segmented approach is based on the horizontal segmentation of a faulted coil and other methods for its mathematical development. This means that it is more sophisticated.

Therefore, it is important to investigate whether the Cylindrical approach would satisfy the model requirements and produce sufficient results.

The third objective is to incorporate the core circuit into the model. The core provides the interphase coupling in the overall model. This allows simulating transformer internal faults during unbalanced operation and core saturation. Core saturation may occur due to different types of inrush currents, overexcitation, and geomagnetically induced currents. It is expected that the core effect should lead to an increase in the transformer zero-sequence inductance causing a reduction of the delta current magnitude.

To differentiate between two options of the coil volume internal fault model, the option without the core circuit is named the coil volume internal fault leakage model and the option with the core circuit is named the coil volume internal fault full model.

The core circuit used in this dissertation is adopted from the Hybrid model. This means that the coil volume internal fault full model utilizes the core circuit of a healthy three-legged transformer. Incorporation of the core representation is based on the same method developed for the Hybrid model of ATPDraw – involvement of the artificial infinitely thin $(N+1)^{th}$ coil located on each core leg.

It is important to investigate whether the core circuit of the Hybrid model would be sufficient for the purposes of the coil volume internal fault model.

The results produced by the coil volume internal fault leakage and full models are compared with the results produced by a proprietary higher-order $R-L$ internal fault model developed by a transformer manufacturer.

1.3 Planned Publications

The coil volume internal fault model requires further development and extensive laboratory tests before several publications can be offered to the scientific world. After the coil volume internal fault model is ready, the planned publications are the following:

- A coil volume method for EMTP-type simulations of internal faults in core-type transformers with concentric coils. This paper will describe the methodology and show the preliminary results.
- Benchmarking of the coil volume internal fault model versus the proprietary model. This paper should include extensive laboratory tests. This paper is possible if the transformer manufacturer wants to cooperate and share details of their model.
- Transformer internal faults during core saturation conditions: inrush currents, overvoltages, geomagnetically induced currents. This paper will show the modeling results on the internal fault cases. Improvement of protection schemes for transformers vulnerable to geomagnetic disturbances can be investigated.

1.4 Dissertation Outline

Chapter 2 offers a background overview on transformer protection, transformer modeling both for healthy and faulted transformers, methods for calculation of the binary leakage reactances in core-type transformers and formulation of the leakage admittance matrices.

Chapter 3 develops the coil volume internal fault model for simulation of internal faults in a three-phase three-winding autotransformer. It is based on the Cylindrical approach for estimation of the binary reactances. This chapter also develops the leakage field pattern representation during internal faults. Ideas for the Segmented approach development are shown, and issues encountered are described. Advantages and limitations of each approach are defined.

Chapter 4 describes implementation of the coil volume internal fault model in ATP software. The results demonstrated in this chapter are based on the steady-state single T2T fault located on the phase A common coil at the 10% height counting from the coil bottom. These results are compared with the ones of the proprietary model for the same transformer. Difficulties encountered during implementation are also specified.

Chapter 5 contains the simulation results for steady-state single T2T faults located at different heights on the phase A common coil of the studied autotransformer. These results are presented in the form of comparisons with the results obtained from the proprietary internal fault model for the same transformer.

Chapter 6 offers conclusions and recommendations for future work.

2 BACKGROUND

This chapter offers an overview of transformer protection, general thoughts on transformer modeling and highlights the modeling of transformer internal faults described in the literature. Methods for calculation of the binary short-circuit reactances for core-type transformers with cylindrical and sandwich windings are described, followed by a transformer matrix construction method. Introduction of EMTP, EMTP-ATP, and ATPDraw software is followed by more detailed information on the transformer modeling in ATP/ATPDraw.

2.1 Transformer Protection

Transformer protection is an important part of a power system protection strategy. Detailed information on transformer protection can be found in [4]. There are three major signs of an internal fault [5]:

- change in operating current,
- change in phase currents, and
- gas accumulation due to an arc.

Usually, a combination of electrical and nonelectrical protection methods is applied to protect very large transformers. Protection minimizes the effects of thermal stress and electrodynamic forces by minimizing the time of a fault event.

Due to many design features of the transformer design, such as tap changers, phase shifters, multiple windings, etc., special considerations should be taken to develop protection system design for an individual transformer. The challenges of transformer protection are the following:

- detect turn-to-turn and ground faults with high sensitivity,
- detect internal faults during transformer inrush currents quickly,
- avoid tripping for external faults with current transformer saturation,
- avoid tripping for overexcitation,
- avoid tripping or provide tripping for some transformers in response to geomagnetically induced currents.

2.1.1 Electrical Protection

The main protective unit for large power transformers is a microprocessor differential relay providing multi-level protection [6], [7]. Such a relay provides protection from turn-to-turn (T2T) and turn-to-ground (T2G) faults, although it cannot always be set with high sensitivity without sacrificing dependability. The transformer backup protection usually also consists of a microprocessor differential relay obtained from a different manufacturer.

2.1.1.1 Percentage Restraint Differential Relay

The typical percentage differential protection diagram with current transformers (CTs) is shown in Fig. 2.1. This representation is common across the literature on transformer protection.

Fig. 2.1. Transformer current percentage differential relay [6].

The relays should trip if the differential (operating) current I_{OP} exceeds a predetermined percentage of the through (restraint) current I_{RT} . Commonly, relays calculate these currents according to (2.1) – (2.4), choosing one of the alternatives for the restraining current [4]:

$$|\overline{I_{OP}}| = |\overline{I_1} + \overline{I_2}|, \quad (2.1)$$

$$|\overline{I_{RT}}| = k |\overline{I_1} + \overline{I_2}|, \quad (2.2)$$

$$|\overline{I_{RT}}| = k (|\overline{I_1}| + |\overline{I_2}|), \quad (2.3)$$

$$|\overline{I_{RT}}| = \max(|\overline{I_1}|, |\overline{I_2}|). \quad (2.4)$$

Equations (2.3) and (2.4) are used in relays with more than two restraints.

Based on the relay design, the percentage difference can be fixed or variable, depending on the relay manufacturer. Details on setting the minimum pickup, restraint currents and characteristic slope are defined by a manufacturer and fine-tuned by the utilities. Depending on design of the percentage restraint system, the slope can be a straight line or a curve up. The slope characteristic provides security for external faults with CT saturation. The varieties of slopes are shown in Fig. 2.2, adopted from [4] with permission.

Such relays are reliable for most internal faults, except in ungrounded and high-impedance grounded systems. The differential element is not sensitive for ground faults close to the grounding point in solidly grounded transformers [8].

Fig. 2.2. Characteristics of percentage differential relays. Image source: [4]. ©2008 IEEE

Another challenging task for a relay is detection of a single shorted turn. In most cases, such a fault cannot be detected until it evolves in a larger fault which produces high enough currents to be detected by a relay. According to [4], if the number of shorted turns is less than 10% of the total winding turns, terminal currents do not change significantly, making the fault transparent for protection.

The changing core flux induces the electromotive force (emf) in each turn, including the shorted turns. Since the impedance of a single shorted turn is lower compared to several shorted turns, very high current flows in the shorted turn loop. According to Lenz's law, the induced fault current flows in a direction so that its magnetic field opposes the core flux. To compensate for the core flux change and keep it constant, input current in the faulty winding increases [9]. However, due to the interphase coupling provided by a magnetic core, currents in the other phases also change. If the current imbalance is high enough, a relay declares an internal fault.

According to [8], [10], the T2T faults are detected with the negative-sequence percentage restraint differential element of a microprocessor relay. Negative-sequence currents are independent of the transformer winding connection and loading. In this case, the operating current I_{OPQ} is the sum of the $3I_2$ vectors from each transformer side, and the restraint current I_{RTQ} is the maximum magnitude out of all $3I_2$ vectors:

$$I_{OPQ} = \left| \sum 3I_2 \right|, \quad (2.5)$$

$$I_{RTQ} = \max(|3I_2|). \quad (2.6)$$

This method improves the relay speed by overriding all the restraint and blocking signals in case of an internal fault.

The T2G faults in transformers with the wye-connected winding and solidly-grounded or impedance-grounded neutral are detected with the restricted-earth fault (REF) element. It compares the zero-sequence current from the wye-connected CTs on the wye-connected winding to the current from the neutral connection. The operating current I_{OP} is the current I_N from the neutral CT, and the restraint current is the current $3I_0$ from the line CTs:

$$I_{OP} = I_N = I_A + I_B + I_C, \quad (2.7)$$

$$I_{RT} = 3I_0 = I_A + I_B + I_C. \quad (2.8)$$

The REF protection of autotransformers with the delta tertiary winding is described in [11]. An interesting fact is that in such autotransformers, it is possible to have the zero-sequence current in the delta winding and none in the neutral. This is due to the magnetic and electric coupling between the high-voltage and medium-voltage windings. A residual relay can be used to detect faults within the delta winding.

2.1.1.2 Inrush Currents

The transformer differential relay should not misoperate for any type of inrush current. The relay should operate quickly if an internal fault is detected during energization or the other type of inrush current. Inrush currents have the following major characteristics:

- presence of dc offset, odd and even harmonics,
- unipolar or bipolar pulses separated by intervals of very low current values (typically, those intervals are $\frac{1}{4}$ cycle), and
- the time constant of a decaying dc offset is longer than that of the fault currents.

CT saturation generates odd harmonics, therefore, even harmonics are better to adopt for magnetizing inrush discrimination. Usually, the 2nd and 4th harmonic restraints are applied to provide secure relay operation during inrush currents [12]:

$$I_{OP} \geq I_{RT} \cdot f(SLP_1, SLP_2) + K_2 \cdot I_2 + K_4 \cdot I_4. \quad (2.9)$$

Usually, the 2nd-harmonic content of inrush currents is above 16% of the fundamental. However, if a transformer is energized at a reduced voltage, variations of the point-on-wave initiation may generate a lower percentage of the 2nd harmonic which will lead to relay misoperation [12]. Also, transformers with low-loss steel have a lower percentage of the 2nd harmonic. Some additional measures to overcome these issues are shown in [13]. The differences in inrush currents between three-phase units and transformer banks are described in [14].

2.1.1.3 Overexcitation Protection

The transformer core can become overexcited at an increased voltage or/and reduced frequency from their rated values. During overexcitation, the core operates in a nonlinear magnetic region causing the rise of odd harmonics followed by transformer heating, increased excitation current, noise, and vibration. This is a concern for unit transformers mostly, directly connected to generators because during startup and shutdown the voltage and frequency values are far from the rated ones. In such cases, the transformer overexcitation protection is usually provided by the generator overexcitation protection. The details can be found in [15].

CT secondary currents have odd harmonics mainly either due to overexcitation of the transformer core or an internal/external fault followed by CT saturation. Line current amplitudes for the transformer overexcitation cases are not as high as for the internal/external faults which cause CT saturation.

Transformer differential relays should not operate for overexcitation cases. The 3rd harmonic cannot be used by relays since it can represent both the zero-sequence behavior and overexcitation, and it is difficult to discriminate between both. Besides, the 3rd harmonic is filtered out either by a relay algorithm or a CT/transformer delta connection [12]. That is why the 5th harmonic is used by relays: it is not filtered out and it is the next largest in values after the 3rd harmonic. The 5th harmonic is a more dependable measure of overexcitation to use for relay blocking:

$$I_{OP} < K_5 \cdot I_5. \quad (2.10)$$

2.1.2 Nonelectrical Protection

This type of protection operates independently of the transformer currents and voltages. Its operation is based on the physical or chemical conditions defining the design of the temperature relays, Buchholz relays, and pressure relays. These relays are mostly used for alarming. More information is in [4]. Both the Buchholz and pressure relays operate on accumulation of gases in the oil due to internal faults, but they have different applications. Usually, the power transformer protection set combines both electrical and nonelectrical types of protection, e.g., a differential relay and a pressure relay.

2.1.3 Transformer Monitoring and Diagnostics

Transformer monitoring can be carried out online and offline. Parameters that are typically monitored are gases in the oil, partial discharges, winding and oil temperatures, insulation power factor, pump/fan operation, load tap changer operation, conservator membrane, etc. The details can be found in [16]. The most popular two diagnostic methods are the dissolved gas analysis (DGA) and the frequency response analysis (FRA). More information on DGA can be found in [17], and on FRA in [18].

2.2 Overview of Transformer Modeling

Power transformers encounter different events during their service life which can be classified into four frequency groups according to [19] and [20], see Table 2.1 which is slightly modified from the indicated sources. The most frequent events are torsional oscillations, turbine blade variations, fast bus transfers, harmonic interactions, inrush currents, ferroresonance, short-circuits, lightning and switching transients, internal resonances, and overvoltages. Other events include geomagnetically induced currents (GIC). Their influence on transformer operation is a relatively new research topic in the transformer community. Depending on the type of event, the transformer model requires a different set of parameters. A universal model which can be used for simulation of all the possible transient phenomena in transformers does not exist.

Table 2.1 summarizes the importance of the transformer parameters for modeling in each of the four frequency ranges.

Table 2.1. Frequency groups for power transformer modeling. Table source: [19], [20].

PARAMETER/ EFFECT	LOW FREQUENCY TRANSIENTS	SLOW FRONT TRANSIENTS	FAST FRONT TRANSIENTS	VERY FAST TRANSIENTS
	0.1 Hz – 3 kHz	50/60 Hz – 20 kHz	10 kHz – 3 MHz	100 kHz – 50 MHz
	Geomagnetically induced currents 0.001 – 0.1 Hz	Switching transients	Lightning strikes	Events in gas insulated substations
	Torsional oscillations, turbine blade variations, fast bus transfers, harmonic interactions, resonances			
Short-circuit impedance	Very important	Very important	Important	Negligible
Saturation	Very important	Very important ¹⁾	Negligible	Negligible
Iron losses	Important ²⁾	Important	Negligible	Negligible
Eddy currents	Very important	Important	Negligible	Negligible
Capacitive coupling	Negligible	Important	Very important	Very important

1) Only for transformer energization phenomena, otherwise important

2) Only for resonance phenomena

2.2.1 Modeling of Power Transformers

The simplest and most famous transformer model found in the literature is the one which does not require the core representation. This model is used for steady-state load flow and short-circuit studies at power frequencies. In this case, a transformer can be represented by a lumped parameter coupled-winding model.

More sophisticated models require the representation of nonlinear behavior of different core topologies for events like inrush currents, ferroresonance, GIC, and others. For low-frequency studies, transformer modeling is challenging because both frequency- and voltage-dependent nonlinear effects must be considered. The classical textbook power-frequency model of a single-phase transformer is shown in Fig. 2.3, where L_1 and L_2 are the leakage reactances of each winding, R_1 and R_2 are the corresponding winding resistances. Parallel elements represent the simplified core model, where R_m is responsible for the core losses and L_m is responsible for the magnetic properties of the core.

Fig. 2.3. Transformer representation [6].

Switching and lightning surges require detailed modeling of capacitive coupling in a transformer, thus, a distributed parameter model is required [21]. For fast front transient studies, a transformer can be represented by its stray capacitances to ground [22].

According to [23], the following categories of transformer models exist:

- The black box approach has an admittance or inductance matrix and is suitable for a wide frequency range. Detailed design information is not needed, the model data are based on terminal measurements.
- The white box approach has a complex network of lumped elements. Each element represents a circuit equivalent for a small piece of a conductive or dielectric portion of the transformer. Since it requires detailed design information, mainly manufacturers use this model. High level of details in this model allows simulations in a wide frequency range.
- The grey box approach has a simplified network of lumped elements. Design details are not used, but the main features of frequency dependent admittance are covered.

A description of different approaches for low-frequency transformer modeling is presented in [22]. A transformer model for transient studies should combine three main elements:

- the winding structures representing the transfer characteristics (the short-circuit response),
- the iron core controlling the flux balance (the no-load response),
- the phase-to-phase coupling which is defined by the transformer topology.

Based on the research in [22], the two main approaches for modeling in the low-frequency range that retain a topological representation of the magnetic structure are as follows:

- the core is represented with an equivalent magnetic circuit,

- the core is represented with an equivalent electric circuit derived from the magnetic circuit through the duality transformations.

To implement either of these two methods, the flux paths should be well defined. However, the leakage flux paths are defined with more approximations than the core flux paths.

The most popular software packages for modeling transients are EMTP-ATP (further referred to as ATP), EMTP-RV, PSCAD-EMTDC, MT-EMTP, RTDS, SymPowerSystems (MATLAB), SPICE.

2.2.2 Transformer Inductance, Capacitance, and Resistance

A transformer is a simple device conceptually. However, nonlinearities in the core, inductances and capacitances make the mathematics and complete visualization of the processes complex. Below is a brief discussion on the concepts of transformer inductances and capacitances to enhance understanding of the internal fault model based on the transformer leakage admittance matrix. The faulted turn resistance increases due to a high magnitude fault current, and calculation of an approximate resistance change is offered in this section as well.

2.2.2.1 Inductance

Suppose, there is a three-phase three-legged two-winding transformer as in Fig. 2.4 a). If the core is ideal, having infinite permeability, then the full flux is retained inside the core. Therefore, the full core leg flux links two coils. Each turn of a coil has its own inductance, and their sum is the total self-inductance of this coil. Both coils, located on one leg, have their own self-inductance and a mutual inductance due to the same flux linking both coils.

This idea is simple; however, in real transformers, the core permeability is finite, which allows some flux to leak out of the iron core. In this case, not all the flux is linking both coils and, therefore, the mutual inductance of the two coils in a real transformer is lower compared to the ideal case. In the case of Fig. 2.4 a), all the three phases share the same magnetic core, and a mutual inductance exists between all the coil pairs. For a three-phase transformer consisting of a bank of three single-phase units, like in Fig. 2.4 b), there is no mutual inductance between coils of different phases.

Fig. 2.4. Three-phase transformers: a) three-legged core, b) triplex configuration [24].

When an internal fault occurs, it alters the leakage flux distribution and, therefore, the mutual inductances between all the coil pairs located on the same core as well as the self-inductance of the faulted coil is changed. A shorted turn (portion) introduces a high amount of new leakage flux. The major fraction of this flux links neither with the healthy turns of the faulted coil nor with the healthy coils and can be thought of as a “do-nothing” leakage flux. However, some fraction of this new leakage flux yet links with the healthy turns of the faulted coil and with the healthy coils. An internal fault model should predict new values of the mutual and self-inductances of a faulted transformer, paying special attention to estimation of the self-inductance value of the shorted turn (portion).

2.2.2.2 Capacitance

The transformer capacitance can be visualized easier as shown for each turn in Fig. 2.5. Fig. 2.6, adopted from [25] with permission, shows the capacitive effects in a typical core-type transformer with a cylindrical configuration of windings. For internal fault modeling at the power system frequency, the transformer capacitance can be neglected, see Table 2.1. The capacitive coupling is of great importance for high-frequency studies like switching and lightning surges as well as events in gas insulated substations.

Fig. 2.5. Distributed capacitance representation [27].

Fig. 2.6. Representation of capacitive effects for a typical core-type transformer with cylindrical windings. Image source: [25].

2.2.2.3 Resistance

According to [26], contamination of the transformer oil with acetylene gas is associated with sustained internal arcing in the windings. The evolution of acetylene gas occurs at temperatures above 700 °C as the oil begins to break down. Therefore, the copper

conductor temperature in a shorted portion is above 700 °C. Often, copper conductors melt during internal faults and, as per [27], pure copper melts at 1083 °C. This temperature rise causes the copper resistance to increase, as discussed below.

The material resistance R is defined in [27] as

$$R = \frac{\rho(T) \times \text{length}}{\text{area}}, \quad (2.11)$$

where ρ is resistivity and T is temperature.

The material resistivity changes due to temperature variations. According to [28], resistivity of copper at 85 °C (358.15 K) is

$$\rho_{\text{copper}}(\text{at } 85 \text{ } ^\circ\text{C}) = 2.1 \times 10^{-8} \text{ } \Omega \times \text{m}, \quad (2.12)$$

and resistivity of copper at 1000 °C (1273.15 K) is

$$\rho_{\text{copper}}(\text{at } 1000 \text{ } ^\circ\text{C}) = 9.5 \times 10^{-8} \text{ } \Omega \times \text{m}. \quad (2.13)$$

The ratio of these two resistivities is

$$\frac{\rho_{\text{copper}}(\text{at } 1000 \text{ } ^\circ\text{C})}{\rho_{\text{copper}}(\text{at } 85 \text{ } ^\circ\text{C})} = \frac{9.5}{2.1} = 4.5. \quad (2.14)$$

These calculations demonstrate that the copper conductor resistivity at 1000 °C is 4.5 times higher than that at 85°C. Therefore, the copper conductor resistance increases in the same proportion.

2.2.3 Transformer Modeling for Simulation of Internal Faults

The main challenge in building any model for simulation of internal faults is to come up with a method for calculation of new parameters corresponding to a fault, or so-called parameter estimation. An internal fault modifies the overall transformer magnetic field distribution and, therefore, the transformer short-circuit behavior changes [29]. Internal fault modeling is usually performed at the power frequency.

The literature describes many models for simulation of transformer internal faults. A number of papers present transformer internal fault modeling as making changes in the leakage admittance matrix. This can be achieved through calculation of the leakage factors. The basis for this approach is [30] since it is listed among references in many papers on transformer internal fault modeling. Parameter estimation and model implementation

assume that the leakage factors between the portions of the faulted coil and healthy coils are known. A method for their evaluation is based on design data and is also proposed in [30]. The mutually coupled R - L branches are derived as $[R]$ and $[L]$ matrices of 6×6 size for healthy transformers using the BCTRAN routine of ATP. According to [30], a three-phase transformer with a T2G fault is defined by a 7×7 matrix, and a T2T fault by an 8×8 matrix.

Ideally, the leakage factors should be calculated using the core and winding design data as described in [30], but such detailed information is rarely available outside of the manufacturer. However, the approximate estimation can be found in the following references: [31] suggests using the value of 1 for the unknown ratio of two leakage factors, and [32] suggests an approach for calculation of the leakage factors between new portions. Thus, the combination of these two suggestions makes the factory test report data sufficient for building a model for simulation of internal faults.

Ideas from [33] are also based on [30] and describe modeling internal faults in single-phase distribution transformers. In [33], it is assumed that all the leakage factors are equal to zero. This assumption is true for large transformers where windings are tightly coupled, but not true for small transformers. The precision of the leakage factors affects the model precision.

Other references related to the leakage admittance matrix reconfiguration for internal fault modeling are [34], [35], [36], [37], [38], and [39].

Alternative models available in the literature require the transformer design data which are unavailable in most cases, although such methods give the most accurate results. In [40] and [41], a combination of two models is described: a finite element method (FEM) model and a deteriorating insulation model. This synthesis targets the detection of internal incipient faults. The deteriorating insulation model consists of an aging model and an arcing model which can be connected either in parallel or in series. In the overall model, the shorting switch is replaced with the model of degraded insulation before breakdown.

The leakage inductance-based models, shown in the literature, usually do not include the core representation. Such a simplification is acceptable to some extent; however, the core effect influences the estimation of the fault current. In [42], it is stated that the fault current has a complex dependency on several factors simultaneously: resistance and inductance of the shorted loop, leakage flux distribution due to a fault location with respect to the healthy windings and healthy portions of the faulted winding, proximity to the core, fault severity, and a possible local saturation of the core. Therefore, a major improvement in internal fault modeling is addition of the magnetic core circuit. Incorporation of the core circuit allows studying the effects of the core magnetization states on internal faults, i.e., internal faults during normal operation and energization under geomagnetic disturbances.

There is an opinion that the leakage models are not as accurate as the FEM models [29], [43], [44] or models based on Maxwell equations [45], [46] because the modified transformer parameters cannot be predicted well enough without design data. However,

only the leakage inductance-based models offer the advantage of being built from the test report data and are incorporated in transient simulation software packages, like ATP, for public benefit.

For two-winding transformers, the leakage admittance matrix can be built as in [21] without interphase coupling. The method from [21] is described further in Subsection 2.4.4. The leakage admittance matrix of the BCTRAN support routine in ATP can also be modified for simulation of internal faults, e.g., as in [47] for faults in three-winding transformers.

For three-winding transformers with auto connection, BCTRAN is not accurate due to the “black box” approach: the leakage admittance matrix is built by looking into the terminals. A study on internal fault modeling in [48] presents a faulted three-phase three-winding autotransformer modeled using BCTRAN. However, the Hybrid model of ATPDraw suggests a more appropriate estimation of the matrix for transformers with auto connection [49]. Its algorithm recalculates the binary short-circuit reactances from a test report to produce the binary short-circuit reactances between the pairs of actual coils.

Application of differential equations to describe the physics in single-phase two-winding transformers during an internal fault is used in the model from [50]. In this model, the terminal voltages are the inputs, the winding currents are the outputs, and the flux linkages are the internal variables.

One of the advantages for modeling internal faults in EMTP-like software packages is that currents can be directly injected into transformer microprocessor relays in binary COMTRADE or .pl4 formats. Also, relay algorithms can be developed in the same file as the transformer model with the purpose of exploration for advancements.

Software packages mentioned in Subsection 2.2.1 allow building a transformer model for internal fault studies; however, development of a built-in model is a demanding task due to complex properties of each transformer unit: geometric, (di-)electric and magnetic.

The main limitation of the models described above is that most of them require detailed design data and the core effect is incorporated in the leakage admittance matrix. The latter does not allow performing modifications of the core flux easily. This offers an opportunity for the development of a new internal fault model, where the data for modeling come from a typical test report and basic design data. The new model should incorporate both the leakage and core representations as in the hybrid approach.

2.2.4 Calculation of the Binary Short-Circuit Reactances for Core-Type Transformers with Cylindrical and Sandwich Windings

The flux which is confined in the steel core and links all the coils located on the same core leg is called the main flux. It excites the transformer and transfers energy between transformer sides by means of the magnetic field concentrated in the steel core. However,

the magnetic permeability of the core steel is finite and some amount of the main flux leaks outside the magnetic core. This flux, flowing outside the core and not linking all the coils on the same core leg, is called the leakage flux and results in a voltage being induced in each coil that opposes the flow of current. This effect is represented as the leakage impedance in the equivalent transformer circuit [42].

Estimation of the leakage reactance depends on the leakage flux distribution and the resulting flux leakages of each winding [51]. The distribution and amplitude of the leakage flux also depend on the geometrical configuration of the coils and neighboring iron masses. This defines the transformer short-circuit impedance between the windings [51], [52].

Fig. 2.7 shows the leakage field distribution in core-type transformers with cylindrical and sandwich windings. Sandwich windings are used both in core-type and shell-type transformers. In this dissertation, the transformers of the core type only are studied.

The leakage field is mainly concentrated in the space between windings. In cylindrical windings, the leakage flux is parallel to the core leg for nearly the full coil height. In sandwich windings, the leakage flux is mainly parallel to the coil width, i.e., perpendicular to the core. In both cases, the field is symmetrical having a geometry which allows introducing several approximations when describing it mathematically [51].

Fig. 2.7. Leakage flux in transformer windings: a) cylindrical coils, b) sandwich coils [52].

2.2.4.1 Approximations for Calculation of the Binary Reactances in Core-Type Transformers

Calculations of the binary leakage reactances are greatly simplified by making the following assumptions [51] with reference to Fig. 2.8:

1. Both windings have an equal axial height.

2. The flux paths are parallel to the windings and core along the axial height.
3. The permeance of the leakage flux path external to the winding height is very large, so negligible magnetomotive force (mmf) is required on that path. Therefore, the whole winding mmf is expended along the winding height and the flux path goes entirely through the duct. This also means that the mmf required for the steel path is negligible.
4. The primary winding mmf is equal to the secondary winding mmf, meaning that the magnetizing mmf and the magnetizing current are zero.
5. The leakage flux links with each winding equally, i.e., one half of the leakage flux links with one winding and the other half links with another winding.
6. The reluctance of the flux path through yokes is negligible and yields no effect on the flux distribution of yokes.
7. The windings are uniformly distributed, leading to a linear distribution of the winding mmf from zero at one end and to the full ampere-turns at the other end.

Based on these assumptions, two well-known methods for calculation of the leakage reactances in core-type transformers yield the same formulas:

- the flux linkages per unit current method, and
- the energy method.

Fig. 2.8. Simplified leakage flux representation and mmf distribution for two cylindrical coils of core-type transformers [51], [52].

2.2.4.2 Flux Linkages per Unit Current Method

An easy to understand the derivation of the reactance formula is given in [51], although this treatment lacks information on the Rogowski factors. A derivation for cylindrical windings with more details than in any textbook is suggested in [53]. A detailed explanation of the necessity and types of the Rogowski factors is given in [52]. Reference [24] introduces the Rogowski factor formula for cylindrical windings but omits it for sandwich windings.

2.2.4.2.1 Cylindrical Windings

The derivation is given below with reference to [51] and Fig. 2.8, where:

- Φ_p and Φ_s are the leakage fluxes of the primary and secondary windings respectively,
- Φ_a is the leakage flux through the duct,
- b_p and b_s are the radial widths of the primary and secondary windings respectively,
- a is the width of the radial duct,
- h is the axial height of the windings,
- h_R is the winding height h modified by the Rogowski factor for cylindrical windings K_{cyl} ,
- τ_l is the radial size of the two windings and the duct in between,
- D_p and D_s are the mean diameters of the primary and secondary windings,
- D_a is the mean diameter of the duct,
- L_{mtp} and L_{mts} are the mean circumferences of the primary and secondary windings,
- L_{mta} is the mean circumference of the duct ($L_{mt} = 2\pi R_{mt}$ for circular coils),
- dx is the width of an infinitesimal strip at a distance x from the edge of the secondary winding along its width, and
- N_p and N_s are the numbers of turns in the primary and secondary windings.

The per-phase leakage flux channel consists of three regions:

- the secondary winding,
- the duct between the windings, and
- the primary winding.

First, the flux linkages of the strip are calculated for the secondary winding:

- Mmf acting across the strip: $F_{strip} = \left(\frac{x}{b_s}\right) I_s N_s$.
- Permeance of the strip: $p_{strip} = \mu_0 \left(\frac{L_{mts}}{h}\right) dx$, where $\mu_0 = 4\pi \times 10^{-7}$ H/m.
- Flux in the strip: $\Phi_{strip} = F_{strip} \cdot p_{strip} = \mu_0 I_s N_s \left(\frac{L_{mts}}{h}\right) \left(\frac{x}{b_s}\right) dx$.

- This flux links with $N_{strip} = \left(\frac{x}{b_s}\right) N_s$ turns of the secondary winding.
- The flux linkages of the strip: $d\lambda_1 = \Phi_{strip} \cdot N_{strip} = \mu_0 I_s N_s^2 \left(\frac{L_{mts}}{h}\right) \left(\frac{x}{b_s}\right)^2 dx$.

And thus, the flux linkages of the secondary winding due to the flux in the strip are

$$\lambda_1 = \int d\lambda_1 = \mu_0 I_s N_s^2 \frac{L_{mts}}{h} \int_0^{b_s} \left(\frac{x}{b_s}\right)^2 dx = \mu_0 I_s N_s^2 \frac{L_{mts}}{h} \frac{b_s}{3}. \quad (2.15)$$

Second, the flux linkages of the secondary winding are calculated for the duct portion:

- Mmf acting across the duct: $F_a = I_s N_s$.
- Permeance of the duct: $p_a = \mu_0 \left(\frac{L_{mta}}{h}\right) a$.
- Flux in the duct: $\Phi_a = F_a \cdot p_a = \mu_0 I_s N_s \left(\frac{L_{mta}}{h}\right) a$.
- Half of the duct flux links with each of the two windings. The duct flux linking with the secondary winding: $\Phi_{a-s} = \frac{1}{2} \mu_0 I_s N_s \left(\frac{L_{mta}}{h}\right) a$.
- The flux linkages of the secondary winding due to the duct flux are

$$\lambda_a = \frac{1}{2} \mu_0 I_s N_s^2 \frac{L_{mta}}{h} a. \quad (2.16)$$

Thus, the total flux linkages of the secondary winding are

$$\lambda_s = \lambda_1 + \lambda_a = \mu_0 \frac{I_s N_s^2}{h} \left(\frac{L_{mts} \cdot b_s}{3} + \frac{L_{mta} \cdot a}{2} \right). \quad (2.17)$$

The leakage inductance of the secondary winding is then

$$L_s = \frac{\lambda_s}{I_s} = \mu_0 \frac{N_s^2}{h} \left(\frac{L_{mts} \cdot b_s}{3} + \frac{L_{mta} \cdot a}{2} \right). \quad (2.18)$$

And the leakage reactance of the secondary winding is

$$x_s = 2\pi f \mu_0 \frac{N_s^2}{h} \left(\frac{L_{mts} \cdot b_s}{3} + \frac{L_{mta} \cdot a}{2} \right). \quad (2.19)$$

Third, the leakage reactance of the primary winding is similarly calculated as

$$x_p = 2\pi f \mu_0 \frac{N_p^2}{h} \left(\frac{L_{mp} \cdot b_p}{3} + \frac{L_{mta} \cdot a}{2} \right). \quad (2.20)$$

The leakage reactance of the secondary winding referred to the primary side is

$$x'_s = x_s \left(\frac{N_p}{N_s} \right)^2 = 2\pi f \mu_0 \frac{N_p^2}{h} \left(\frac{L_{ms} \cdot b_s}{3} + \frac{L_{mta} \cdot a}{2} \right). \quad (2.21)$$

And therefore, the total per phase short-circuit reactance of a core-type transformer referred to the primary side is

$$X_p = x_p + x'_s = 2\pi f \mu_0 \frac{N_p^2}{h} \left(\frac{L_{mp} \cdot b_p + L_{ms} \cdot b_s}{3} + L_{mta} \cdot a \right), \quad (2.22)$$

where the expression in parenthesis defines the area of the reduced leakage channel, and the width of the reduced leakage channel is defined as

$$a_{RED} = \frac{b_p + b_s}{3} + a. \quad (2.23)$$

The reference of a reactance to either winding is defined by the choice of the number of turns N of the corresponding winding in (2.22). This takes care of the turns ratios.

The Rogowski factor for cylindrical windings

The leakage flux lines are parallel to the core legs almost along the full winding height and only at the ends they decline as shown in Fig. 2.7 a). This means that most of the leg mmf is expended along the winding height because the magnetic reluctance drops outside the windings. For transformers, where the winding height is much more than the radial size τ_l (width of two windings and a duct in between), the short-circuit reactance is usually calculated considering only the axial component of the leakage flux that equals the winding height. The radial component produced by the leakage flux fringing on the winding ends is usually neglected. In this case, estimation of the short-circuit reactance is still close to the test report data [51], [52].

However, if the winding height is of the same order as the radial size τ_l , it is necessary to use coefficients considering the magnetic reluctance outside the windings, i.e., the radial component of the leakage flux should also be brought into the reactance formula. Such coefficients, distinct for cylindrical and sandwich windings, were derived by professor V.

Rogowski at the beginning of the 20th century and are named after him [52]. In [24], the Rogowski factor for cylindrical windings is also introduced.

Application of the Rogowski factors allows a mathematical representation of the real leakage field. Application of the Rogowski factor for cylindrical windings elongates their height from h to h_R by reaching the yokes as shown in Fig. 2.8.

Thus, (2.22) becomes:

$$X_p = 2\pi f \mu_0 \frac{N_p^2}{h_R} \left(\frac{L_{mp} \cdot b_p + L_{ms} \cdot b_s}{3} + L_{ma} \cdot a \right), \quad (2.24)$$

where $h_R = h/K_{cyl}$ and $K_{cyl} < 1$ is the Rogowski factor for cylindrical windings.

From [24], [52] and with reference to Fig. 2.8, the Rogowski factor for cylindrical windings is calculated as

$$K_{cyl} = 1 - \frac{1 - e^{-\pi(h/\tau_1)}}{\pi(h/\tau_1)}. \quad (2.25)$$

2.2.4.2.2 Sandwich Windings

Symmetrical sandwich winding is shown in Fig. 2.7 b), where coils of the primary and secondary windings alternate with each other. As per [51], if the primary winding has n coils, then each of these coils is sandwiched between the coils of the secondary winding. This requires two half coils of the secondary winding be located at each end of the sandwich winding with the number of turns also halved. In this case, the secondary winding has $(n - 1)$ full coils spread in between. The whole sandwich winding can be thought of as consisting of $2n$ equal units connected in series, where each unit has one half-coil of the primary winding and one half-coil of the secondary winding. Each unit can be treated as a pair of cylindrical windings, and the sandwich winding width w is analogous to the height h of cylindrical windings as shown in Fig. 2.9.

The simplified leakage flux and its mmf distribution in a sandwich winding unit are shown in Fig. 2.9, where:

- $\Phi_{l/2p}$ and $\Phi_{l/2s}$ are the leakage fluxes of half coils of the primary and secondary windings respectively,
- Φ_a is the leakage flux through the duct,
- b_p and b_s are the heights of the full coils of the primary and secondary windings respectively,
- a is the height of the radial duct,
- w is the coil width,

w_R is the coil width w modified by the Rogowski factor for sandwich windings K_{snw} ,
 τ_2 is the width of a half wave of the radial field mmf,
 δ is the distance between the core and the winding,
 D is the mean diameter of the sandwich winding,

N_p and N_s are the numbers of turns in the primary and secondary windings, and
 L_{mt} is the mean circumference of the winding ($L_{mt} = 2\pi R_{mt}$ for circular coils).

Fig. 2.9. Simplified leakage flux representation and mmf distribution in a sandwich winding unit of core-type transformers [52].

The leakage field, in this case, is called radial since the flux lines are perpendicular to the core legs. By analogy with the derivation for cylindrical windings and (2.22), the per phase leakage reactance of each unit, referred to the primary side is

$$X_{p_unit} = 2\pi f \mu_0 \frac{L_{mt}}{w} \left(\frac{N_p}{2n} \right)^2 \left(\frac{0.5b_p + 0.5b_s}{3} + a \right), \quad (2.26)$$

where $N_p / 2n$ is the number of turns in each half coil of the primary winding.

If the units are connected in series, the total per phase short-circuit reactance of a transformer referred to the primary side is

$$X_p = 2n \times X_{p_unit} = \pi f \mu_0 \frac{N_p^2 \cdot L_{mt}}{n \cdot w} \left(\frac{0.5b_p + 0.5b_s}{3} + a \right). \quad (2.27)$$

The reference of a reactance to either winding is defined by the choice of the number of turns N of the corresponding winding in (2.26) and (2.27). This takes care of the turns ratios.

The Rogowski factor for sandwich windings

The effect of the steel core is more prominent for sandwich windings than for cylindrical windings, therefore neglecting the Rogowski factor in the case of sandwich windings can lead to considerable error [52]. For sandwich windings of shell-type and core-type transformers, the Rogowski factor differs due to the steel effect: for shell-type transformers, some parts of a coil have steel on two sides, and the rest – only on one side; for core-type transformers, a coil has steel only on one side. Application of the Rogowski factor in the case of sandwich windings elongates their width from w to w_R as shown in Fig. 2.9.

Thus, (2.26) and (2.27) become (2.28) and (2.29), respectively

$$X_{p_unit} = 2\pi f \mu_0 \frac{L_{mt}}{w_R} \left(\frac{N_p}{2n} \right)^2 \left(\frac{0.5b_p + 0.5b_s}{3} + a \right), \quad (2.28)$$

$$X_p = \pi f \mu_0 \frac{N_p^2 \cdot L_{mt}}{n \cdot w_R} \left(\frac{0.5b_p + 0.5b_s}{3} + a \right), \quad (2.29)$$

where $w_R = w/K_{snw}$ and $K_{snw} < 1$ is the Rogowski factor for sandwich windings.

From [52] and with reference to Fig. 2.9, the Rogowski factor for sandwich windings is calculated as

$$K_{snw} = 1 - \frac{1 - e^{-\pi(w/\tau_2)}}{\pi(w/\tau_2)} \left[1 - 0.5e^{-2\pi(\delta/\tau_2)} (1 - e^{-\pi(w/\tau_2)}) \right]. \quad (2.30)$$

In [24], it is also suggested that the width w of sandwich windings should be modified by the Rogowski factor. However, in [24], the Rogowski factor for sandwich windings is incorrectly given by (2.25), and not by (2.30).

2.2.4.3 Energy Method

The derivation of the reactance formula is given below with reference to [52]. Here, the Rogowski factor is applied immediately to the winding height h_R for cylindrical windings and the winding width for sandwich windings w_R . In [24], a similar derivation is offered.

2.2.4.3.1 Cylindrical Windings

The energy of the magnetic leakage field is related to the short-circuit reactance by

$$W = \frac{1}{2} L_{s.c.} i^2, \quad (2.31)$$

where $L_{s.c.}$ is the transformer short-circuit inductance and i is the instantaneous current.

Restating (2.31), the transformer short-circuit inductance is obtained as

$$L_{s.c.} = \frac{2W}{i^2} = \frac{2W_m}{I_m^2} = \frac{W_m}{I^2}, \quad (2.32)$$

where W_m is the energy field corresponding to the current amplitude I_m , and I is the effective current in one of the windings. The short-circuit reactance is then

$$X_{s.c.} = 2\pi f L_{s.c.} = 2\pi f \frac{W_m}{I^2}. \quad (2.33)$$

Alternatively,

$$W = \frac{1}{2} \int_V B H dV = \frac{1}{2\mu_0} \int_V B^2 dV, \quad (2.34)$$

where $B = \mu_0 H$ since the leakage field is located mainly in the nonmagnetic medium.

Here, instead of referring the secondary winding reactance to the primary side, the assumption is made that $N_p = N_s = N$, which is equivalent to referring the windings to the common number of turns and stating $I_p = I_s = I$.

Mmf of the leakage field in any cross section has a trapezoidal shape as shown in Fig. 2.8. This comes from the fact that mmf at any point of the cross section is equal to the full current inside the contour defined by a magnetic tube through that point. On the left side of the secondary winding, mmf is zero and linearly grows from left to right until it reaches the right edge. This is because the current encircled by a contour increases proportionally to the distance from the left side of the secondary winding.

In the duct between the secondary and primary windings, mmf is constant because any contour in the duct completely encircles the primary or secondary winding. From the left side of the primary winding, mmf decreases because the current in the primary winding has an opposite sign and becomes zero on the right side of the primary winding.

The flux density B of the leakage field has the same shape as the mmf since for a contour defined by any magnetic tube

$$H \cdot l = I \cdot N, \quad (2.35)$$

where H is the magnetic field intensity, l is the length where this field acts, I is the current in the coil surrounding the length l , and N is the number of turns in the coil. For the example shown in Fig. 2.8, (2.35) can be re-written as

$$\frac{B_x}{\mu_0} h_R = (iN)_x, \quad (2.36)$$

where the subscript x defines location along the coil width. The flux density B is proportional to ampere-turns $(iN)_x$ defined by a magnetic tube through the point x :

$$B_x = \frac{\mu_0}{h_R} (iN)_x. \quad (2.37)$$

The amplitude of the magnetic flux density in (2.37) is defined as

$$B_m = \frac{\mu_0 I_m N}{h_R}. \quad (2.38)$$

For the vertical duct between the windings, $B = B_m$ at the current maximum, and the volume of the leakage field in the duct is

$$V = \pi D_a a h_R. \quad (2.39)$$

The magnetic field energy in the duct between the windings from (2.34) is

$$W_{m_a} = \frac{B_m^2}{2\mu_0} V. \quad (2.40)$$

Substituting (2.38) and (2.39) into (2.40) and setting $I_m = \sqrt{2}I$, the magnetic field energy in the duct is

$$W_{m_a} = \frac{\mu_0 (IN)^2 \pi D_a a}{h_R}. \quad (2.41)$$

To find the magnetic field energy in the space occupied by the secondary winding, consider again the strip of width dx as illustrated in Fig. 2.8. However, now, it should be thought of as a tube. The flux density at the distance x is

$$B_x = B_m \frac{x}{b_s} = \frac{\mu_0 I_m N x}{h_R b_s}. \quad (2.42)$$

And the element of volume dV is

$$dV = \pi(D_s - b_s + 2x)h_R dx. \quad (2.43)$$

Substituting (2.42) and (2.43) into (2.34), the magnetic energy in the space occupied by the secondary winding is

$$\begin{aligned} W_{m_s} &= \frac{1}{2\mu_0} \int_V B_x^2 dV = \frac{1}{2\mu_0} \int_0^{b_s} \frac{\pi(D_s - b_s + 2x)\mu_0^2 I_m^2 N^2 x^2 h_R}{h_R^2 b_s^2} dx = \\ &= \frac{\mu_0 \pi (IN)^2 b_s}{3h_R} \left(D_s + \frac{b_s}{2} \right) \approx \frac{\mu_0 \pi (IN)^2 b_s D_s}{3h_R}. \end{aligned} \quad (2.44)$$

By analogy, the magnetic energy in the space occupied by the primary winding is

$$W_{m_p} = \frac{\mu_0 \pi (IN)^2 b_p}{3h_R} \left(D_p - \frac{b_p}{2} \right) \approx \frac{\mu_0 \pi (IN)^2 b_p D_p}{3h_R}. \quad (2.45)$$

The sum of (2.41), (2.44), and (2.45) provides the total leakage field magnetic energy in the space occupied by the two cylindrical windings and the duct in between:

$$\begin{aligned} W_{m_p} &= W_{m_p} + W_{m_a} + W_{m_s} \approx \frac{\mu_0 \pi (IN)^2}{h_R} \left(\frac{D_p \cdot b_p + D_s \cdot b_s}{3} + D_a \cdot a \right) = \\ &= \frac{\mu_0 (IN)^2}{h_R} \left(\frac{L_{mp} \cdot b_p + L_{ms} \cdot b_s}{3} + L_{ma} \cdot a \right). \end{aligned} \quad (2.46)$$

Substituting (2.46) into (2.33), the total per phase short-circuit reactance of a transformer with cylindrical windings referred to either side is

$$X_{s.c.} = 2\pi f \mu_0 \frac{N^2}{h_R} \left(\frac{L_{mp} \cdot b_p + L_{ms} \cdot b_s}{3} + L_{ma} \cdot a \right). \quad (2.47)$$

The reference of a reactance to either winding is defined by the choice of the number of turns N of the corresponding winding in (2.47). This takes care of the turns ratio.

2.2.4.3.2 Sandwich Windings

By analogy with the derivation of the short-circuit reactance formula for sandwich windings using the flux linkages per unit current method in Subsection 2.2.4.2.2, modification of (2.47) is sufficient. Referring to Fig. 2.9, equation (2.29) will be obtained here again if the Rogowski factor K_{snw} for sandwich windings is applied.

2.2.4.4 *Other Methods for Calculation of the Binary Reactances in Core-Type Transformers*

The two methods described above in Subsections 2.2.4.2 and 2.2.4.3 are known from the beginning of the 20th century and use the approximate geometry of the transformer windings, i.e., conductor insulation materials, winding cooling ducts, and other nonconductive spaces are all considered as the current carrying parts of transformer windings, participating in mmf generation. Such an approximation made the task of leakage reactance calculation simple but not precisely accurate. Addition of other details makes the task more complex but hopefully more accurate as well [54].

For calculation of the leakage inductance of complex transformer windings, references [55], [56], [57] suggest using the concept of the geometric mean distances (GMD), known from power systems theory for estimation of inductances between transmission lines. However, the weakest part of the proposed method is in the incorrect application of the concept for the mean turn diameter of two windings. Better solutions for calculation of the mean turn diameter of two windings in various transformers is studied in [58].

Reference [59] simplifies the work performed in [58] for concentric uniform windings of equal height by means of moving away from the problematic concept mentioned above. The well-known leakage reactance formula requires calculation of the area of the reduced leakage channel through the separate calculation of the mean turn diameter of two windings and the reduced leakage channel width. This results in excessive computational effort. The author suggests alternative formulas for the computation of the area of the reduced leakage channel that offers identical results to the conventional method, but with significantly less computational effort.

Further improvements of [55], [56], [57], [58], and [59] lead to [60], where the author suggests new formulas, based on the GMD method, for calculation of the leakage inductance of two complex windings, which can have a nonuniform height distribution. The core effects are taken into consideration by applying image theory.

In [54], the GMD concept is also suggested for computation of the leakage reactances. The proposed method is valid for step-lap core-type transformers with helical and disc windings. The authors propose two methods based on design data availability:

- block mode with the windings discretized into small square conductors perfectly joined to each other (insulation details are not known and omitted in the calculations) – with a maximum of 5% deviation from the test reports of 12 transformers used in the study,
- sections mode with real discretization between conductors (conductor size, spaces between individual conductors, etc. are known) – with a maximum of 3% deviation from the test reports of 12 transformers used in the study.

The results of [54] showed that the energy method is much less accurate than the suggested methods: a maximum of 13% deviation from the test reports of 12 studies transformers.

FEM methods can also be applied for calculation of the leakage reactances [61], [62]. However, FEM methods are computationally intensive and require extensive memory.

Other procedures applied to compute the leakage reactances of power transformers are shown in [63], [64], [65], and [66].

2.2.4.5 Calculation of the Binary Reactances for Windings with Nonuniform MMF Distribution

For concentric coils of equal height with uniform mmf distribution, the leakage field is mainly axial along the winding height, excluding the fringing effect on the ends. However, the mmf distribution becomes nonuniform due to taps, different insulation thickness along the height, etc. In this case, the radial flux component becomes substantial and should be considered in calculation of the leakage reactance. Two cases from [52] for calculation of the leakage reactance between windings of unequal height due to different tap positions are described below.

2.2.4.5.1 Nonsymmetrical Tap Zone Position

Fig. 2.10 shows two concentric windings of unequal height – the tap is located on the top of the outer winding. The tap zone occupies β of the winding height h_1 , where $h_1\beta = h_1 - h_2$, and β is the shortening coefficient. If $\beta = 1$, then $h_1\beta = h_1$. Suppose the outer and inner windings have an equal number of turns which is equivalent to referring the windings to the common number of turns.

Fig. 2.10. Leakage field decomposition into radial and axial components when the tap zone is positioned on one side of the outer winding [52].

To represent both axial and radial components of the leakage field, two original windings should be interchanged with the windings A and B, as in Fig. 2.10. The windings A have even mmf distribution and establish the axial leakage field. The winding B of thickness $\tau_1 = b_p + a + b_s$ has an mmf distribution so that, together with the windings A, it yields the mmf distribution corresponding to the original windings. By observation, the winding B represents a sandwich winding unit with the number of turns $N\beta$ responsible for the radial leakage field. The axial size of this unit is $\tau_2 = h_1$.

According to (2.47), the leakage reactance of the windings A is

$$X_A = 2\pi f \mu_0 \frac{N^2}{h_1/K_{cyl}} \left(\frac{L_{mp} \cdot b_p + L_{mts} \cdot b_s}{3} + L_{mta} \cdot a \right), \quad (2.48)$$

where the expression in parenthesis is the area of the mmf diagram of the windings A:

$$\sum ATD_A = \frac{L_{mp} \cdot b_p + L_{mts} \cdot b_s}{3} + L_{mta} \cdot a. \quad (2.49)$$

The leakage reactance of the winding B is described according to (2.28), where the leakage channel width now is $h_1/3$ and $w = \tau_1$:

$$X_B = 2\pi f \mu_0 \frac{(N\beta)^2}{\tau_1/K_{snw}} \cdot \frac{L_{mta} \cdot h_1}{3}. \quad (2.50)$$

Both X_A and X_B represent the leakage fields created by the same currents and, therefore, should be added algebraically to provide the total short-circuit reactance between the windings of Fig. 2.10:

$$X_{TOT} = X_A + X_B = 2\pi f \mu_0 \frac{N^2 K_{cyl} \sum ATD_A}{h_1} \left(1 + \frac{\beta^2 K_{snw} L_{mta} h_1^2}{3 K_{cyl} \tau_1 \sum ATD_A} \right) = X_A \cdot k_{q1}, \quad (2.51)$$

where the expression in parenthesis k_{q1} is the coefficient that increases the short-circuit reactance due to the nonuniform mmf distribution shown in Fig. 2.10.

2.2.4.5.2 Symmetrical Tap Zone Position

Fig. 2.11 shows two concentric windings of unequal height where the tap zone of the same size from the previous case is located symmetrically on both sides of the outer winding.

The windings A have the same reactance as in the previous case. However, the winding B represents two units of sandwich winding, each having the number of turns $(N\beta)/2$. The axial size of each unit is $\tau_2 = h_1/2$. The leakage channel width here is $(h_1/2)/3$ and $w = \tau_1$.

According to (2.28), the leakage reactance of the windings B is

$$X_B = 2 \times \left[2\pi f \mu_0 \frac{(N\beta)^2}{4\tau_1/K_{snw}} \cdot \frac{L_{mta} \cdot h_1}{6} \right] = 2\pi f \mu_0 \frac{(N\beta)^2}{\tau_1/K_{snw}} \cdot \frac{L_{mta} \cdot h_1}{12}. \quad (2.52)$$

Fig. 2.11. Leakage field decomposition into radial and axial components when the tap zone is symmetrically positioned on both sides of the outer winding [52].

Again, X_A and X_B should be added algebraically to provide the total short-circuit reactance between the windings of Fig. 2.11:

$$X_{TOT} = X_A + X_B = X_A \cdot k_{q2} = X_A \left(1 + \frac{\beta^2 K_{snw} L_{mta} h_1^2}{12 K_{cyl} \tau_1 \sum ATD_A} \right). \quad (2.53)$$

The coefficients k_{q1} and k_{q2} differ only by the constant in the denominator. For the case in Fig. 2.10, $k_{q1} = 3$ and for the case in Fig. 2.11, $k_{q2} = 12$. This result suggests that at the symmetrical distribution of the tap zone on both ends of the outer winding, the transformer short-circuit reactance is lower compared to the case in Fig. 2.10, when the tap zone is located only at one end of the outer winding.

2.2.5 Transformer Matrix Representation

Mathematically, any transformer can be represented with a symmetrical matrix because a transformer is a set of mutual inductances depending on the number of windings. This matrix represents the mutual coupling between winding coils per phase. According to [67] transformers can be mathematically represented with two types of matrices: the leakage inductance matrix $[L]$ and the leakage admittance matrix $[A]$.

2.2.5.1 Leakage Inductance Matrix $[L]$

As explained in [67], transformers can be represented as coupled $[R]$ - $[L]$ branches if the excitation current is not ignored. The resistance matrix $[R]$ consists of the branch resistances and the inductance matrix $[L]$ consists of the self- and mutual inductances. The analysis for steady-state conditions is defined by

$$[V] = [Z][I], \quad (2.54)$$

where $[Z]$ is the impedance matrix.

For analysis of transients, (2.54) is rewritten as a differential equation:

$$[v] = [R][i] + [L]\left[di/dt\right]. \quad (2.55)$$

The elements of the reactance matrix $[X] = j\omega[L]$ contain information about the transformer excitation current by means of the short-circuit reactances indirectly represented through the small difference between each pair X_{ii} and X_{ij} . The short-circuit reactances might be lost unless a very high accuracy is used during measurements and then carried out into the data file. The reactance matrix $[X]$ is almost singular and therefore ill-conditioned. This is more ill-conditioned for smaller excitation currents [67].

2.2.5.2 Leakage Admittance Matrix $[A]$

An alternative transformer representation is known as the admittance formulation [67]. The admittance matrix is the inverse of the reactance matrix for steady-state conditions. Therefore, (2.54) transforms into (2.56):

$$[I] = [Y][V], \quad (2.56)$$

$$[A] = j\omega[Y]. \quad (2.57)$$

For zero excitation current, $[A]$ does exist and its elements are obtained through a series of standard binary short-circuit tests. For transient studies, the branch matrices $[R]$ and $[L]^{-1}$ must be separated:

$$\left[di/dt\right] = [L]^{-1}[v] - [L]^{-1}[R][i]. \quad (2.58)$$

This option is preferable since matrices $[R]$ and $[L]^{-1}$ are not ill-conditioned. If a user wants to include magnetizing effects of the core, this can be done by attaching the core circuit to the transformer terminals of the winding which is closest to the core. This assumes that since the unsaturated magnetizing reactance is much higher than the short-circuit reactance, they can be separated. Even though such a core model is not topologically correct, the

results are acceptable in many cases, and such an attachment is also a means to avoid instabilities of the transformer model [68]. For three-winding transformers, the core representation can also be connected to the star point, although this might give unstable results if one of the leakage inductances is negative which occurs in many cases [69].

2.2.6 Calculation of $[A]$ from Transformer Test Report Data

The transformer leakage admittance matrix $[A]$ is formulated as the bus admittance matrix $[Y_{bus}]$:

- the diagonal elements Y_{jj} equal the sum of the admittances directly connected to node j , and
- the off-diagonal elements Y_{ij} equal the negative of the net admittance connected between nodes i and j .

More information on formulation of $[Y_{bus}]$ can be found in [70].

The details of calculation of the leakage admittance matrix $[A]$ are described in [67] and [71]. A brief overview is given below in Subsections 2.2.6.1, 2.2.6.2, and 2.2.6.3.

2.2.6.1 Two-Winding Transformers

The separated branch matrices $[R]$ and $[\omega L]^{-1}$ are written in per unit (p.u.) as follows if the excitation branch is omitted:

$$[R_{pu}] = \begin{bmatrix} R_{1pu} & 0 \\ 0 & R_{2pu} \end{bmatrix} \text{ and } [\omega L_{pu}]^{-1} = \begin{bmatrix} 1/X_{pu} & -1/X_{pu} \\ -1/X_{pu} & 1/X_{pu} \end{bmatrix}, \quad (2.59)$$

where subscripts 1 and 2 refer to the primary and secondary transformer windings, X_{pu} is the transformer short-circuit reactance from the test report in p.u., and $[R_{pu}]$ is the diagonal matrix containing resistance values of the windings. Further, the leakage admittance matrix $[A_{pu}]$ is formed by simply removing the angular frequency ω :

$$[A_{pu}] = \omega [\omega L_{pu}]^{-1}. \quad (2.60)$$

The p.u. matrices are converted to Ohms values following the rules below:

- for obtaining $[R]$ – divide each matrix element of $[R_{pu}]$ by the common power base and multiply each diagonal element by its corresponding voltage squared, and
- for obtaining $[A]$ – multiply each matrix element of $[A_{pu}]$ by the common power base and divide each row and column i by V_i .

Both $[R]$ and $[A]$ matrices are of 2×2 size and $[A]$ has no zeros. The leakage admittance matrix $[A]$ contains the correct turns ratios and, together with $[R]$, describes a single-phase two-winding transformer.

The $[A]$ matrix for a three-phase two-winding transformer is represented with three such blocks located on the main block diagonal and with zeros in the off-diagonal blocks if the interphase coupling (the zero sequence) is not included in the matrix. If both the positive and zero sequences are included in $[A]$, then non-zero values fill the off-diagonal blocks. The $[R]$ matrix for a three-phase two-winding transformer is a diagonal matrix holding resistance values of individual windings in each phase. Both the $[R]$ and $[A]$ matrices for a three-phase two-winding transformer are of 6×6 size.

2.2.6.2 Three-Winding Transformers

A three-winding transformer should be represented with the star circuit first. The star point does not exist physically and is needed for the mathematical description. Reactances for the star circuit are calculated from the p.u. binary short-circuit reactances, taken from the test report, using the voltage ratings and one common power base.

Next, the wye-delta transformation is applied to convert the star circuit into the delta circuit, which is represented with susceptances between the pairs of windings. From this point, $[\omega L]^{-1}$ is calculated as the bus admittance $[Y_{bus}]$ matrix:

$$[\omega L_{pu}]^{-1} = \begin{bmatrix} B_{HLpu} + B_{HTpu} & -B_{HLpu} & -B_{HTpu} \\ -B_{HLpu} & B_{HLpu} + B_{LTpu} & -B_{LTpu} \\ -B_{HTpu} & -B_{LTpu} & B_{HTpu} + B_{LTpu} \end{bmatrix}, \quad (2.61)$$

where B is a susceptance between corresponding pairs of windings in p.u. among the high H, low L, and tertiary T windings. Considering the relationship between inverse inductance, reactance, and susceptance, the elements of the leakage admittance matrix are calculated as

$$a = \frac{1}{L} = \frac{\omega}{X} = \omega \cdot B. \quad (2.62)$$

The leakage admittance matrix in p.u. $[A_{pu}]$ is formed by simply removing the angular frequency ω as in (2.60). Conversion of the p.u. admittance matrix $[A_{pu}]$ to the actual $[A]$ is performed as in Subsection 2.2.6.1. The obtained matrix contains correct turns ratios. The matrix $[R]$ holding the resistance values of each winding is diagonal and of 3×3 size. The $[A]$ matrix is of 3×3 size also but with no zeros. Together, these matrices $[R]$ and $[A]$ describe a single-phase three-winding transformer.

Formulation of the matrices for a three-phase three-winding transformer follows the same concepts and, if core effects (the zero sequence) are omitted, produces the block diagonal

leakage admittance matrix $[A]$ of 9×9 size with zeros in the off-diagonal blocks and a diagonal matrix $[R]$ of the same size:

$$[A] = \begin{bmatrix} \text{phase } A & \text{phase } B & \text{phase } C \\ \begin{matrix} a_{HH} & a_{HL} & a_{HT} \\ & a_{LL} & a_{LT} \\ & & a_{TT} \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \hline & \begin{matrix} a_{HH} & a_{HL} & a_{HT} \\ & a_{LL} & a_{LT} \\ & & a_{TT} \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \hline & & \begin{matrix} a_{HH} & a_{HL} & a_{HT} \\ & a_{LL} & a_{LT} \\ & & a_{TT} \end{matrix} \end{bmatrix} \quad (2.63)$$

$$[R] = \begin{bmatrix} \text{phase } A & \text{phase } B & \text{phase } C \\ \begin{matrix} R_H & 0 & 0 \\ & R_L & 0 \\ & & R_T \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \hline & \begin{matrix} R_H & 0 & 0 \\ & R_L & 0 \\ & & R_T \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \hline & & \begin{matrix} R_H & 0 & 0 \\ & R_L & 0 \\ & & R_T \end{matrix} \end{bmatrix}. \quad (2.64)$$

Both for three-phase two- and three-winding transformers, the zero-sequence reluctances can be included in the overall transformer model by either of the following methods:

- In a topologically correct nonlinear core circuit connected externally to the leakage admittance matrix $[A]$. Such a setup defines the hybrid approach, where the no-load and short-circuit transformer behaviors are decoupled [20], [49], [72], [73], [74]. The appropriate connections between the coils are also implemented outside of $[A]$.
- As mutual reactances between the windings of different phases, directly included in $[A]$. Then, non-zero values fill the off-diagonal blocks [67]. In this case, if the core circuit is attached externally, it does not correspond to the correct topology.

2.2.6.3 Transformers with Any Number of Coils

Transformers, usually, have a maximum of three windings. However, one winding can be represented with an assembly of coils. If a number of coils is more than three, then a transformer model different from that described in Subsection 2.2.6.2 is needed [67]. Also, if the number of transformer coils is more than three, then the system of equations defining the leakage admittance matrix becomes overdetermined. For such cases, the realization that the leakage admittance matrix $[A]$ is singular can be used to help to build this matrix. Singularity in this case means:

- the p.u. elements of each row and column sum to zero,
- $[A]$ has no connection to ground (floating network), and
- $[A]$ represents the short-circuit response only.

The procedure to build $[A]$ for a single-phase N -coil transformer is called Dommel's method and is described below. For more details refer to [67]. This method was originally presented in [75] and [76]. Dommel's method produces an accurate transformer short-circuit representation and can be extended to any number of windings.

Formulation of the branch admittance matrix $[Y]$, as in (2.56), forms the basis of this method. One common base power should be used throughout the calculations. First, to obtain $[Y]$, the transfer characteristics between coils are expressed as voltage drops between a coil i and the last coil N :

$$\begin{bmatrix} V_1 - V_N \\ V_2 - V_N \\ \vdots \\ V_{N-1} - V_N \end{bmatrix} = \begin{bmatrix} Z_{11}^{red} & Z_{12}^{red} & \cdots & Z_{1,N-1}^{red} \\ Z_{21}^{red} & Z_{22}^{red} & \cdots & Z_{2,N-1}^{red} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N-1,1}^{red} & Z_{N-1,2}^{red} & \cdots & Z_{N-1,N-1}^{red} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}, \quad (2.65)$$

where $[Z^{red}]$ is a p.u. symmetric matrix whose elements are found directly from the transformer short-circuit test data. Since the excitation current has a negligible influence on transfer characteristics, it is ignored. In this case, the sum of the p.u. currents is zero:

$$\sum_{k=1}^N I_{k\ pu} = 0. \quad (2.66)$$

The diagonal elements of (2.65) are the short-circuit impedances between the pairs of coils i and N :

$$Z_{ii\ pu}^{red} = Z_{iN\ pu}^{short}. \quad (2.67)$$

The off-diagonal elements are calculated by

$$Z_{ik\ pu}^{red} = \frac{1}{2} \left(Z_{iN\ pu}^{short} + Z_{kN\ pu}^{short} - Z_{ik\ pu}^{short} \right). \quad (2.68)$$

Equation (2.65) cannot be expanded to include all the coils since the matrix becomes singular with no inverse due to the omitted excitation current. Second, to obtain $[Y]$, an inversion of (2.65) is required:

$$[Y_{pu}^{red}] = [Z_{pu}^{red}]^{-1}. \quad (2.69)$$

Third, remembering that $[A]$ is singular, the N^{th} row of the full $[Y]$ is created by taking the negative sum of rows 1, ..., $N - 1$:

$$Y_{iN\ pu}^{red} = Y_{Ni\ pu}^{red} = - \sum_{k=1}^{N-1} Y_{ik\ pu}^{red} \text{ for } i \neq N. \quad (2.70)$$

The last element is defined by

$$Y_{NN\ pu} = - \sum_{i=1}^{N-1} Y_{iN\ pu}. \quad (2.71)$$

And the other elements of the full $[Y]$ are taken from (2.69):

$$Y_{ik\ pu} = Y_{ik\ pu}^{red} \text{ for } i, k \leq N - 1. \quad (2.72)$$

Representation of the full branch admittance matrix $[Y]$ of a single-phase N -coil transformer is defined as

$$\begin{bmatrix} I_{1\ pu} \\ I_{2\ pu} \\ \vdots \\ I_{N\ pu} \end{bmatrix} = \begin{bmatrix} Y_{11\ pu} & Y_{12\ pu} & \cdots & Y_{1N\ pu} \\ Y_{21\ pu} & Y_{22\ pu} & \cdots & Y_{2N\ pu} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N1\ pu} & Y_{N2\ pu} & \cdots & Y_{NN\ pu} \end{bmatrix} \begin{bmatrix} V_{1\ pu} \\ V_{2\ pu} \\ \vdots \\ V_{N\ pu} \end{bmatrix}. \quad (2.73)$$

Fourth, conversion from p.u. values to Ohms values is performed as described in Subsection 2.2.6.1.

Following the recommendation to separate the resistive and inductive parts for transient studies, $[Z^{red}]$ should be built from reactances of the short-circuit test data. Finally, the transformer admittance matrix $[A]$ is obtained by removing the angular frequency ω from $[Y]$:

$$[A] = [L]^{-1} = j\omega[Y]. \quad (2.74)$$

The winding resistances form the diagonal matrix $[R]$ as shown in Subsections 2.2.6.1 and 2.2.6.2.

Extension to the three-phase model is achieved by including the zero-sequence reluctances using one of the two options described in Subsection 2.2.6.2.

2.3 ElectroMagnetic Transient Program

2.3.1 EMTP

The main tool for investigation of transient phenomena of electromagnetic and electromechanical nature in power systems is time domain simulation. The most common software packages are ElectroMagnetic Transient Programs (EMTP-like programs).

Typical EMTP applications, as per [77], are insulation coordination, lightning overvoltage studies, switching transients and faults, statistical and systematic overvoltage studies, very fast transients in GIS and grounding, machine modeling, transient stability, motor startup, shaft torsional oscillations, transformer and shunt reactor/capacitor switching, ferroresonance, power electronic applications, circuit breaker duty (electric arc), current chopping, FACTS devices: STATCOM, SVC, UPFC, TCSC modeling, harmonic analysis, network resonances, and protection device coordination/testing.

2.3.2 EMTP-ATP

EMTP-ATP, or Alternative Transient Program (ATP), is a free software package (although requiring licensing) that has been developed through international research contributions. This software originated in 1984 from the BPA's version of EMTP available in the public domain. When the EMTP Development Coordination Group (DCG) and the Electric Power Research Institute (EPRI) decided to commercialize EMTP, Dr. W. Scott Meyer started a new program, now well known as ATP. Since then, ATP has been continuously developed through international contributions.

ATP can simulate complex networks and control systems of arbitrary structure. It has extensive modeling capabilities and other important features besides computation of transients. The major models of ATP are rotating machines, transformers, surge arresters, transmission lines, cables, and recently added protection relays. Operating principles of ATP are listed in [78].

2.3.3 ATPDraw

ATPDraw [79] is a graphical preprocessor to ATP allowing building digital circuits from an extensive palette using a mouse. Also, a user can create new objects using the programming language MODELS or the data base modules \$INCLUDE. The output of an ATPDraw file is an input to ATP. The author of ATPDraw is Dr. Hans Kristian Høidalen

at Norwegian University of Science and Technology, and his work is supported by BPA, USA.

2.4 Overview of Transformer Modeling in ATP

Representation of transformers for digital simulations is a complex task due to [80]:

- many options of core and winding designs,
- some parameters are nonlinear and frequency dependent.

A transformer model can be considered well developed if it correctly represents the core and winding configurations, self- and mutual inductances of windings, magnetic core saturation, hysteresis and eddy current losses, and capacitive effects. Irrespective of a transformer model type, the parameter estimation is based mainly on the nameplate data and standard tests [80]. Usually, the following data are available for transformer modeling: ratings, excitation and short-circuit data for positive- and zero-sequence impedances, saturation characteristic, and capacitances between terminals and windings. Other sources of data can be the design data (material type, core geometry, and turns numbers) and typical values (textbook data with tabulated leakage impedances, copper/core losses, and magnetizing currents).

Currently, the ATP built-in transformer models can be used for low and mid-frequency power system transients, i.e., up to 3-5 kHz. ATP has the following built-in transformer components [77]:

- Single-phase and three-phase ideal transformer,
- Single-phase two- and three-winding saturable transformer,
- Three-phase saturable transformer (three- and five-legged, shell form) with two- and three-windings (delta, wye, zig-zag, and auto-transformer connections),
- XFORMER,
- TRELEG routine,
- BCTRAN routine: single- and three-phase, two- and three-winding (delta, wye, and auto-transformer connections),
- The Hybrid Transformer model (XFRM) with the topologically correct core options: three-phase (triplex, three- and five-legged, shell form), two- and three-windings (delta, wye, and auto-transformer connections).

The details on the above-mentioned models except the Hybrid model can be found in [67] and [81]. The Hybrid model is described in Subsections 2.4.1 – 2.4.3 below.

2.4.1 Hybrid Approach Development

The success of a transformer modeling approach depends on the data available to engineers and researches, and usually, these data consist of a typical test report. The mutual reactances between the windings of different phases are not directly specified in a typical

test report, however, the open-circuit test data can be converted into a magnetic core representation. Thus, the interphase coupling information can be retrieved.

A strategy of combining two separate models to represent the overall transformer model is defined as hybrid in the literature. If a core is not saturated, then the core impedance is much higher than the short-circuit impedance and, therefore, a transformer can be represented with two separate but interconnected models [22]:

- A model containing the transformer short-circuit response (leakage admittance matrix $[A]$). The leakage formulation can be represented as a star circuit or a mesh circuit as shown in Fig. 2.12 [22].
- A model containing the magnetic core effects and the interphase coupling (a magnetic circuit or a duality derived electric circuit).

Fig. 2.12. The hybrid approach: a) an equivalent star circuit, b) an equivalent mesh circuit [22].

In the case of the star circuit, the core representation can be connected to the star point or to the terminals of the winding closest to the core. In the case of the mesh circuit, the number of winding is not limited to three, and an additional artificial winding can be mathematically formulated to connect the core model [22]. The concept of a fictitious turn for interfacing the core and leakage models was proposed in [72], [73]. This approach allows modeling of different core types according to their topology. These models are known as “topologically correct” core models. This offers an advantage of easy access to the core parameters containing the saturation characteristic allowing easy modification of the residual flux levels in individual core limbs. Although, at high core saturation the overall model might behave incorrectly since the core inductance approaches the air-core inductance, and the assumption of decoupling is not accurate [22].

Therefore, the physical behavior of a transformer based on the available information in a typical test report is better represented by the hybrid approach.

2.4.2 The Hybrid Transformer Model (XFMR) of ATPDraw

The Hybrid model (XFMR) was developed at Michigan Technological University and incorporated in ATPDraw in 2005. The Hybrid model is designed for low to mid

frequencies (from sub 60 Hz up to 3-5 kHz), which is suitable for switching transients. The details of the Hybrid model can be found in [20], [25], [49], [71], [74], and [82]. The main advantages of the Hybrid model over the other built-in ATP models include:

- incorporation of the leakage effects between the windings and the core,
- existence of an interface between the leakage model and the core model,
- representation of different core topologies and winding configurations,
- frequency dependency of winding resistances,
- incorporation of the capacitive effects to extend validity of the model for frequencies up to 3-5 kHz, and
- separation of the positive-sequence reactances and the zero-sequence reluctances.

The structure of the model, illustrated in Fig. 2.13, adopted from [20] with permission, consists of four main parts:

- the leakage admittance matrix $[A]$,
- the capacitance matrix $[C]$,
- the winding resistance matrix $[R]$, where resistances can be frequency dependent $R(f)$, and
- the duality-based core circuit attached to the fictitious infinitely thin $(N+1)^{th}$ coil through the nodes α - β - γ .

Fig. 2.13. A structure of the Hybrid model (XFMR). Image source: [20]. ©2007 IEEE

Below, Subsections 2.4.2.1 and 2.4.2.2 offer an overview of the key points of the Hybrid model for two- and three-winding transformers.

2.4.2.1 Leakage Admittance Formulation

Leakage reactance is the main transfer characteristic of a power transformer. During a short circuit, the system fault current is limited by the transformer short-circuit impedance which is defined by the leakage flux and resistances of the windings. The exact representation of the leakage flux for modeling is a complex task because it passes through the air, oil and other nonmagnetic materials and also may flow through the tank and other metallic fittings.

The leakage flux linked by the innermost winding but not flowing in the core is not measured during factory tests, however, is important for accurate modeling. This path becomes especially important during core saturation. The popular BCTRAN support routine does not include this effect, but the Hybrid model incorporates this portion of the leakage flux by assuming a fictitious infinitely thin $(N+1)^{th}$ coil located on the surface of each core leg as shown in Fig. 2.14, adopted from [20] with permission. The same winding serves as an interface between the leakage inductance matrix $[A]$ and the core circuit. The $(N+1)^{th}$ coil allows for the complete transformer leakage representation [20]. The Hybrid model is valid for cylindrical windings.

Fig. 2.14. Conceptual implementation of the $(N+1)^{th}$ coil in two-winding transformers. Image source: [20]. ©2007 IEEE

By definition, the leakage admittance matrix $[A]$ is singular, refer Subsection 2.2.6.3. But when it is interconnected with the other three layers of the Hybrid model (the grounded core circuit, winding resistances $[R]$, coupling capacitances $[C]$), the multi-layered transformer model becomes non-singular. Such a model can be implemented in EMTP-type software packages.

2.4.2.1.1 Two-Winding Transformers

The transformer short-circuit reactance between the high- and low-voltage windings X_{HL} is given in a factory test report. The reactance value between the core and the low-voltage

winding X_{L-Core} is estimated employing the proportionality factor K times the short-circuit reactance between the windings [20]:

$$X_{L-CORE} \approx K \cdot X_{HL}, \quad (2.75)$$

where $K = 0.5$ is derived empirically to represent the additional effect of the leakage between the innermost coil and the core. Then, the reactance between the core and the high-voltage winding X_{H-Core} is

$$X_{H-CORE} \approx X_{L-CORE} + X_{HL} = K \cdot X_{HL} + X_{HL} = (1 + K) X_{HL}. \quad (2.76)$$

The fictitious winding voltage is assumed to have the voltage of the innermost winding. For three-phase two-winding transformers, $[A]$ is a full three-phase representation with 3×3 submatrices, similar to (2.63):

$$[A] = \begin{bmatrix} A_w & 0 & 0 \\ 0 & A_w & 0 \\ 0 & 0 & A_w \end{bmatrix}, \text{ where } [A_w] = \begin{bmatrix} a_{HH} & a_{HL} & a_{H-Core} \\ a_{HL} & a_{LL} & a_{L-Core} \\ a_{H-Core} & a_{L-Core} & a_{Core-Core} \end{bmatrix}. \quad (2.77)$$

The turns ratios are incorporated in $[A]$, but the winding coupling (wye, delta or auto) is incorporated outside of $[A]$. The algorithm for building the submatrices $[A_w]$ is given in Subsection 2.2.6.2.

2.4.2.1.2 Autotransformers

Most of the U.S. grid transformers are autotransformers [1]. They are often employed to connect two transmission lines of different voltage levels [70]. Autotransformers have smaller p.u. leakage impedance compared to other transformers, which offers the advantage of a lower series voltage drop and the disadvantage of higher short-circuit currents. The other characteristics of autotransformers are lower excitation current, lower p.u. losses and lower cost if the turns ratio is not too high [83]. Usually, autotransformers have tertiary winding connected in delta which is kept unloaded to stabilize the wye-wye connection by providing a path for the 3rd harmonic of the excitation current as well as for the zero-sequence current.

The proper modeling of autotransformers should address the actual coil topology: series, common, and delta coils. Fig. 2.15, adopted from [84] with permission, illustrates in a) a three-winding autotransformer scheme and in b) the concept of the leakage model for a three-winding transformer. The methodology to calculate the binary reactances between the coils and build the leakage admittance matrix is presented in [49] and [71]. It is briefly discussed below.

The leakage reactances between the core and each winding are estimated as follows:

$$X_{D-CORE} \approx K \cdot X_{CD}, \quad (2.78)$$

$$X_{C-CORE} \approx (K+1) \cdot X_{CD}, \quad (2.79)$$

$$X_{S-CORE} \approx (K+1) \cdot X_{CD} + X_{SC}, \quad (2.80)$$

where $K = 0.5$ as shown in Subsection 2.4.2.1.1.

Fig. 2.15. A three-winding transformer with the delta tertiary: a) an autotransformer, b) conceptual implementation of the $(N+1)^{th}$ coil. Image source: [84].

Usually, the windings are positioned as follows: series is the outer winding, common is the middle one and delta tertiary is the innermost winding. Therefore, the delta winding has the lowest voltage rating and the least coil-to-core leakage.

The winding reactances are calculated through the short-circuit reactive power available from the binary short-circuit tests between pairs of terminals. The complete derivation is given in [49] and [71]. The derived formulas are as follows:

$$X_C = \frac{I_{HL}^2 \cdot (Q_{HT}/3 - Q_{LT}/3) - I_{HT}^2 \cdot Q_{HL}/3}{I_{HT}^2 \cdot I_{HL}^2 - I_{LT}^2 \cdot I_{HL}^2 - I_C^2 \cdot I_{HT}^2}, \quad (2.81)$$

$$X_S = \frac{Q_{HL}/3 - X_C \cdot I_C^2}{I_{HL}^2}, \quad (2.82)$$

$$X_D = \frac{Q_{LT}/3 - X_C \cdot I_{LT}^2}{I_D^2}. \quad (2.83)$$

The short-circuit reactances between the actual windings $X_{S-C \text{ pu}}$, $X_{S-D \text{ pu}}$, and $X_{C-D \text{ pu}}$ are obtained through the wye-delta conversion, as shown in Fig. 2.16, where $X_{SC \text{ pu}}$, $X_{SD \text{ pu}}$, and $X_{CD \text{ pu}}$ are the reactances for each pair of windings on the delta diagram.

Fig. 2.16. Calculation of the binary reactances in an autotransformer [49].

The corresponding formulas from [49] and [71] are given below without derivation:

$$X_{S-C \text{ pu}} = \left[\frac{1}{X_{SC \text{ pu}}} + \frac{1}{X_{SD \text{ pu}} + X_{CD \text{ pu}}} \right]^{-1}, \quad (2.84)$$

$$X_{S-D \text{ pu}} = \left[\frac{1}{X_{SD \text{ pu}}} + \frac{1}{X_{SC \text{ pu}} + X_{CD \text{ pu}}} \right]^{-1}, \quad (2.85)$$

$$X_{C-D \text{ pu}} = \left[\frac{1}{X_{CD \text{ pu}}} + \frac{1}{X_{SC \text{ pu}} + X_{SD \text{ pu}}} \right]^{-1}. \quad (2.86)$$

Based on (2.78) – (2.80) and (2.84) – (2.86), the leakage admittance matrix $[A]$ for three-winding autotransformers is calculated using the procedure described in Subsection 2.2.6.3 and is similar to (2.77), where each submatrix $[A_w]$ is of 4×4 size since the $(N+1)^{th}$ coil is included.

2.4.2.2 Core Representation

A realistic hysteresis model, which can accommodate plotting numerous loops, is very complex to design and implement [80]. To model the simplified magnetization curves, engineers usually work with the λ - i piecewise linear characteristics. The B - H

characteristics, usually, are not used as they include material properties and should be scaled according to the actual area and length of each core section to obtain the λ - i data for implementation in EMTP inductors [71]. Recent developments on hysteresis modeling in ATPDraw can be found in [85], [86], [87].

The true λ - i trajectory depends not only on the initial flux but also on history. Each core limb has its own λ - i characteristic and flux history λ_0 . The λ - i characteristic is derived from the measurements at the transformer terminals: the current is measured directly, and the flux-linked is derived by integrating the terminal voltage.

Both the core and winding losses are frequency dependent. Usually it is assumed that the magnetic field penetrates the core uniformly; however, it is not true in general. The changing magnetic field induces eddy currents in the core steel causing the flux density to drop. The flux distribution in the core depends on frequency. The higher the frequency, the more flux is pushed in the outer lamination surface and the thinner this surface is.

The core modeling in the Hybrid model is suitable for stacked cores with three and five legs. The inductive and resistive core representations are treated independently. The core losses are assumed to be linear, and the nonlinear inductances are modeled with the Frölich equation to represent the magnetization characteristic of each limb [82].

The core representations are developed using various transformer topologies: the electric core circuit is obtained from the original magnetic core circuit using a duality transformation [88]. This approach formulates the topologically correct nonlinear representation of the core structure [20], which is attached to the $(N+1)^{th}$ coil through the α - β - γ terminals.

All the indicated references related to the Hybrid model have an outdated core circuit. Fig. 2.17 shows the latest version of the core circuit for three-phase three-legged transformers. The default core grounding is provided, and the off-core inductances (the zero-sequence flux paths Z_{0A} , Z_{0B} , and Z_{0C}) are distributed between the three legs for three-legged transformers or between the three main legs for five-legged transformers [79]. The lack of default core grounding in the ATPDraw versions below v.6.1p8 leads to floating subnetwork problems in some cases.

The part of the core circuit which represents the magnetic characteristic is modeled as a parallel combination of a linear resistor (core losses) and a saturable inductor (λ - i characteristic) for each leg and yoke.

The core losses are modeled with a constant resistance which lumps hysteresis, eddy currents, and anomalous losses. This is based on [89]: a nonlinear inductance in parallel with a constant resistance is accurate within about 5% up to about 3 kHz. The saturable inductor element contains either a reversible anhysteretic magnetization curve, as Type-93/98, or a simplified hysteresis loop, as Type-96, suitable for setting residual flux in inrush studies. The final slope inductance, calculated from the air-core inductance or design data, is important in inrush studies [90].

Fig. 2.17. The duality derived core circuit of three-legged transformers. Implemented in XFMR starting from v.6.1p8 of ATPDraw [79].

The Advanced Core Settings tab of the Hybrid model allows the user to enter the transformer zero-sequence inductance. This value is not usually available from manufacturers. However, the ATPDraw developer suggests a zero-sequence inductance of 750 mH for a 300 MVA three-legged transformer. After entering this value in the Advanced Core Settings, the Hybrid model of a three-legged 60 MVA transformer gives the value of 500 Ω for each of the off-core inductors X_{0A} , X_{0B} , and X_{0C} at the power frequency of 60 Hz.

The off-core impedances Z_{0A} , Z_{0B} , and Z_{0C} do not model the air flux paths exactly and the flux paths through the tank are not included in the Hybrid model. Although the core model includes the zero-sequence branches, their parameters are not exactly defined for the Hybrid model. In the case of five-legged or shell-type transformers, the zero-sequence reluctances are represented with nonlinear saturable inductors. The research on transformer zero-sequence flux modeling, where both the air and tank paths are included, is reflected in [91], [92], [93], [94].

2.4.3 BCTRAN versus XFMR

The BCTRAN support routine and the Hybrid model are the two most popular built-in ATPDraw components for transformer modeling [79].

The leakage admittance matrix $[A]$ of BCTRAN is built from the positive- and zero-sequence components, and therefore the interphase coupling is embedded in $[A]$, i.e., a part of the core representation is included in $[A]$. The core circuit can be connected externally to the leakage model to simulate inrush currents but without the correct core topology. Modeling of three-winding autotransformers in BCTRAN is based on the “black box” approach, where the series and common coils are not separated and, therefore, it is not possible to simulate faults on discrete series or common coil.

The leakage admittance matrix $[A]$ of the Hybrid model requires the positive-sequence components only but accommodates the zero-sequence data if available in the test report. The interphase coupling is formed by addition of the core circuit, containing the zero-sequence flux paths providing the correct topological representation of the core. Such an

approach decouples the leakage and the core models, as per the hybrid approach, and at the same time provides proper interaction of the zero-sequence flux with the main flux. Therefore, better results for the cases where core saturates, e.g., inrush currents, are provided. However, high saturation cases might produce incorrect results due to the limitations of the decoupling assumption.

Neither the BCTRAN support routine nor the Hybrid model is capable of simulating internal faults. However, the leakage admittance matrix in the .atp file of each can accommodate the necessary changes for simulation of internal faults. In [47], the BCTRAN support routine is applied for modeling of internal faults in transformers.

2.4.4 Internal Fault Model for Three-Phase Two-Winding Transformers

The test-report-based model for simulation of internal winding faults from [21], developed at Michigan Technological University, expands the leakage admittance matrix of a healthy two-winding transformer to obtain a higher-order form through segmentation of windings. Application of this model for relay performance testing is shown in [95].

Fig. 2.18 illustrates the number of winding segments necessary to simulate T2G and T2T faults [21], [30]. If the segments are connected in series and a fault is not applied, the model should reproduce the original short-circuit behavior [21].

Fig. 2.18. Segmentation of the faulted winding 1 for internal faults:
a) T2G fault, b) T2T fault [21], [30].

The internal fault model from [21] is based on the method of winding proportions, which is more accurate for smaller transformers because their nature is resistive. For large power transformers, reactance is predominant and affects the model accuracy. Also, this model is more accurate for cylindrical windings of equal height. Although the leakage flux

distribution during an internal fault can be more accurately estimated given the design information, such information is rare. The fault current in the shorted loop varies depending on a fault location and its severity [21], [42]. A leakage admittance model based on test report data only might not be very accurate, but it is a practical alternative to other options for internal fault modeling. For example, for the 500-kVA 11430Y/235Y V layer-layer cylindrical winding core-form distribution transformer, the fault current was underestimated by at least 25% for T2T and T2G faults; however, for the 15-kVA 240Δ/208Y V layer-layer cylindrical winding core-form distribution transformer, the highest fault current deviation was 7.8% for a T2G fault [21].

2.4.4.1 Calculation of the Binary Reactances

The winding proportions model is built on the principle of dividing the short-circuit reactance equally between the windings [21]. This is true for a conventionally designed transformer, where the primary and secondary windings are of the same type, e.g., one of the windings is not a sheet winding. Then, the p.u. short-circuit equivalent impedance $Z_{s.c.}$ is assumed to be split equally between the high-voltage and low-voltage windings. This approximation is based on an engineering rule of thumb [24], [67] and also is an intuitive thought. If a transformer operates in balanced conditions, then the leakage flux produced by the primary winding should be balanced out by the leakage flux produced by the secondary winding. This means that the corresponding leakage impedances should be equal on the p.u. basis.

From [21], an option of the winding proportions method below shows the high-voltage winding H consisting of N portions: $a, b, \dots, N$ for simulation of an internal fault. The low-voltage winding L remains healthy but can be formulated in the same manner for internal faults. This representation can be applied to the winding H in the faulted phase only or to the winding H in each phase for regularity.

For the Ohms representation, the turns ratio should be considered:

$$Z_H = Z_{s.c.}/2, \quad (2.87)$$

$$Z_L = Z_H/t^2, \quad (2.88)$$

where t is the turns ratio $t = N_H/N_L \approx V_H/V_L$ (per phase).

The winding H is divided into a through N portions, a coefficient for each portion is obtained:

$$\begin{aligned} n_a &= N_a/N_H, \\ n_b &= N_b/N_H, \end{aligned} \quad (2.89)$$

$$\vdots$$

$$n_N = N_N / N_H,$$

where $N_a, N_b, \dots, N_N$ are the numbers of turns in the $a, b, \dots, N$ portions, and

$$N_a + N_b + \dots + N_N = N_H,$$

$$n_a + n_b + \dots + n_N = 1. \quad (2.90)$$

Therefore, the winding proportions method calculates the individual impedance of each portion as

$$Z_a = n_a \cdot Z_H,$$

$$Z_b = n_b \cdot Z_H,$$

$$\vdots$$

$$Z_N = n_N \cdot Z_H. \quad (2.91)$$

The voltage of each portion is obtained using the same coefficients:

$$V_a = n_a \cdot V_H,$$

$$V_b = n_b \cdot V_H,$$

$$\vdots$$

$$V_N = n_N \cdot V_H. \quad (2.92)$$

For the low-voltage winding L, Z_L and V_L keep the original values. The real part of each impedance from (2.91) forms the resistance of an individual portion. The imaginary part of each impedance from (2.91) should be used for calculation of the equivalent binary reactance between any two portions i and j in the same phase:

$$X_{i-j} = X_i + \left(\frac{V_i}{V_j} \right)^2 \cdot X_j. \quad (2.93)$$

The Ohms value of each binary reactance should be represented as a p.u. value applying the common power base and the base voltage. Finally, the leakage admittance matrix of a faulted transformer is calculated by adopting the procedure described in Subsection 2.2.6.2.

2.4.4.2 Limitations of Winding Proportions Model

The winding proportions method for simulation of internal faults works for two-winding transformers and does not work for three-winding transformers since one of the reactances from the star diagram is negative in many cases. Dividing a negative reactance into several portions causes a simulation to blow up. Such a result exposes an opportunity for the development of an internal fault model which is suitable for transformers with any number of cylindrical windings and which can be built from easily available transformer data.

3 MODEL DEVELOPMENT

Based on the literature overview shown in Chapter 2, this chapter develops the coil volume internal fault model for simulation of two types of transformer internal faults: turn-to-turn (T2T) and turn-to-ground (T2G).

Any transformer model is based on an algorithm for estimation of the leakage flux represented with the binary reactances necessary for calculation of the leakage admittance matrix $[A]$. For healthy transformers, it is a well-known and widely used procedure. For faulted transformers, $[A]$ requires addition of one (T2G faults) or two (T2T faults) rows and columns. The coil volume model for simulation of internal faults can utilize one of two approaches for estimation of the binary reactances: Cylindrical or Segmented. Developing the methodology for these two approaches is the goal of Chapter 3. Their mathematical formulation is presented along with the advantages and limitations of each approach.

3.1 Modification of the Design Data of the Original Autotransformer

The test report data and design information of a real three-winding autotransformer have been modified for two reasons:

- Disclosing data of a real utility power transformer is not allowed.
- Design of a real transformer is complex and can be simplified for model development. After the coil volume internal fault model of the simplified transformer produces sufficient results, the complexity level of the model can be increased by shifting focus to real transformer data and working around the technical data security concern.

The modified transformer data, used in this dissertation, have been developed based on the existing transformer operated by a utility. The nondisclosure agreement signed between the utility and Michigan Technological University does not allow sharing information about the existing transformer with the public. The modified three-phase three-winding autotransformer available for this project is called “Mirage” since in this form it does not exist, and such a name reflects its artificiality. Its ratings are 144.9/72.5/13.8 kV, 60/60/12 MVA, YNa0d1, and its simplified core window is shown in Fig. 3.1. The modified transformer assumes that each winding contains only one cylindrical coil and all the coils have equal height.

The made-up test report data for the Mirage autotransformer are shown in Table 3.1. Based on the dimensions shown in Fig. 3.1, the binary short-circuit reactances between the windings are calculated as per (2.22), forming the data block 1. The data block 2 is calculated based on the data block 1 and the actual transformer resistances from the data block 4.

Fig. 3.1. Simplified core window of the Mirage autotransformer with ratings 144.9/72.5/13.8 kV, 60/60/12 MVA, YNa0d1.

Since the series coil of the original autotransformer has built-in taps, the Mirage's series coil is assumed to have full taps in order to represent the series coil as shown in Fig. 3.1. This simplification forces the use of the voltage rating of the original series winding at full taps in the Mirage's series coil. The same idea is used for the Mirage's common coil. Since the common winding of the original autotransformer has a tap coil, for simplification, these two coils are combined into one coil in the Mirage as shown in Fig. 3.1. This simplification also forces the use of the voltage rating of the original common winding at full taps in the Mirage's common coil. The rating of the Mirage's delta coil is the same as that of the original autotransformer. The voltage ratings of the Mirage are given in the data block 5.

Modification of the original autotransformer preserves the MVA rating, the full tap voltage ratings of the series and common coils, the voltage rating and size of the delta coil, and the core design of the original transformer. Therefore, the data blocks 3, 4, 6, and 7 of the Mirage are taken from the original test report corresponding to the full taps of the original transformer, in spite of the modifications for the series and common windings. The data block 8 is calculated based on the design data of the original transformer.

The data blocks 1, 4, 5, and 6 from Table 3.1 have been given to a transformer manufacturer who has their own internal fault model, later referred to as the proprietary model. Benchmarking of the coil volume internal fault model with the proprietary model is described in Chapter 4.

The exact transformer data are not strictly important in this research since the data of the modified autotransformer have been given to the transformer manufacturer. Therefore, benchmarking of the proposed model with the proprietary model is based on the same data. The main idea of this research is to develop the methodology and implementation of the coil volume internal fault model. Benchmarking of the proposed model with a trustable model will help to adjust the methodology and implementation of the proposed model for

the simplified autotransformer. After results from the proposed model, which is based on the simplified design data, become satisfactory, the proposed model can be developed further to use the original design of the autotransformer.

Table 3.1. The made-up test report data for the Mirage autotransformer.

1	X_{SC}, Ω	22.534	Calculated from Fig. 3.1 by (2.22), the Rogowski factor for cylindrical windings K_{cyl} is neglected.		
	X_{SD}, Ω	41.371			
	X_{CD}, Ω	12.185			
2	$Z_{HL}, \%$	6.441	The above reactances are used to calculate the corresponding impedances between the terminals.		
	$Z_{HT}, \%$	4.837			
	$Z_{LT}, \%$	2.786			
3	P_{HL} , kW at 60 MVA, 85°C	96.610	Taken from the original test report of the existing 60 MVA transformer, corresponding to the full taps of the original series and common windings.		
	P_{HT} , kW at 12 MVA, 85°C	18.117			
	P_{LT} , kW at 12 MVA, 85°C	18.329			
4	R_{SERIES}, Ω at 85°C	0.292781	Taken from the original test report of the existing 60 MVA transformer, corresponding to the full taps of the original series and common windings.		
	R_{COMMON}, Ω at 85°C	0.166339			
	R_{DELTA}, Ω at 85°C	0.034170			
5	V_H , kV _{L-L}	144.9	The series and common windings of the original transformer are assumed at full taps to give these nonstandard terminal voltages.		
	V_L , kV _{L-L}	72.5			
	V_T , kV _{L-L}	13.8			
6	Number of turns N_{SERIES}	444	Taken from the original test report of the existing 60 MVA transformer, corresponding to the full taps of the original series and common windings.		
	Number of turns N_{COMMON}	444			
	Number of turns N_{DELTA}	140			
7	Open circuit test data	$V, \%$	Losses, kW	$I_{average}, \%$	Taken from the original test report of the existing 60 MVA transformer, corresponding to the full taps of the original series and common windings.
		90	16.778	0.039	
		100	21.898	0.047	
		105	25.365	0.062	
		110	31.193	0.124	
8	The final slope inductance, mH	$L_a = 3.26$			Calculated based on the design data of the existing 60 MVA transformer.

3.2 Leakage Admittance Matrix of the Healthy Mirage

The two models of the healthy Mirage, without the core circuit $[A(N)]$ and with the core circuit $[A(N+I)]$, are created as an intermediate step towards formulation of the coil volume internal fault model. The $[A(N+I)]$ model for the healthy Mirage fully resembles the Hybrid model of the healthy Mirage. The latter is referred to as the XFMR Mirage. Therefore, the test results for these two models should be identical. These tests include the base load test, short-circuit tests, open-circuit test, and voltages across the α - β - γ terminals. Benchmarking of the $[A(N)]$ and $[A(N+I)]$ models versus the XFMR Mirage is given in Chapter 4.

3.2.1 Leakage Representation Only

If the transformer model includes the leakage representation only and omits the core effect, such a model is built from the positive-sequence reactances and can be used for studying balanced events, like balanced normal operation or balanced three-phase faults. This model is easy to build, and it is useful for initial verifications. The leakage model $[A(N)]$ of a three-phase three-winding autotransformer built from the positive-sequence reactances is illustrated in Fig. 3.2, where the number of windings is $N=3$ per phase. The elements combined by a dashed rectangle in each phase describe the independent phase leakage admittance submatrices $[A_A(N)]$, $[A_B(N)]$, and $[A_C(N)]$ of the total leakage admittance matrix $[A(N)]$.

Fig. 3.2. The leakage model $[A(N)]$ of a three-phase three-winding autotransformer.

3.2.1.1 Calculation of the Binary Reactances

The per-phase mmf diagram of the three-winding autotransformer from Fig. 3.2 is illustrated in Fig. 3.3. All the necessary data to calculate the binary short-circuit reactances

between the pairs of windings are given in Fig. 3.1. The winding pairs are series – common, series – delta, and common – delta.

As mentioned in Subsection 2.2.4.2.1, when the height of two cylindrical windings is much more than their radial size τ_l (winding depths and the duct in between), the radial component of the leakage field can be neglected. In this case, estimation of the binary reactances is based on the axial component of the leakage field mathematically described by (2.22).

Application of the Rogowski factor for cylindrical windings K_{cyl} as per (2.24) and (2.25) slightly increases the height of the windings from h to h_R , as shown in Fig. 2.8, leading to a slight reactance reduction since the Rogowski factor is less than 1.

Fig. 3.3. Per phase mmf diagram of a three-winding autotransformer: the leakage model $[A(N)]$.

Equations (3.1) – (3.3), describing the mmf diagram of Fig. 3.3 mathematically, should be used to calculate the binary reactances between the pairs of windings of a three-winding autotransformer:

$$X_{SC} = 2\pi f \mu_0 \frac{N^2}{h_{R_SC}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,C} \cdot b_C}{3} + L_{mt,a3} \cdot a_3 \right), \quad (3.1)$$

$$X_{SD} = 2\pi f \mu_0 \frac{N^2}{h_{R_SD}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,D} \cdot b_D}{3} + L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right), \quad (3.2)$$

$$X_{CD} = 2\pi f \mu_0 \frac{N^2}{h_{R_CD}} \left(\frac{L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D}{3} + L_{mt,a2} \cdot a_2 \right), \quad (3.3)$$

where h_R is the increased coil height due to application of the Rogowski factor K_{cyl} . The reference of a reactance to either winding is defined by the choice of the number of turns N of the corresponding winding. This takes care of the turns ratio.

It is important to remember that a typical transformer test report does not offer information on binary short-circuit reactances between the actual windings of a three-winding autotransformer. Instead, the binary short-circuit reactances between the terminals are measured. This is because in three-winding autotransformers, the series and common coils have an electrical connection and the terminals are brought out on a tank top in such a way that tests between the actual windings are not possible. The leakage reactances from a test report should be recalculated to obtain the leakage reactances between the pairs of the actual windings as described in [49].

3.2.1.2 $[A(N)]$ Formulation

The circuit, illustrated in Fig. 3.4, is another representation of the phase leakage admittance submatrices $[A_A(N)]$, $[A_B(N)]$, and $[A_C(N)]$ from Fig. 3.2. Fig. 3.4 directly represents each binary short-circuit test. The corresponding turns ratios are included.

Fig. 3.4. Per phase admittance formulation for a three-winding autotransformer: the leakage model $[A(N)]$.

The admittance formulation based on Fig. 3.4 is represented mathematically as

$$[A(N)] = \begin{bmatrix} A_A & 0 & 0 \\ 0 & A_B & 0 \\ 0 & 0 & A_C \end{bmatrix}, \quad (3.4)$$

$$\text{where } [A_A(N)] = [A_B(N)] = [A_C(N)] = \begin{bmatrix} a_{SS} & a_{SC} & a_{SD} \\ a_{SC} & a_{CC} & a_{CD} \\ a_{SD} & a_{CD} & a_{DD} \end{bmatrix}.$$

The procedure for calculation of the leakage admittance matrix $[A(N)]$ from the binary reactances is given in Subsections 2.2.6.2 and 2.2.6.3. They give the same result for three-winding transformers. The resistance matrix $[R(N)]$ should have the form as in (2.64).

This leakage model of a three-phase three-winding autotransformer should reproduce the test report data in binary short-circuit tests.

3.2.2 Full Model

If the grounded core model is connected externally to $[A]$, this is equivalent to connecting $[A]$ to ground, and the overall transformer model becomes nonsingular, refer to Subsection 2.4.2.1 for more details. Such a model can be used to study both balanced and unbalanced events in a system since incorporation of the core circuit brings the interphase coupling in the model. Fig. 3.5 illustrates the full model $[A(N+1)]$ of a three-phase three-winding autotransformer. This model consists of the leakage admittance matrix, built from the positive sequence reactances, and the core circuit attached to the artificial $(N+1)^{th}$ coil as implemented in the Hybrid model. In this case, the per phase number of windings becomes $N+1=4$. The elements within the dashed rectangular in each phase describe the independent phase leakage admittance submatrices $[A_A(N+1)]$, $[A_B(N+1)]$, and $[A_C(N+1)]$ of the total leakage admittance matrix $[A(N+1)]$. These submatrices are interconnected outside of the leakage model by including the zero-sequence flux paths in the core circuit, which are represented as Z_{0A} , Z_{0B} , and Z_{0C} in Fig. 3.5.

The core circuit, illustrated in Fig. 3.5, is the existing default core representation in the Hybrid model of the ATPDraw software, v.6.1p8. The voltage rating and the number of turns of the artificial $(N+1)^{th}$ coil are equal to those of the winding closest to the core, i.e., of delta winding.

Fig. 3.5. The full model $[A(N+1)]$ of a three-phase three-winding autotransformer.

3.2.2.1 Calculation of the Binary Reactances

The per-phase mmf diagram of the three-winding autotransformer from Fig. 3.5 is illustrated in Fig. 3.6. Again, all the necessary data are given in Fig. 3.1.

Fig. 3.6. Per phase mmf diagram of a three-winding autotransformer: the full model $[A(N+1)]$.

The binary leakage reactances between the pairs of N windings are calculated as in Subsection 3.2.1.1. The binary leakage reactances between each actual winding and the $(N+1)^{th}$ coil in the Hybrid model are estimated by (2.78) – (2.80). However, if the core window information, as in Fig. 3.1, is available, equation (2.22) can be used if the Rogowski factor K_{cyl} is not applied or (2.24) and (2.25) if the Rogowski factor is applied. Since the $(N+1)^{th}$ coil is infinitely thin, its thickness is zero in the equations. Equations (3.1) – (3.3) and (3.5) – (3.7) describe the mmf diagram of Fig. 3.6 mathematically:

$$X_{S-Core} = 2\pi f \mu_0 \frac{N^2}{h_{R_S-Core}} \times \left(\frac{L_{mt,S} \cdot b_S}{3} + L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right), \quad (3.5)$$

$$X_{C-Core} = 2\pi f \mu_0 \frac{N^2}{h_{R_C-Core}} \left(\frac{L_{mt,C} \cdot b_C}{3} + L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 \right), \quad (3.6)$$

$$X_{D-Core} = 2\pi f \mu_0 \frac{N^2}{h_{R_D-Core}} \left(\frac{L_{mt,D} \cdot b_D}{3} + L_{mt,a1} \cdot a_1 \right), \quad (3.7)$$

where h_R is the increased coil height due to application of the Rogowski factor K_{cyl} . The reference of a reactance to either winding is defined by the choice of the number of turns N of the corresponding winding. This takes care of the turns ratios.

The MATLAB code calculating the binary short-circuit reactances between the pairs of actual windings and between each winding and the $(N+1)^{th}$ coil with and without application of the Rogowski factor K_{cyl} for the healthy Mirage is given in Appendix A.

3.2.2.2 $[A(N+1)]$ Formulation

The circuit, illustrated in Fig. 3.7, is another representation of the phase leakage admittance submatrices $[A_A(N+1)]$, $[A_B(N+1)]$, and $[A_C(N+1)]$ from Fig. 3.5. Fig. 3.7 directly represents each binary short-circuit test. The corresponding turns ratios are included.

Fig. 3.7. Per phase admittance formulation for a three-winding autotransformer: the full model $[A(N+1)]$.

The assumption that the voltage rating of the $(N+1)^{th}$ coil is equal to the one of the winding closest to the core is adopted from the Hybrid model.

The admittance formulation based on Fig. 3.7 is represented mathematically as in (3.4), where the expanded submatrices are calculated as

$$[A_A(N+1)] = [A_B(N+1)] = [A_C(N+1)] = \begin{bmatrix} a_{SS} & a_{SC} & a_{SD} & a_{S-Core} \\ a_{SC} & a_{CC} & a_{CD} & a_{C-Core} \\ a_{SD} & a_{CD} & a_{DD} & a_{D-Core} \\ a_{S-Core} & a_{C-Core} & a_{D-Core} & a_{Core-Core} \end{bmatrix} \quad (3.8)$$

The procedure for calculation of the leakage admittance matrix $[A(N+I)]$ is given in Subsection 2.2.6.3. The resistance matrix $[R(N+I)]$ should have the form in (2.64) but with three zeros on the main diagonal, each corresponding to the zero resistance of the $(N+1)^{th}$ coil in each phase.

The MATLAB code calculating the $[A(N+I)]$ and $[R(N+I)]$ matrices of the healthy Mirage is given in Appendix B, where the input parameters are

- resistances of the individual windings (Table 3.1),
- binary reactances between pairs of windings (Table 3.1 and formulas from Subsection 3.2.2.1),
- rated voltage of the individual windings (Table 3.1),
- common base power (usually 100 MVA).

3.3 Representation of the Shorted Turn Self-Reactance and the Core Effect

This section describes the faulted turn representation for modeling purposes and the effect which the magnetic core presence exerts on the delta current.

3.3.1 Representation of the Shorted Turn Self-Reactance

Consider magnetic and electric circuits of a two-winding transformer with a single T2T fault on the secondary side as shown in Fig. 3.8 a). According to Lenz's law, the fault current I_F creates a counter flux $\Phi_{counter}$ opposing the main flux Φ_{pri} and, therefore, causes an additional reduction of Φ_{pri} together with Φ_{sec} .

Consider now an electric circuit of the shorted turn as shown in Fig. 3.8 b) with the following circuit parameters: an induced electromotive force (emf) e_{ind_F} , a voltage drop V_{drop_F} , a turn resistance R_{turn} , and a turn self-reactance X_{L_turn} . The induced emf e_{ind_F} is responsible for the flow of current in the shorted turn. It is an active element. The voltage V_{drop_F} is a potential difference between two points. It is a passive element. The voltage between the shorted terminals A and B is zero:

$$V_{AB} = e_{ind_F} + V_{drop_F} = 0. \quad (3.9)$$

According to Faraday's law, an induced voltage in a coil is defined as

$$e_{ind} = -\frac{d\lambda}{dt} = -N \frac{d\Phi}{dt}, \quad (3.10)$$

where λ is a flux linked, N is a number of the coil turns, and Φ is the flux through the coil.

Fig. 3.8. A single T2T fault on the secondary side of a two-winding transformer:
a) electric and magnetic circuits, b) an electric circuit of the faulted turn.

In the faulted turn, an induced emf e_{ind_F} is close to its rated value of volts per turn e_{ind_rated} since insulation in the faulted turn does not break immediately: it takes time to evolve into a larger fault. The voltage drop existing across the R - L circuit of Fig. 3.8 b) is

$$V_{drop_F} = L_{turn} \frac{di_F}{dt} + R_{turn} i_F = -e_{ind_F} \approx -e_{ind_rated}, \quad (3.11)$$

where L_{turn} and R_{turn} are the self-inductance and resistance of the shorted turn. Therefore, the fault current in this turn is simply a ratio of the induced emf and impedance of the turn Z_{turn} :

$$I_F \approx \frac{e_{ind_F}}{Z_{turn}} \approx \frac{e_{ind_rated}}{R_{turn} + jX_{L_turn}}. \quad (3.12)$$

The shorted turn reactance X_{L_turn} is a complex function consisting of:

- the dominating self-inductance of the shorted turn, and
- the mutual inductances between the faulted turn and the healthy portions of the faulted coil as well as between the faulted turn and the healthy coils, refer to Subsection 2.2.2.1. The mutual inductances should be incorporated in the leakage admittance matrix.

The overall reactance value X_{L_turn} of the shorted turn depends on the fault location, proximity to the core and possible saturation of the core [42].

Mathematically, the shorted turn can be connected anywhere along the faulted coil, and Fig. 3.9 illustrates this visualization.

Fig. 3.9. A single shorted turn connected to the faulted winding.

The faulted portion inductance is

$$L = \frac{N^2}{\mathfrak{R}}, \quad (3.13)$$

where $\mathfrak{R}$ is a reluctance of the flux path. The more turns N are involved in the shorted portion, the higher the impedance of the shorted loop, and, therefore, the fault current reduces according to (3.12). Of course, the induced emf e_{ind_F} of the shorted portion also increases, but the overall effect from the increased number of turns involved in a fault is the fault current reduction: the increase of impedance outweighs the increase of the induced emf.

Theoretically, the highest fault current for T2T type of faults is when only one turn is shorted and ironically this is the type of a fault which protective relays cannot detect until it evolves into a larger fault. This is explained by the fact that even if the fault current of a single shorted turn is large, the transformer turns ratio almost does not change. Therefore, an increase in the phase currents is small, leading to only a slight current imbalance between the phases. This imbalance does not exceed the relay sensitivity setting [42].

Referring again Fig. 3.8 b), if the fault current amplitude of the shorted turn is available from the proprietary model, the self-reactance of the shorted turn X_{DNF} can be calculated from the steady-state form of (3.11):

$$X_{DNF} = \frac{V_{drop_F} - R_{turn} \cdot I_F}{I_F} = \frac{e_{ind_F} - R_{turn} \cdot I_F}{I_F} \approx \frac{e_{ind_rated} - R_{turn} \cdot I_F}{I_F}. \quad (3.14)$$

X_{DNF} describes a large fraction of the leakage flux produced due to a fault, which links neither with the healthy portions of the faulted coil nor with the healthy coils. This reactance should be a part of the faulted turn representation in the proposed model and can be placed outside the leakage admittance matrix. The subscript *DNF* stands for a “do-nothing” flux.

Perhaps, a more precise estimation of the shorted portion reactance can be achieved if the shorted portion (turn) is represented as an iron-core solenoid. Also, the GMD method used in [54], [60] might be applied for this estimation, refer to Subsection 2.2.4.4.

The other physical effects influencing the fault current in a shorted turn (portion) are the resistance increase due to a high fault current amplitude (refer to Subsection 2.2.2.3) and the fault arc. These are not considered to be a part of the proposed model, although both provide an additional effect on the fault current estimation.

According to [21], a simplistic evaluation of the fault current, with the assumption of the low-voltage side being open can be performed using the turns ratio for an autotransformer with reference to Fig. 3.10:

$$I_{short} = \left(1 + \frac{N_a + N_c}{N_b} \right) \cdot I_{line}. \quad (3.15)$$

Fig. 3.10. A shorted portion in a two-winding transformer with the low-voltage side open [21].

In this simplistic case, the core effect and the delta winding are not included.

For a T2G fault, consider Fig. 3.11. Connection of the secondary windings and the core effect are not important. For easier visualization, consider a wye-wye two-winding transformer grounded on both sides.

Fig. 3.11. A T2G fault in a wye-wye transformer.

By observation, a T2G fault is a T2T fault which is connected to ground. This is the difference between these two types of faults. Connection of a shorted turn (portion) to ground helps to detect these faults faster than T2T faults, since the fault current circulates through the neutral and any changes in the neutral current are monitored, refer to Subsection 2.1.1.1 for more details.

The more turns are involved in a T2G fault, the farther the fault is from the neutral point. As a consequence, the fault current reduces and the phase currents of the windings, that are on the same core leg as the faulted winding, experience more change. T2G faults that are close to the transformer neutral have lower driving voltage, but the fault current in the shorted loop is larger [42].

For T2G faults, the X_{DNF} value can also be estimated by (3.14).

3.3.2 The Core Effect in Three-Legged Transformers

A fault current creates an ampere-turn (AT) distortion on the core leg of the faulted phase. The delta winding produces a coupling effect between the faulted and healthy phases: an AT distortion of the faulted phase is transferred to the healthy phases. In three-legged transformers, the balancing ATs are created by the zero-sequence current flowing in the delta winding. This means that the fluxes sum to zero on the core leg of the faulted phase. The zero-sequence current circulating in the delta winding brings the balancing ATs of the faulted phase into the healthy phases and causes an overall AT change on the core legs of the healthy phases. However, as per [42], the fluxes in each core leg are balanced in three-legged transformers since ATs on any pair of core legs are always equal. This rule is commonly known as “ATs on each core leg should sum to zero in three-legged transformers.”

Addition of the iron core in five-legged transformers offers a low reluctance path for the zero-sequence flux and, according to (3.13), the zero-sequence inductance increases. In three-legged transformers, the iron core partially offers a low reluctance path for the zero-sequence flux, which closes through the air, oil, and tank. This still considerably increases the zero-sequence inductance compared to the pure air core. In three-legged transformers with the delta winding, the zero-sequence flux is supported by the delta current. If the zero-

sequence reactance is high, then the zero-sequence current (the delta current) is low and, consequently, the zero-sequence flux is low.

Therefore, the presence of an iron core offers less AT distortion during imbalances, e.g., during faults:

- the effect of an iron core is a reduced current circulating in the delta winding, and
- the effect of the delta winding is establishment of the balancing ATs between the phases during imbalances. This always causes fluxes to sum to zero on all core legs.

With an iron core and the delta winding, an AT distortion due to a fault is less, forcing the fault current in a shorted loop to be less compared to transformers with an air core and without the delta winding.

3.4 Internal Fault Representation

This section describes the development of the two approaches of the coil volume internal fault model: Cylindrical and Segmented. Either can be applied for formulation of the leakage admittance matrix of a faulted phase. The faulted phase contains only one faulted winding: series, common, or delta. Since the coil volume internal fault model is suggested for low-frequency studies, the capacitive effects are omitted except for T2G faults on the delta winding. In this case, the fault current closes through the inherent transformer capacitances because the delta winding has no connection to ground. The winding resistances are constant. Limitations of the coil volume internal fault leakage model, i.e., without the core representation, are described followed by the advantages of the coil volume internal fault full model, where the leakage representation is complemented with the core effect.

3.4.1 Assumptions for Portraying Internal Faults

Assumptions for the coil volume internal fault model are as follows:

- The insulation thickness is neglected.
- The cross-sectional area of the transformer coils is uniformly filled with the winding turns to form cylindrical one-layered uniform coils.
- All the coils have equal height.

To model T2T and T2G faults, assumptions should be established for representation of each type of fault.

3.4.1.1 T2T Faults

Consider a single T2T fault in a cylindrical one-layered uniform winding illustrated in Fig. 3.12, where the insulation thickness is neglected. Simulation of T2T faults requires

segmentation of the faulted coil into three portions connected in series. The middle portion $C2$ is shorted, and the portions $C1$ and $C3$ remain healthy. C stands for the common coil. This representation can be used for T2T faults on the common, series and delta coils.

Fig. 3.12. Segmentation of a cylindrical one-layered coil for a T2T fault.

3.4.1.2 T2G Faults

Again, consider a cylindrical one-layered uniform winding with the insulation thickness neglected. Simulation of T2G faults requires segmentation of the faulted coil into two portions. Fig. 3.13 illustrates series, common, and delta coils of a three-winding autotransformer, where each of the coils has a T2G fault. The representations in Fig. 3.13 can be used for T2G faults on the common, series and delta coils.

Fig. 3.13. Segmentation of cylindrical one-layered coils for a T2G fault in: a) a grounded common coil, b) an indirectly grounded series coil, c) an ungrounded delta coil.

For the beginning, consider a simple case of a T2G fault in the grounded cylindrical one-layered winding shown in Fig. 3.13 a). The coil is represented with two portions connected in series, where the bottom portion $C2$ is shorted to ground. The portion $C1$ remains

healthy, and C stands for the common coil. Here it is shown that one turn, closest to the neutral point, is shorted to ground. This is possible for the common coil; however, the series coil has an indirect connection to ground through the electrical connection with the grounded common coil as illustrated in Fig. 3.13 b). Therefore, a T2G fault on the series coil involves all the turns of the healthy common coil C and the corresponding faulted turns $S2$ of the series coil, where the portion $S1$ represents the healthy part of the series coil.

Although allowing the fault current to flow during T2G faults on the delta winding, coupling capacitances do not offer a direct connection to ground and, therefore, T2G fault currents on the delta winding are much lower than those on windings with a direct path to ground. For this reason, when modeling T2G faults on the delta winding, it is important to include capacitances in the model. For simulation of T2G faults on the delta winding, implementation of the capacitive effects estimated for the corresponding healthy transformer by the Hybrid model should be sufficient and details can be found in Subsection 2.2.2.2. For simulation of T2G faults on the series and common windings, capacitive effects are not important since the currents flowing through the leakage model are high.

Fig. 3.13 c) shows a faulted delta coil with the lumped capacitances: two on each coil end. The portion $D1$ remains healthy, and the portion $D2$ is shorted to ground. The capacitive fault current has several parallel paths to ground and Fig. 3.13 c) shows two.

3.4.1.3 Leakage Field Pattern Representation

In transformers, the magnetic field outside the magnetic materials forms the leakage field represented with the binary short-circuit reactances between the windings. For transient simulations, the active and reactive components of the binary impedances are separated. During an internal fault, the leakage field is formed by the healthy windings and two (T2G fault) or three (T2T fault) portions of the faulted winding.

As explained in Subsection 3.3.1, a shorted turn reactance is a complex function consisting of the dominating self-inductance of the shorted turn (portion) X_{DNF} and the mutual inductances between the faulted turn (portion) and the healthy portions of the faulted coil as well as between the faulted turn (portion) and the healthy coils. The self-reactance of the shorted turn (portion) X_{DNF} cannot be estimated as a part of the binary reactances by (2.22) and should be accounted for separately.

Hence, the transformer leakage field pattern during internal faults is formed by the superposition of two representations:

- The leakage field of the mutual inductances between all the pairs formed by the healthy coils and two (T2G fault) or three (T2T fault) portions of the faulted coil as if they all are healthy. This defines the binary short-circuit reactances and, consequently, the leakage admittance matrix $[A]$ of the faulted transformer.

- The individual leakage field of the shorted turn (portion) links neither with the healthy portions of the faulted coil nor with the healthy coils. The individual leakage field is interpreted as the “do-nothing” flux and described as a stand-alone inductor X_{DNF} located in series with the circuit elements of the shorted turn (portion). Its value depends on the fault location.

The short is applied outside the leakage admittance matrix across all the circuit elements of the dedicated turn (portion): the turn resistance, X_{DNF} , and the mutual reactance (a part of the leakage admittance matrix).

This pattern can be realized in two approaches: Cylindrical and Segmented. Each approach performs:

- segmentation of the cross-sectional area of a faulted coil (the vertical segmentation by the Cylindrical approach and the horizontal segmentation by the Segmented approach),
- estimates the binary reactances of the faulted phase.

The value of X_{DNF} is adjusted for each approach by matching the delta current from the coil volume internal fault model with that from the proprietary model. Ideally, a set of terminal currents from the Cylindrical approach should be similar to that from the Segmented approach. If the X_{DNF} element is neglected, then this difference should be considerable, and the Segmented approach should better predict the terminal currents.

3.4.2 Development of the Cylindrical Approach

The assumptions for this approach are as follows:

- the width of a faulted coil is proportional to the number of turns (the vertical segmentation of the faulted coil),
- this helps to represent the faulted coil as having two (T2G faults) or three (T2T faults) sub-coils with the height equal to the height of the healthy coils.

This set of assumptions helps to represent a faulted transformer as an N -coil transformer where the faulted phase and healthy phases have a different number of windings: the faulted phase has $N+1$ physically present windings for T2G faults and $N+2$ physically present winding for T2T faults. For example:

- a T2T fault on the common coil in a three-winding autotransformer leads to $N+2=5$ physically present windings in the faulted phase and $N=3$ physically present windings in the healthy phases, and
- a T2G fault on the common coil in a three-winding autotransformer leads to $N+1=4$ physically present windings in the faulted phase and $N=3$ physically present windings in the healthy phases.

A T2T fault on the common coil, shown above in Fig. 3.12, is represented by the Cylindrical approach as shown in Fig. 3.14. The faulted common coil consists of three sub-coils, where the middle sub-coil $C2$ is shorted. The healthy sub-coils $C1$ and $C3$ of Fig. 3.14 represent the healthy portions $C1$ and $C3$ of Fig. 3.12.

Fig. 3.14. Representation of T2T faults by the Cylindrical approach.

The T2G fault on the common coil, shown above in Fig. 3.13 a), is represented by the Cylindrical approach as shown in Fig. 3.15. The faulted common coil consists of two sub-coils, where the left sub-coil $C2$ is shorted to ground. The healthy sub-coil $C1$ of Fig. 3.15 represents the healthy portion $C1$ of Fig. 3.13 a).

Fig. 3.15. Representation of T2G faults by the Cylindrical approach.

Such a representation produces a set of parameters, not tightly corresponding to the actual leakage field during internal faults but is an attempt to represent the faulted phase in a simple way preserving some sort of correspondence to the fault. The fraction of the total leakage field corresponding to the leakage admittance matrix $[A]$ is underestimated since

this representation of a faulted winding does not correspond to the real segmentation of a winding during internal faults. As mentioned in Section 2.2.4.2.1, the fraction of the leakage field represented by $[A]$ has a dominating axial component for cylindrical windings. The assumption that the sub-coils of the faulted coil have equal height with each other and with the healthy windings underestimates the radial component represented in $[A]$. The X_{DNF} element offsets inaccuracy of $[A]$ due to the nature of the Cylindrical approach. Its value is adjusted through benchmarking with results from the proprietary model.

To include the radial component in binary reactances, the Rogowski factor for cylindrical windings K_{cyl} should be applied to increase the coil height from h to h_R as explained in Subsection 2.2.4.2.1. Since the sub-coils of the faulted coil are treated as individual coils with the height equal to the height of the healthy coils, the binary reactances for all the pairs are estimated using (2.24) or (2.46). The leakage admittance matrix $[A]$ is calculated by applying Dommel's method described in Subsection 2.2.6.3.

The mmf diagrams of the faulted common coil are shown in Fig. 3.16, both for T2T and T2G faults. The mmf diagrams for T2T and T2G faults on series and delta coils look similar.

Fig. 3.16. Mmf diagrams of the faulted common coil by the Cylindrical approach: a) T2T fault, b) T2G fault.

It is important to realize that these two diagrams do not represent the fault condition fully: they show a faulted coil with the vertical segmentation and consisting of sub-coils which form the binary leakage reactances. The full picture of any T2T or T2G fault is represented with the combination of:

- the binary leakage reactances from the complete mmf diagram,
- insertion of the X_{DNF} element in series with the circuit elements of the sub-coil intended to be shorted (sub-coil C2 both for T2T and T2G faults), and

- the short applied across all the circuit elements of the sub-coil $C2$.

3.4.2.1 *Different Fault Locations and Severity*

For T2T faults, both the fault location and fault severity can change. Modeling different locations of the faulted portion $C2$ along the coil height h of Fig. 3.12, as well as severity of a fault, is achieved by changing the width of each sub-coil. For T2G faults, the fault location and the fault severity are both the same: some number of turns counted from the neutral point is shorted to ground. By default, in this project, the neutral point is located at the coil bottom and, therefore, the severity of a T2G fault grows from the bottom of the coil to the top. Modeling the T2G fault severity of Fig. 3.13 is also achieved by changing the width of each sub-coil.

The width of each sub-coil is calculated in proportion to the number of turns involved. The simplified reality shown in Figs. 3.12 and 3.13, where $h \sim N$, is represented by the Cylindrical approach as $b \sim N$ and illustrated in Figs. 3.14 and 3.15, respectively, where h is the coil height, b is the coil width, and N is the total number of turns of the coil. Fig. 3.17 helps to visualize the changes in fault location and severity for T2T faults. Fig. 3.18 helps to visualize the changes in severity of T2G faults. Every change leads to a recalculation of the binary reactances and, therefore, $[A]$ should be updated.

3.4.2.2 *Advantages and Limitations*

The representation of T2T and T2G faults by the Cylindrical approach provides an inexact estimation of the real leakage field during these faults when the X_{DNF} element is excluded, but it is a simple tool for the initial estimation of the phase currents and the fault current as well as for understanding the overall modeling process. However, when the X_{DNF} element is included, the results are expected to be accurate.

With more statistics obtained for the values of the X_{DNF} element which is necessary for the accurate simulation of internal faults in transformers of different types and sizes, the Cylindrical approach seems promising because it offers a simple representation of a faulted transformer. Such statistics will also help to come up with an independent method for estimation of the X_{DNF} value based on the initial empirical data. An important attraction of the Cylindrical approach is its straightforward implementation in the ATP software.

On the other side, there is a limitation to this approach. A T2T fault current at a certain location in the lower half of a coil should be approximately equal to the T2T fault current at the symmetrical location in the upper half of the coil, i.e., the properties of two T2T faults symmetrically located along the horizontal middle line, dividing a coil into halves,

Fig. 3.17. Different fault locations and severities for T2T faults by the Cylindrical approach.

Fig. 3.18. Different fault severities for T2G faults by the Cylindrical approach.

should be equivalent. The binary reactances are symmetrically equal. Therefore, the currents for these two symmetrical T2T faults are approximately equal.

The main limitation of the Cylindrical approach is that it cannot replicate this behavior due to the vertical segmentation of a faulted coil. Using the vertical segmentation, the bottom-to-top fault transition is represented by the left-to-right fault transition, as shown in Figs. 3.17 and 3.18. Thus, this type of segmentation produces distinct sets of the binary reactances for symmetrical fault locations. This unsymmetrical behavior of the Cylindrical approach, hopefully, can be adjusted with the X_{DNF} parameter.

3.4.3 Development of the Segmented Approach

To overcome the unsymmetrical behavior of the Cylindrical approach, the Segmented approach should be considered. The assumptions for this approach are as follows:

- the height of a faulted coil is proportional to the number of turns (the horizontal segmentation of the faulted coil),
- this helps to represent the faulted coil as having two (T2G faults) or three (T2T faults) portions stacked one on the other.

The T2T fault on the common coil, shown in Fig. 3.12, is represented by the Segmented approach as shown in Fig. 3.19 with three portions, where the middle portion $C2$ is shorted. The healthy portions $C1$ and $C3$ of Fig. 3.19 represent the healthy portions $C1$ and $C3$ of Fig. 3.12.

Fig. 3.19. Representation of T2T faults by the Segmented approach.

The T2G fault on the common coil shown in Fig. 3.13 a) is represented by the Segmented approach as shown in Fig. 3.20 with two portions, where the bottom portion $C2$ is shorted to ground. The healthy portion $C1$ of Fig. 3.20 represents the healthy portion $C1$ of Fig. 3.13 a).

Fig. 3.20. Representation of T2G faults by the Segmented approach.

This representation resembles the reality of an internal fault more closely than the Cylindrical approach. However, it may not be valid to call the faulted phase an N -coil transformer as assumed for the Cylindrical approach. In the Segmented approach, segmentation of a faulted coil is similar to representation of a coil for higher order studies. Therefore, Dommel's method for N -coil transformers, described in Subsection 2.2.6.3, may not be applicable as used in the literature and as used in the Cylindrical approach. Additional research is needed to understand the proper application of Dommel's method and the underlying issues when one of the coils is horizontally segmented into two (T2G faults) or three (T2T faults) portions.

The binary reactances for the pairs of healthy coils are calculated in (2.24) or (2.46), but the binary reactances for the pairs where the portions of a faulted coil are involved can be estimated through calculation of ampere-turns (ATs). At that, both axial and radial ATs sum to zero (ATs between each pair of full windings sum to zero). Equation (3.16), where ATs are applied for calculation of the binary reactances of a healthy transformer, is a slight modification of (2.24) or (2.46) and shown in [24]:

$$X\% = \frac{X}{Z_b} \times 100 = \omega \frac{\mu_0 (NI)}{h_R \cdot (V/N)} \left(\frac{L_{mt,1} \cdot b_1 + L_{mt,2} \cdot b_2}{3} + L_{mt,air} \cdot b_{air} \right) \times 100. \quad (3.16)$$

Calculation of the binary reactances for the pairs where the portions of a faulted coil are involved is performed as a sum of two reactances. The first reactance represents the axial component (two cylindrical windings of equal height), and the second reactance represents the radial component (a sandwich winding made of two cylindrical coils and the gap in between) as described in Subsection 2.2.4.5.

To include the radial component, the Rogowski factor both for cylindrical windings K_{cyl} and sandwich windings K_{smw} should be applied to increase the coil height from h to h_R and the coil width from w to w_R as explained in Subsections 2.2.4.2.1, 2.2.4.2.2, and 2.2.4.5.

The Segmented approach offers a more realistic representation of the leakage field during internal faults and its results are expected to be closer to the results from the proprietary model compared to the Cylindrical approach. However, even this representation is not able to account for the considerable amount of leakage flux emanating from a shorted turn (portion). Therefore, here, this effect should also be compensated by insertion of the X_{DNF} element in series with circuit elements of the shorted turn (portion). In this case, its values are expected to be smaller than for the Cylindrical approach and should also be adjusted through benchmarking with results from the proprietary model.

The mmf diagrams of the faulted common coil are shown in Fig. 3.21, both for T2T and T2G faults. The mmf diagrams for T2T and T2G faults on series and delta coils look similar.

Fig. 3.21. Mmf diagrams of the faulted common coil by the Segmented approach:
a) T2T fault, b) T2G fault.

As in the case of the Cylindrical approach, here also it is important to remember that these two diagrams do not represent the fault condition fully: they show a faulted coil with the horizontal segmentation and consisting of portions which form the binary leakage reactances. The full picture of any T2T or T2G fault is represented with the combination of:

- the binary leakage reactances from the complete mmf diagram,
- insertion of the X_{DNF} element in series with the circuit elements of the portion intended to be shorted (portion C2 both for T2T and T2G faults), and
- the short applied across all the circuit elements of the portion C2.

3.4.3.1 Different Fault Locations and Severity

For T2T faults, both the fault location and fault severity can change. Modeling different locations of the faulted portion C2 along the coil height h of Fig. 3.12, as well as severity of a fault, is achieved by changing the height of each portion. For T2G faults, the severity

Fig. 3.22. Different fault locations and severities for T2T faults by the Segmented approach.

Fig. 3.23. Different fault severities for T2G faults by the Segmented approach.

of a fault grows from the coil bottom (the default neutral point) towards its top. Modeling the T2G fault severity of Fig. 3.13 is also achieved by changing the height of each portion.

The height of each portion is calculated in proportion to the number of turns involved. The simplified reality, where $h \sim N$, shown in Figs. 3.12 and 3.13 is exactly represented by the Segmented approach and illustrated in Figs. 3.19 and 3.20, respectively. Here, h is the coil height, b is the coil width, and N is the total number of turns of the coil. Fig. 3.22 helps to visualize the changes in the fault location and severity for T2T faults. Fig. 3.23 helps to visualize the changes in severity of T2G faults. Every change leads to a recalculation of the binary reactances and, therefore, $[A]$ should be updated.

3.4.3.2 *Advantages and Limitations*

The Segmented approach overcomes the limitation of the Cylindrical approach and avoids the Cylindrical approach's unsymmetrical representation of faults.

The representation of T2T and T2G faults by the Segmented approach provides a better estimation of the real leakage field during faults than the Cylindrical approach. The results without application of the X_{DNF} element are expected to be closer to the results from the proprietary model than in the case of the Cylindrical approach. However, for tuning, the X_{DNF} element is also required for the Segmented approach.

The Segmented approach can be further extended for estimation of the phase currents and fault current during T2T and T2G faults for the cases when one winding is represented with two coils. For example, the common winding can be represented as a combination of the main coil and the tap coil, or when a winding has built-in taps in the middle, which is typical for the series winding.

Here, similar to the Cylindrical approach, more statistics should be collected on the values of the X_{DNF} element, which is necessary for the accurate simulation of internal faults in transformers of different types and sizes. Since additional research is needed to verify the applicability of Dommel's method to the Segmented approach, it is early to determine whether the implementation in ATPDraw will be a straightforward task.

3.4.3.3 *Issues Encountered and Observations*

Initial implementation of the Segmented approach revealed the following issues:

- The binary reactances for the coil pairs where the portions of a faulted coil are involved can be calculated through estimation of ATs. However, ready-made formulas are not available in the literature. The preliminary formulas for these calculations might be inaccurate and require further research. This is beyond the scope of the initial project development. Meanwhile, the preliminary formulas for estimation of the binary reactances through ATs are shown in this chapter.

- Dommel's method, described in Subsection 2.2.6.3, requires further exploration concerning its applicability to the Segmented approach due to the horizontal segmentation of a faulted coil. Dommel's method has been developed for N -coil transformers, and additional research should be carried out to determine whether the horizontal segmentation of one of the coils still keeps the faulted transformer in the same category.

The vision for the development of the Segmented approach is presented later in this chapter. Due to the issues described above, the Segmented approach is not implemented in this dissertation.

3.4.4 Highlights

For development of any new model, benchmarking with a reliable model or verification with test cases on a real transformer is desired.

Both the Cylindrical and Segmented approaches estimate the binary reactances of a faulted transformer based on segmentation of the cross-sectional area of a faulted coil: vertical for the Cylindrical approach and horizontal for the Segmented approach. Both the Cylindrical and Segmented approaches require limited data about a transformer: test report data and the core window information (widths of the core legs, widths of the windings, air gaps, the average height of the windings, and the number of turns in each winding). Both methods have potential for accurate prediction of currents during internal faults. With sufficient empirical data and defined rules on estimation of the values for the X_{DNF} element for each approach, the resulting models will yield simple and handy tools for simulation of T2T and T2G faults in transformers of different types and sizes.

Overall representation of the leakage field during internal faults for each approach consists of the components: the leakage admittance matrix and the self-inductance of the shorted turn X_{DNF} . The short is applied outside of the leakage admittance matrix across all the circuit elements of the shorted turn: resistance, X_{DNF} , and the mutual inductance.

The main advantage of the Cylindrical approach is the straightforward implementation in ATP. It is not yet known if the Segmented approach can be simply implemented in ATP.

The logical progression from easy to more complex development of the coil volume internal fault model is the following sequence:

- the Cylindrical approach without the core circuit,
- the Cylindrical approach with the core circuit,
- the Segmented approach without the core circuit,
- the Segmented approach with the core circuit.

The leakage admittance matrix of a faulted phase generated by each approach is substantially different. Insertion of the X_{DNF} element should produce phase currents of

similar amplitudes, but the fault current amplitude is distinct for each approach. The importance of an accurate prediction of the fault current amplitude is lower compared to that of the phase currents. This is because the phase currents are used by the protection relay and they must be of accurate prediction for relay performance studies.

The Cylindrical approach offers a rough estimate of the leakage field during internal faults, but it is an important intermediate step in advancing development of the coil volume internal fault model. The Segmented approach offers an improved estimate of the leakage field during internal faults and, therefore, better estimation of the phase and fault currents is expected. This approach can be advanced further to include more complex transformer windings.

The advantages and limitations of the Cylindrical approach are described in Subsection 3.4.2.2, and of the Segmented approach in Subsection 3.4.3.2. The issues encountered during the initial development of the Segmented approach are described in Subsection 3.4.3.3.

The complete mmf diagrams of the faulted phase and calculation of the binary reactances are given for each approach in the following subsections of this chapter. Internal faults are applied only to one winding at a time in a faulted phase: series, common, or delta. The faulted phase is chosen to be the phase A for all the cases.

The schematic of the input/output for the proposed coil volume internal fault model is illustrated in Fig. 3.24.

Fig. 3.24. Schematic of the input/output for the proposed coil volume internal fault model.

3.4.5 Core Model Omitted versus Core Model Included

The coil volume internal fault model without the core representation is the first step towards obtaining the overall model with either approach. At this stage, the initial benchmarking of the coil volume internal fault model can be performed. Initial estimates of the phase currents and fault current can be obtained.

The coil volume internal fault model and the model for healthy transformers with the core representation omitted possess the same properties:

- built from the positive sequence reactances,
- interphase coupling is omitted,
- only the leakage field between the windings is represented,
- the delta winding effect is included,
- the core effect is omitted.

The coil volume internal fault model and the model for healthy transformers with the core representation included possess the same properties:

- built from the positive sequence reactances,
- interphase coupling is included (the zero-sequence flux paths are located in the core circuit),
- leakage field between the windings and the core is represented,
- the delta winding effect is included,
- the core effect is included.

Attachment of the core circuit to the leakage representation should reduce the delta current and, consequently, lead to a lower fault current amplitude for each approach, refer to Subsection 3.3.2 for more details.

The developed coil volume internal fault model is suggested for low-frequency studies and, therefore, the capacitive effects (except for T2G faults in the delta winding) and the frequency dependency of the winding resistances are omitted.

3.5 The Coil Volume Internal Fault Model for T2T Faults

The T2T faults are applied in phase A on each winding: common, series and delta. The other two phases B and C remain healthy and are represented as described in Subsection 3.2.1. The formulas for calculation of the binary reactances in the faulted phase and the corresponding mmf diagrams are given for both the Cylindrical and Segmented approaches. This formulation is given in detail for the common coil. Formulations for the series and delta coils are shown in Appendix C.

3.5.1 Leakage Representation

The leakage representation is the first step in development and implementation of the coil volume internal fault model for simulation of T2T faults. The core circuit is omitted.

3.5.1.1 T2T Faults on the Common Coil

Fig. 3.25 illustrates the leakage model for simulation of T2T faults on the common coil of phase A. The common coil is sectioned into three portions, and the middle portion $C2$, in series with the X_{DNF} element, is shorted to represent T2T faults. For this case, the number of windings is $N = 5$ in the faulted phase and $N = 3$ in the healthy phases. The elements within the dashed rectangles in each phase describe the phase leakage admittance submatrices $[A_A(N)]$, $[A_B(N)]$, and $[A_C(N)]$ of the total leakage admittance matrix $[A(N)]$. These submatrices are independent of each other since the core model is not included at this stage.

If the X_{DNF} element is removed from the circuit and the short is not applied, the model should produce the short-circuit behavior of the healthy Mirage and, thus, confirm that segmentation is performed correctly. If this step is completed successfully, then the next step is an attachment of the core circuit. The coil volume internal fault leakage model based on Fig. 3.25 includes only the positive-sequence behavior.

Fig. 3.25. Simulation of T2T faults on the phase A common coil of the Mirage: the leakage model $[A(N)]$.

3.5.1.1.1 Calculation of the Binary Reactances

Referring to Section 3.4, the leakage field during a T2T fault can be represented by the Cylindrical or Segmented approaches. Each leads to a distinct set of the binary leakage reactances as described below. The mmf distribution diagrams for each approach describe the binary leakage reactances from which the corresponding leakage admittance matrix $[A(N)]$ is calculated and neither matrix incorporates the effect of X_{DNF} . The fault condition is simulated with insertion of the X_{DNF} element in series with the circuit elements of the

portion C2 in the implemented circuit. Application of the short across all the circuit elements of the portion C2 outside of $[A(N)]$ is illustrated in Fig. 3.25.

The number of windings in the faulted phase $N = 5$ defines the number of the elements on the main diagonal of $[A_A(N)]$. The number of binary reactances in the faulted phase A, which corresponds to the number of the off-diagonal elements m in $[A_A(N)]$, is calculated by (3.17), leading to 10 binary reactances in the faulted phase A for this case:

$$m = \frac{1}{2} N(N-1). \quad (3.17)$$

To keep track of the reactance reference, the binary reactances in this dissertation are always referred to the winding whose designation comes first, e.g., X_{SD} is the binary reactance between the series and delta windings and it is referred to the series winding.

A. The Cylindrical approach

The related mmf distribution diagram of the faulted phase A, where the common coil is sectioned for simulation of T2T faults, is illustrated in Fig. 3.26. Most of the data used to calculate the binary short-circuit reactances are given in Fig. 3.1. The rest of the data, i.e., the width of each sub-coil C1, C2, and C3, is defined by the number of turns involved and depends on the fault location.

Fig. 3.26. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the common coil: the leakage model $[A(N)]$, the Cylindrical approach.

Equations (3.18) – (3.27), describing the mmf distribution diagram of Fig. 3.26 mathematically, are used to calculate the set of the binary reactances in phase A for the Cylindrical approach. The Rogowski factor for cylindrical windings K_{cyl} , as per (2.25), is

included. K_{cyl} can be omitted (if neglecting the radial component of the leakage flux) by substituting the modified coil height h_R for the actual coil height h , refer to Subsection 2.2.4.2.1 for more details.

$$X_{SC1} = \omega\mu_0 \frac{N_S^2}{h_{R_SC1}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,C1} \cdot b_{C1}}{3} + L_{mt,a3} \cdot a_3 \right), \quad (3.18)$$

$$X_{SC2} = \omega\mu_0 \frac{N_S^2}{h_{R_SC2}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,C2} \cdot b_{C2}}{3} + L_{mt,C1} \cdot b_{C1} + L_{mt,a3} \cdot a_3 \right), \quad (3.19)$$

$$X_{SC3} = \omega\mu_0 \frac{N_S^2}{h_{R_SC3}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,C3} \cdot b_{C3}}{3} + L_{mt,C1} \cdot b_{C1} + \right. \\ \left. L_{mt,C2} \cdot b_{C2} + L_{mt,a3} \cdot a_3 \right), \quad (3.20)$$

$$X_{SD} = \omega\mu_0 \frac{N_S^2}{h_{R_SD}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,D} \cdot b_D}{3} + L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right), \quad (3.21)$$

$$X_{C1C2} = \omega\mu_0 \frac{N_{C1}^2}{h_{R_C1C2}} \left(\frac{L_{mt,C1} \cdot b_{C1} + L_{mt,C2} \cdot b_{C2}}{3} \right), \quad (3.22)$$

$$X_{C1C3} = \omega\mu_0 \frac{N_{C1}^2}{h_{R_C1C3}} \left(\frac{L_{mt,C1} \cdot b_{C1} + L_{mt,C3} \cdot b_{C3}}{3} + L_{mt,C2} \cdot b_{C2} \right), \quad (3.23)$$

$$X_{C1D} = \omega\mu_0 \frac{N_{C1}^2}{h_{R_C1D}} \left(\frac{L_{mt,C1} \cdot b_{C1} + L_{mt,D} \cdot b_D}{3} + L_{mt,C2} \cdot b_{C2} + \right. \\ \left. L_{mt,C3} \cdot b_{C3} + L_{mt,a2} \cdot a_2 \right), \quad (3.24)$$

$$X_{C2C3} = \omega\mu_0 \frac{N_{C2}^2}{h_{R_C2C3}} \left(\frac{L_{mt,C2} \cdot b_{C2} + L_{mt,C3} \cdot b_{C3}}{3} \right), \quad (3.25)$$

$$X_{C2D} = \omega\mu_0 \frac{N_{C2}^2}{h_{R_C2D}} \left(\frac{L_{mt,C2} \cdot b_{C2} + L_{mt,D} \cdot b_D}{3} + L_{mt,C3} \cdot b_{C3} + L_{mt,a2} \cdot a_2 \right), \quad (3.26)$$

$$X_{C3D} = \omega\mu_0 \frac{N_{C3}^2}{h_{R_C3D}} \left(\frac{L_{mt,C3} \cdot b_{C3} + L_{mt,D} \cdot b_D}{3} + L_{mt,a2} \cdot a_2 \right), \quad (3.27)$$

where $\omega=2\pi f$, f is the power frequency and the reactances have units of Ohms.

Equations (3.18) – (3.27) form the set of the binary reactances estimated by the Cylindrical approach for the coil volume internal fault leakage model for simulation of T2T faults on the common coil without the core effect.

B. The Segmented approach

It is necessary to include both the axial and radial components of the leakage field in estimation of the binary reactances since, in this case, the radial component is not negligible. Therefore, the Rogowski factors both for cylindrical and sandwich windings, K_{cyl} and K_{snw} respectively, should be applied. The related mmf distribution diagrams of the faulted phase A, where the common coil is sectioned for simulation of T2T faults, are illustrated in Figs. 3.27 – 3.33. Most of the data used to calculate the binary short-circuit reactances are given in Fig. 3.1. The rest of the data, i.e., the height of each portion $C1$, $C2$, and $C3$, is defined by the number of turns involved.

Fig. 3.27 shows the mmf distribution for four pairs of windings in the faulted phase A: between the healthy series S and delta D coils, between the portions $C1$ and $C2$, $C1$ and $C3$, $C2$ and $C3$ of the faulted common coil.

Fig. 3.27. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the common coil: the leakage model $[A(N)]$, the Segmented approach – SD , $C1C2$, $C1C3$, and $C2C3$ pairs.

The binary reactance between the healthy series and delta coils is estimated by (3.21), where the Rogowski factor for cylindrical windings K_{cyl} is calculated by (2.25). The reactances between the portions $C1$ and $C2$, $C1$ and $C3$, $C2$ and $C3$ are estimated for sandwich windings by (2.28) leading to (3.28) – (3.30). The Rogowski factor for sandwich windings K_{snw} is calculated by (2.30) and incorporated in the coil width w_R by expanding the original coil width w as in Fig. 2.9:

$$X_{C1C2} = \omega\mu_0 \frac{N_{C1}^2 L_{mt,C}}{w_{R_C1C2}} \left(\frac{h_{C1} + h_{C2}}{3} \right), \quad (3.28)$$

$$X_{C1C3} = \omega\mu_0 \frac{N_{C1}^2 L_{mt,C}}{w_{R_C1C3}} \left(\frac{h_{C1} + h_{C3}}{3} + h_{C2} \right), \quad (3.29)$$

$$X_{C2C3} = \omega\mu_0 \frac{N_{C2}^2 L_{mt,C}}{w_{R_C2C3}} \left(\frac{h_{C2} + h_{C3}}{3} \right). \quad (3.30)$$

Fig. 3.28 shows the mmf distribution between the healthy series coil S and the portion CI of the faulted common coil. In this case, the two windings are of unequal heights which produces a higher radial leakage flux compared to the case of windings with equal heights. Then, the binary reactance is estimated according to Fig. 2.10 through calculation of ATs, leading to the set of equations (3.31) – (3.34).

Fig. 3.28. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the common coil: the leakage model $[A(N)]$, the Segmented approach – SCI pair.

The axial component X_{A_SCI} of the binary reactance between S and CI is calculated as if they have full and equal heights. Since this reactance is referred to the series winding S , it is equal to the binary reactance between the healthy S and C windings estimated by (3.1).

The radial component X_{B_SCI} of the binary reactance between S and CI is estimated as for sandwich winding keeping in mind the fact that ATs density is constant across each winding cross-section and ATs of the coil pair sum to zero. This sandwich winding is formed by the two coils of equal height and the air gap in between. The added ATs, which make the portion CI have the full height, are calculated as a proportion: ATs of the healthy coil cross-section versus ATs related to the cross-section of the addition:

$$ATs_{added} = \frac{\text{full ATs} \times \text{cross-section of ATs}_{added}}{\text{full cross-section}}. \quad (3.31)$$

Volts per turn in the faulted coil are calculated as if it is a healthy coil, since if only one turn is shorted, they are equal, refer to Section 3.3. Then, (3.16) is modified to correspond to a sandwich coil as per (2.28) and applied to calculate the radial component of the binary reactance between S and CI in percent:

$$X_{B_SCI} \% = \omega \frac{\mu_0 \cdot ATs_{added\ C1}}{w_{R_SCI} \cdot (V_C / N_C)} \times 100 \times \frac{L_{mt,a3} \cdot h}{3}. \quad (3.32)$$

Equation (3.32) uses the data of the faulted common coil, but its result in Ohms should be referred to the healthy series coil to correspond to the *SC1* designation. This is achieved by multiplying the result of (3.32) with the base impedance of the healthy series coil:

$$X_{B_SC1\ ref\ S} = X_{B_SC1} \% \times \frac{kV_S^2}{MVA_S}. \quad (3.33)$$

Therefore, the total binary reactance between *S* and *C1* is

$$X_{SC1} = X_{A_SC1} + X_{B_SC1\ ref\ S} = X_{SC} + X_{B_SC1\ ref\ S}. \quad (3.34)$$

It is important to remember that the coil voltage and current should be used, and not the terminal values. Also, the MVA value should be the MVA value of the winding pair, i.e., the MVA values for the pairs such as series – common coils, series – delta coils, and common – delta coils are all different and this should be kept in mind when calculating the base impedances.

The process for calculation of the binary reactance between *S* and *C3* is similar to that between *S* and *C1*. The only difference is that the added ATs have a different value since the corresponding cross-section is different. Fig. 3.29 shows the mmf distribution diagram for the *SC3* pair.

Fig. 3.29. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the common coil: the leakage model $[A(N)]$, the Segmented approach – *SC3* pair.

Therefore, equations (3.35) – (3.37) for calculation of the binary reactance between *S* and *C3* are as follows:

$$X_{B_SC3} \% = \omega \frac{\mu_0 \cdot ATs_{added\ C3}}{w_{R_SC3} \cdot (V_C / N_C)} \times \frac{L_{mt,a3} \cdot h}{3} \times 100, \quad (3.35)$$

$$X_{B_SC3\ ref\ S} = X_{B_SC3} \% \times \frac{kV_S^2}{MVA_S}, \quad (3.36)$$

$$X_{SC3} = X_{A_SC3} + X_{B_SC3\ ref\ S} = X_{SC} + X_{B_SC3\ ref\ S}. \quad (3.37)$$

Fig. 3.30 shows the mmf distribution between the portion $C1$ of the faulted common coil and the healthy delta coil D .

Fig. 3.30. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the common coil: the leakage model $[A(N)]$, the Segmented approach – $C1D$ pair.

The process for calculating the binary reactance between $C1$ and D is similar to the one between S and $C1$, with the following differences:

- The axial component X_{A_C1D} is referred to the portion $C1$ to correspond to the $C1D$ designation. Therefore, X_{A_C1D} cannot be equal to the binary reactance X_{CD} . The axial component X_{A_C1D} is calculated for the two coils of full and equal heights by (2.24).
- ATs for this pair are different from ATs of the other pairs, and thus, the added ATs to make the portion $C1$ of the full height are different from the ones of the $SC1$ pair.
- The air gap between $C1$ and D is different from the one between S and $C1$.

Therefore, equations (3.38) – (3.40) should be applied to calculate the binary reactance between $C1$ and D :

$$X_{B_C1D} \% = \omega \frac{\mu_0 \cdot ATs_{added\ C1}}{w_{R_C1D} \cdot (V_C / N_C)} \times \frac{L_{mt,a2} \cdot h}{3} \times 100, \quad (3.38)$$

$$X_{B_C1D\ ref\ C1} = X_{B_C1D} \% \times \frac{kV_{C1}^2}{MVA_D}, \quad (3.39)$$

$$X_{C1D} = X_{A_C1D} + X_{B_C1D \text{ ref } C1}. \quad (3.40)$$

The process for calculation of the binary reactance between $C3$ and D is similar to the one between $C1$ and D with the only difference that the added ATs have a different value since the corresponding cross-section is different. Fig. 3.31 shows the mmf distribution diagram for the $C3D$ pair.

Fig. 3.31. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the common coil: the leakage model $[A(N)]$, the Segmented approach – $C3D$ pair.

Therefore, equations (3.41) – (3.43) for calculation of the binary reactance between $C3$ and D are as follows:

$$X_{B_C3D} \% = \omega \frac{\mu_0 \cdot ATs_{added\ C3}}{w_{R_C3D} \cdot (V_C / N_C)} \times \frac{L_{mt,a2} \cdot h}{3} \times 100, \quad (3.41)$$

$$X_{B_C3D \text{ ref } C3} = X_{B_C3D} \% \times \frac{kV_{C3}^2}{MVA_D}, \quad (3.42)$$

$$X_{C3D} = X_{A_C3D} + X_{B_C3D \text{ ref } C3}. \quad (3.43)$$

For the cases, when the portion $C2$ is involved, this estimation becomes slightly more complex since this portion is not located at either coil end. Fig. 3.32 shows the mmf distribution between the healthy series coil S and the portion $C2$ of the faulted common coil. In this case, the radial component is represented with a sandwich winding consisting of two units of different heights, and their reactances are calculated separately: X_{B_SC2} (corresponds to the top unit) and X_{C_SC2} (corresponds to the bottom unit).

Fig. 3.32. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the common coil: the leakage model $[A(N)]$, the Segmented approach – SC2 pair.

The axial component X_{A_SC2} is also equal to the binary reactance between the healthy S and C windings estimated by (3.1), the way it was established for the reactances X_{SC1} and X_{SC3} .

Equations (3.44) – (3.48) are applied to calculate the binary reactance between S and $C2$:

$$X_{B_SC2} \% = \omega \frac{\mu_0 \cdot AT_s^t \text{ added } c2}{w_{R_SC2}^t \cdot (V_C / N_C)} \times \frac{L_{mt,a3} \cdot (h_{C1} + h_{C2} / 2)}{3} \times 100, \quad (3.44)$$

$$X_{B_SC2 \text{ ref } S} = X_{B_SC2} \% \times \frac{kV_S^2}{MVA_S}, \quad (3.45)$$

$$X_{C_SC2} \% = \omega \frac{\mu_0 \cdot AT_s^b \text{ added } c2}{w_{R_SC2}^b \cdot (V_C / N_C)} \times 100 \times \frac{L_{mt,a3} \cdot (h_{C3} + h_{C2} / 2)}{3}, \quad (3.46)$$

$$X_{C_SC2 \text{ ref } S} = X_{C_SC2} \% \times \frac{kV_S^2}{MVA_S}, \quad (3.47)$$

$$X_{SC2} = X_{A_SC2} + X_{B_SC2 \text{ ref } S} + X_{C_SC2 \text{ ref } S} = X_{SC} + X_{B_SC2 \text{ ref } S} + X_{C_SC2 \text{ ref } S}, \quad (3.48)$$

where superscripts t and b mean top and bottom, respectively.

Fig. 3.33 shows the mmf distribution between the portion $C2$ of the faulted common coil and the healthy delta coil D . Here, the radial component is also represented with a sandwich winding consisting of two units of different heights, and their reactances are calculated separately: X_{B_C2D} (corresponds to the top unit) and X_{C_C2D} (corresponds to the bottom unit).

Fig. 3.33. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the common coil: the leakage model $[A(N)]$, the Segmented approach – C2D pair.

The process for calculation of the binary reactance between C2 and D is similar to the one between S and C2, with the following differences:

- The axial component X_{A_C2D} is referred to the portion C2 that corresponds to the C2D designation. Therefore, X_{A_C2D} cannot be equal to the binary reactance X_{CD} . The axial component X_{A_C2D} is estimated for two coils of full and equal heights by (2.24).
- ATs for this pair are different from ATs of the other pairs and, thus, the added ATs on the top and bottom to make the portion C2 of the full height are different from those of the SC2 pair.
- The air gap between C2 and D is different from the one between S and C2.

Therefore, equations (3.49) – (3.53) are applied to calculate the binary reactance between C2 and D:

$$X_{B_C2D} \% = \omega \frac{\mu_0 \cdot ATs_{added\ C2}^t}{w_{R_C2D}^t \cdot (V_C/N_C)} \times \frac{L_{mt,a2} \cdot (h_{c1} + h_{c2}/2)}{3} \times 100, \quad (3.49)$$

$$X_{B_C2D\ ref\ C2} = X_{B_C2D} \% \times \frac{kV_{C2}^2}{MVA_D}, \quad (3.50)$$

$$X_{C_C2D} \% = \omega \frac{\mu_0 \cdot ATs_{added\ C2}^b}{w_{R_C2D}^b \cdot (V_C/N_C)} \times 100 \times \frac{L_{mt,a2} \cdot (h_{c3} + h_{c2}/2)}{3}, \quad (3.51)$$

$$X_{C_C2D\ ref\ C2} = X_{C_C2D} \% \times \frac{kV_{C2}^2}{MVA_D}, \quad (3.52)$$

$$X_{C2D} = X_{A_C2D} + X_{B_C2D\ ref\ C2} + X_{C_C2D\ ref\ C2}, \quad (3.53)$$

where superscripts t and b mean top and bottom, respectively.

Equations (3.28) – (3.53) form the set of the binary reactances estimated by the Segmented approach for the coil volume internal fault leakage model for simulation of T2T faults on the common coil without the core effect.

3.5.1.1.2 $[A(N)]$ Formulation

The leakage admittance representation of the faulted phase is formulated from the set of binary reactances estimated by either the Cylindrical or Segmented approach. The two sets of binary reactances produce different numbers. However, both lead to the same circuit as illustrated in Fig. 3.34. This circuit represents the faulted phase A leakage admittance submatrix $[A_A(N)]$ for the cases where T2T faults are applied on the common coil. The coil volume internal fault leakage model consists of the leakage representation only. The related turns ratios are included.

Fig. 3.34. Admittance formulation of the faulted phase A for simulation of T2T faults on the common coil: the leakage model $[A(N)]$.

The admittance formulation based on Fig. 3.34 is represented mathematically by (3.4), where the healthy submatrices $[A_B(N)]$ and $[A_C(N)]$ are defined as shown in (3.4) and the faulted phase submatrix $[A_A(N)]$ is defined as

$$[A_A(N)] = \begin{bmatrix} a_{SS} & a_{SC1} & a_{SC2} & a_{SC3} & a_{SD} \\ a_{SC1} & a_{C1C1} & a_{C1C2} & a_{C1C3} & a_{C1D} \\ a_{SC2} & a_{C1C2} & a_{C2C2} & a_{C2C3} & a_{C2D} \\ a_{SC3} & a_{C1C3} & a_{C2C3} & a_{C3C3} & a_{C3D} \\ a_{SD} & a_{C1D} & a_{C2D} & a_{C3D} & a_{DD} \end{bmatrix}. \quad (3.54)$$

The procedure for calculation of the leakage admittance matrix $[A(N)]$ from the binary reactances is given in Subsection 2.2.6.3. The resistance matrix $[R(N)]$ is a diagonal matrix similar to (2.64) and representative of the resistances for all the windings in each phase: 5 in the faulted phase and 3 in the healthy phases. The resistances of the portions $C1$, $C2$, and $C3$ are calculated in proportion to the number of turns involved.

3.5.1.2 T2T Faults on the Series Coil

Discussions in Subsection 3.5.1.1 are valid here, as well, with the difference that T2T faults are applied on the phase A series coil. All the related figures and equations can be found in Appendix C.

3.5.1.3 T2T Faults on the Delta Coil

Discussions in Subsection 3.5.1.1 are valid here, as well, with the difference that T2T faults are applied on the phase A delta coil. All the related figures and equations can be found in Appendix C.

3.5.2 The Full Model for T2T Faults

The main distinction between the full model and the leakage model, described in Subsection 3.5.1, is the presence of the artificial $(N+1)^{th}$ coil located on the core leg surface in each phase. This means that in the coil volume internal fault full model, each per phase set of the mutually coupled inductances, representing the existing windings in each phase, has an additional mutually coupled inductance. This additional inductance is not associated with a physically present winding but is formulated mathematically to serve as an interface between the core model and the leakage model. This solution is adopted from the Hybrid model.

The core model, adopted for development and implementation of the coil volume internal fault full model for simulation of T2T faults, is the existing core representation implemented in the Hybrid model of ATPDraw starting from v.6.1p8. This core model is described in Subsection 2.4.2. The voltage rating and the number of turns of the artificial $(N+1)^{th}$ coil are equal to those of the winding closest to the core, i.e., of the healthy delta winding.

3.5.2.1 T2T Faults on the Common Coil

Fig. 3.35 illustrates the full model for simulation of T2T faults on the phase A common coil. This coil is sectioned into three portions, and the middle portion $C2$, in series with the X_{DNF} element, is shorted to represent T2T faults. For this case, the number of windings is $N+1=6$ in the faulted phase and $N=4$ in the healthy phases. The elements within the dashed rectangles in each phase describe the phase leakage admittance submatrices $[A_A(N+1)]$, $[A_B(N+1)]$, and $[A_C(N+1)]$ of the total leakage admittance matrix $[A(N+1)]$. These

3.5.2.1.1 Calculation of the Binary Reactances

Two sets of equations for estimation of the binary reactances, defined by the Cylindrical and Segmented approaches, for the coil volume internal fault full model are given below. The mmf distribution diagrams for each approach describe the binary leakage reactances from which the corresponding leakage admittance matrix $[A(N+I)]$ is calculated and neither matrix incorporates the effect of X_{DNF} . The fault condition is simulated by insertion of the X_{DNF} element in series with the circuit elements of the portion C2 in the implemented circuit. Application of the short across all the circuit elements of the portion C2 outside of $[A(N+I)]$ is shown in Fig. 3.35. It is expected that incorporation of the core effect should reduce the delta current amplitude and, consequently, reduce the fault current level compared to the air-core model, refer to Subsection 3.3.2.

The number of windings in the faulted phase $N+1=6$ defines the number of the elements on the main diagonal of $[A_A(N+1)]$. The number of the binary reactances in the faulted phase A which corresponds to the number of the off-diagonal elements m in $[A_A(N+1)]$ is calculated by (3.17), leading to 15 binary reactances for this case.

To keep track of the reactance reference, the binary reactances in this dissertation are always referred to the winding whose designation comes first, e.g., $X_{C1-Core}$ is the binary reactance between the portion C1 and the artificial $(N+1)^{th}$ coil and it is referred to the portion C1. The width of the $(N+1)^{th}$ coil is zero and, thus, all the equation components with this thickness vanish.

A. The Cylindrical approach

The related mmf distribution diagram of the faulted phase A, where the common coil is sectioned for simulation of T2T faults and the $(N+1)^{th}$ coil is present, is illustrated in Fig. 3.36. Most of the data used to calculate the binary short-circuit reactances are given in Fig. 3.1. The rest of the data, i.e., the width of each sub-coil C1, C2, and C3, is defined by the number of turns involved.

The mutual coupling between each physically present winding in phase A and the artificial $(N+1)^{th}$ winding defines the difference between Figs. 3.36 and 3.26, and the rest is the same. Therefore, equations (3.18) – (3.27) are also valid for Fig. 3.36 also. The new $N=5$ equations, describing the binary reactances related to the $(N+1)^{th}$ coil, are given in (3.55) – (3.59). The Rogowski factor for cylindrical windings K_{cyl} , as per (2.25), is included and can be omitted (if neglecting the radial component of the leakage flux) by substituting the modified coil height h_R for the actual coil height h , refer to Subsection 2.2.4.2.1 for more details.

Fig. 3.36. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the common coil: the full model $[A(N+1)]$, the Cylindrical approach.

$$X_{S-Core} = \omega\mu_0 \frac{N_S^2}{h_R} \left(\frac{L_{mt,S} \cdot b_S}{3} + L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D + \frac{L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3}{3} \right), \quad (3.55)$$

$$X_{C1-Core} = \omega\mu_0 \frac{N_{C1}^2}{h_R} \left(\frac{L_{mt,C1} \cdot b_{C1}}{3} + L_{mt,C2} \cdot b_{C2} + L_{mt,C3} \cdot b_{C3} + \frac{L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2}{3} \right), \quad (3.56)$$

$$X_{C2-Core} = \omega\mu_0 \frac{N_{C2}^2}{h_R} \left(\frac{L_{mt,C2} \cdot b_{C2}}{3} + L_{mt,C3} \cdot b_{C3} + L_{mt,D} \cdot b_D + \frac{L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2}{3} \right), \quad (3.57)$$

$$X_{C3-Core} = \omega\mu_0 \frac{N_{C3}^2}{h_R} \left(\frac{L_{mt,C3} \cdot b_{C3}}{3} + L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 \right), \quad (3.58)$$

$$X_{D-Core} = \omega\mu_0 \frac{N_D^2}{h_R} \left(\frac{L_{mt,D} \cdot b_D}{3} + L_{mt,a1} \cdot a_1 \right), \quad (3.59)$$

where $\omega = 2\pi f$, f is the power frequency and the reactances have units of Ohms.

Equations (3.18) – (3.27) together with (3.55) – (3.59) form the set of the binary reactances estimated by the Cylindrical approach for the coil volume internal fault full model for simulation of T2T faults on the common coil. This model includes the core effect.

B. The Segmented approach

Since the preliminary implementation of the Segmented approach showed the issues mentioned in Subsection 3.4.3.3 and to save space, the figures and formulas related to calculation of the binary reactances between each of the portions $C1$, $C2$, $C3$ and the artificial $(N+1)^{th}$ coil are omitted. Recommendations on how to calculate them are given below:

- equations (3.38) – (3.43) and (3.49) – (3.53) should be modified keeping in mind that the artificial $(N+1)^{th}$ coil has zero thickness,
- the distances to the artificial $(N+1)^{th}$ coil should be used.

For calculation of the binary reactances between each of the healthy windings S , D and the artificial $(N+1)^{th}$ coil, equations (3.55) and (3.59) should be applied.

3.5.2.1.2 $[A(N+1)]$ Formulation

The leakage admittance representation of the faulted phase is formulated from the set of binary reactances estimated by either the Cylindrical or Segmented approach. The two sets of binary reactances produce different numbers. However, both lead to the same circuit as illustrated in Fig. 3.37. The additional $(N+1)^{th}$ node in Fig. 3.37 compared to Fig. 3.34 requires implementation of the additional row and column corresponding to the $(N+1)^{th}$ coil in $[A(N+1)]$. Therefore, (3.54) transforms to (3.60).

The admittance formulation based on Fig. 3.37 is represented mathematically by (3.8) for the healthy phases. The healthy submatrices $[A_B(N+1)]$ and $[A_C(N+1)]$ are defined as shown in (3.8) and the faulted phase submatrix $[A_A(N+1)]$ is defined as

$$[A_A(N+1)] = \begin{bmatrix} a_{SS} & a_{SC1} & a_{SC2} & a_{SC3} & a_{SD} & a_{S-Core} \\ a_{SC1} & a_{C1C1} & a_{C1C2} & a_{C1C3} & a_{C1D} & a_{C1-Core} \\ a_{SC2} & a_{C1C2} & a_{C2C2} & a_{C2C3} & a_{C2D} & a_{C2-Core} \\ a_{SC3} & a_{C1C3} & a_{C2C3} & a_{C3C3} & a_{C3D} & a_{C3-Core} \\ a_{SD} & a_{C1D} & a_{C2D} & a_{C3D} & a_{DD} & a_{D-Core} \\ a_{S-Core} & a_{C1-Core} & a_{C2-Core} & a_{C3-Core} & a_{D-Core} & a_{Core-Core} \end{bmatrix}. \quad (3.60)$$

The procedure for calculation of the leakage admittance matrix $[A(N+1)]$ from the binary reactances is given in Subsection 2.2.6.3. The resistance matrix $[R(N+1)]$ is a diagonal matrix similar to (2.64). $[R(N+1)]$ contains the resistances for all the physically present windings in each phase: 5 in the faulted phase and 3 in the healthy phases. The resistances

corresponding to the $(N+1)^{th}$ coil are zero in each phase. The resistances of the portions $C1$, $C2$, and $C3$ are calculated in proportion to the number of turns involved.

Fig. 3.37. Admittance formulation of the faulted phase A for simulation of T2T faults on the common coil: the full model $[A(N+1)]$.

3.5.2.2 T2T Faults on the Series Coil

Discussions in Subsection 3.5.2.1 are valid here, as well, with the difference that T2T faults are applied on the phase A series coil. All the related figures and equations can be found in Appendix C.

3.5.2.3 T2T Faults on the Delta Coil

Discussions in Subsection 3.5.2.1 are valid here, as well, with the difference that T2T faults are applied on the phase A delta coil. All the related figures and equations can be found in Appendix C.

3.6 The Coil Volume Internal Fault Model for T2G Faults

The development of the Cylindrical and Segmented approaches from Section 3.5 are valid here as well, except that now the faulted coil is sectioned into two portions instead of three and the faulted portion is connected to ground. T2G faults are applied in phase A on each winding: common, series and delta. The other two phases B and C remain healthy and are represented as described in Subsection 3.2.1. The formulas for calculation of the binary reactances in the faulted phase and the corresponding mmf diagrams are given both for the

Cylindrical and Segmented approaches. This formulation is given in detail for the common coil. Formulations for the series and delta coils are shown in Appendix D.

Since the coil volume internal fault model is used for low-frequency studies, the winding resistances are constant and capacitive effects are omitted. However, capacitive effects become important for T2G faults on the ungrounded delta winding. Although allowing the fault current to flow, coupling capacitances do not offer the direct connection to ground, and therefore, T2G fault currents on the delta winding are much lower than on windings with a direct path to ground. For this reason, when modeling T2G faults on the delta winding, it is important to include capacitances in the model. For simulation of T2G faults on the delta winding, implementation of the capacitance model calculated by the Hybrid model for the corresponding healthy transformer should be sufficient. For simulation of T2G faults on the series and common windings, capacitive effects are not important since the currents flowing through the leakage model are high.

3.6.1 Leakage Representation

The leakage representation is the first step in development and implementation of the coil volume internal fault model for simulation of T2G faults. The core circuit is omitted.

3.6.1.1 T2G Faults on the Common Coil

Fig. 3.38 illustrates the leakage model for simulation of T2G faults on the common coil of phase A. The common coil is sectioned into two portions, and the bottom portion $C2$, in series with the X_{DNF} element is shorted to ground to represent T2G faults. For this case, the number of windings $N=4$ in the faulted phase and $N=3$ in the healthy phases. The elements within the dashed rectangles in each phase describe the phase leakage admittance submatrices $[A_A(N)]$, $[A_B(N)]$, and $[A_C(N)]$ of the total leakage admittance matrix $[A(N)]$. These submatrices are independent of each other since the core model is not included at this stage.

If the X_{DNF} element is removed from the circuit and the short is not applied, the model should produce the short-circuit behavior of the healthy Mirage, and thus confirm that segmentation is performed correctly. If this step is completed successfully, then the next step is an attachment of the core circuit. The coil volume internal fault model based on Fig. 3.38 includes only the positive-sequence behavior.

Fig. 3.38. Simulation of T2G faults on the phase A common coil of the Mirage: the leakage model $[A(N)]$.

3.6.1.1.1 Calculation of the Binary Reactances

Referring to Section 3.4, the leakage field during a T2G fault can be represented by the Cylindrical or Segmented approaches. Each leads to a distinct set of the binary leakage reactances as described below. The mmf distribution diagrams for each approach describe the binary leakage reactances from which the corresponding leakage admittance matrix $[A(N)]$ is calculated and neither matrix incorporates the effect of X_{DNF} . The fault condition is simulated with insertion of the X_{DNF} element in series with the circuit elements of the portion C2 in the implemented circuit. Application of the short across all the circuit elements of the portion C2 outside of $[A(N)]$ is shown in Fig. 3.38.

The number of windings in the faulted phase $N=4$ defines the number of the elements on the main diagonal of $[A_A(N)]$. The number of the binary reactances in the faulted phase A, which corresponds to the number of the off-diagonal elements m in $[A_A(N)]$, is calculated by (3.17), leading to 6 binary reactances for this case. To keep track of the reactance reference, the binary reactances in this dissertation are always referred to the winding whose designation comes first, e.g., X_{CID} is the binary reactance between the portion C1 and delta winding and it is referred to the portion C1.

A. The Cylindrical approach

The related mmf distribution diagram of the faulted phase A, where the common coil is sectioned for simulation of T2G faults, is illustrated in Fig. 3.39. Most of the data used to calculate the binary short-circuit reactances are given in Fig. 3.1. The rest of the data, i.e., the width of each sub-coil C1 and C2, is defined by the number of turns involved.

Fig. 3.39. Mmf distribution diagram of the faulted phase A for simulation of T2G faults on the common coil: the leakage model $[A(N)]$, the Cylindrical approach.

Equations (3.61) – (3.66), describing the mmf distribution diagram of Fig. 3.39 mathematically, should be used to calculate the set of the binary reactances in phase A using the Cylindrical approach. The Rogowski factor for cylindrical windings K_{cyl} , as per (2.25), is included. K_{cyl} can be omitted (if neglecting the radial component of the leakage flux) by substituting the modified coil height h_R for the actual coil height h , refer to Subsection 2.2.4.2.1 for more details.

$$X_{SC1} = \omega\mu_0 \frac{N_S^2}{h_{R_SC1}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,C1} \cdot b_{C1}}{3} + L_{mt,a3} \cdot a_3 \right), \quad (3.61)$$

$$X_{SC2} = \omega\mu_0 \frac{N_S^2}{h_{R_SC2}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,C2} \cdot b_{C2}}{3} + L_{mt,C1} \cdot b_{C1} + L_{mt,a3} \cdot a_3 \right), \quad (3.62)$$

$$X_{SD} = \omega\mu_0 \frac{N_S^2}{h_{R_SD}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,D} \cdot b_D}{3} + L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right), \quad (3.63)$$

$$X_{C1C2} = \omega\mu_0 \frac{N_{C1}^2}{h_{R_C1C2}} \left(\frac{L_{mt,C1} \cdot b_{C1} + L_{mt,C2} \cdot b_{C2}}{3} \right), \quad (3.64)$$

$$X_{C1D} = \omega\mu_0 \frac{N_{C1}^2}{h_{R_C1D}} \left(\frac{L_{mt,C1} \cdot b_{C1} + L_{mt,D} \cdot b_D}{3} + L_{mt,C2} \cdot b_{C2} + L_{mt,a2} \cdot a_2 \right), \quad (3.65)$$

$$X_{C2D} = \omega\mu_0 \frac{N_{C2}^2}{h_{R_C2D}} \left(\frac{L_{mt,C2} \cdot b_{C2} + L_{mt,D} \cdot b_D}{3} + L_{mt,a2} \cdot a_2 \right), \quad (3.66)$$

where $\omega=2\pi f$, f is the power frequency and the reactances have units of Ohms.

Equations (3.61) – (3.66) form the set of the binary reactances estimated by the Cylindrical approach for the coil volume internal fault leakage model for simulation of T2G faults on the common coil without the core effect.

B. The Segmented approach

As in the case of T2T faults, here also it is necessary to include both the axial and radial components of the leakage field for estimation of the binary reactances since, for the Segmented approach, the radial component is not negligible. Therefore, the Rogowski factors both for cylindrical and sandwich windings, K_{cyl} and K_{smw} respectively, should be applied.

Since the explanation for calculation of the binary reactances by the Segmented approach for T2T faults takes a considerable number of pages, here the explanation is given by referring to the figures and equations of Subsection 3.5.1.1.1. Calculation of the binary reactances for T2G faults requires less effort than for T2T faults because the faulted coil is represented with two portions instead of three for T2T faults.

One of the mmf distribution diagrams for the faulted phase A, where the common coil is sectioned for simulation of T2G faults, is illustrated in Fig. 3.40. Most of the data used to calculate the binary short-circuit reactances are given in Fig. 3.1. The rest of the data, i.e., the height of each portion $C1$ and $C2$, is defined by the number of turns involved.

Fig. 3.40. Mmf distribution diagram of the faulted phase A for simulation of T2G faults on the common coil: the leakage model $[A(N)]$, the Segmented approach – SD and $C1C2$ pairs.

Fig. 3.40 shows the mmf distribution for two pairs of windings in the faulted phase A: between the healthy series S and delta D coils and between the portions $C1$ and $C2$ of the faulted common coil. The binary reactances for the healthy windings S and D are calculated by (3.63) and between portions $C1$ and $C2$ by (3.28).

For calculation of the binary reactances between S and $C1$, equations (3.31) – (3.34) should be adopted. The corresponding mmf distribution diagram is similar to Fig. 3.28.

For calculation of the binary reactances between S and $C2$, equations (3.35) – (3.37) should be adopted. The corresponding mmf distribution diagram is similar to Fig. 3.29.

For calculation of the binary reactances between $C1$ and D , equations (3.38) – (3.40) should be adopted. The corresponding mmf distribution diagram is similar to Fig. 3.30.

For calculation of the binary reactances between $C2$ and D , equations (3.41) – (3.43) should be adopted. The corresponding mmf distribution diagram is similar to Fig. 3.31.

Equations (3.28), (3.31) – (3.43) form the set of the binary reactances estimated by the Segmented approach for the coil volume internal fault leakage model for simulation of T2G faults on the common coil without the core effect.

3.6.1.1.2 $[A(N)]$ Formulation

The leakage admittance representation of the faulted phase is formulated from the set of binary reactances estimated by either the Cylindrical or Segmented approach. The two sets of binary reactances produce different numbers. However, both lead to the same circuit as illustrated in Fig. 3.41. This circuit represents the faulted phase A leakage admittance submatrix $[A_A(N)]$ for the cases where T2G faults are applied on the common coil. The coil volume internal fault leakage model consists of the leakage representation only. The related turns ratios are included.

Fig. 3.41. Admittance formulation for the faulted phase A for simulation of T2G faults on the common coil: the leakage model $[A(N)]$.

The admittance formulation based on Fig. 3.41 is represented mathematically by (3.4), where the healthy submatrices $[A_B(N)]$ and $[A_C(N)]$ are defined as shown in (3.4) and the faulted phase submatrix $[A_A(N)]$ is defined as

$$[A_A(N)] = \begin{bmatrix} a_{SS} & a_{SC1} & a_{SC2} & a_{SD} \\ a_{SC1} & a_{C1C1} & a_{C1C2} & a_{C1D} \\ a_{SC2} & a_{C1C2} & a_{C2C2} & a_{C2D} \\ a_{SD} & a_{C1D} & a_{C2D} & a_{DD} \end{bmatrix}. \quad (3.67)$$

The procedure for calculation of the leakage admittance matrix $[A(N)]$ from the binary reactances is given in Subsection 2.2.6.3. The resistance matrix $[R(N)]$ is a diagonal matrix similar to (2.64) and contains the resistances for all the windings in each phase: 4 in the faulted phase and 3 in the healthy phases. The resistances of the portions $C1$ and $C2$ are calculated in proportion to the number of turns involved.

3.6.1.2 T2G Faults on the Series Coil

Discussions in Subsection 3.6.1.1 are valid here, as well, with the difference that T2G faults are applied on the phase A series coil. All the related figures and equations can be found in Appendix D.

3.6.1.3 T2G Faults on the Delta Coil

Discussions in Subsection 3.6.1.1 are valid here, as well, with the difference that T2G faults are applied on the phase A delta coil. All the related figures and equations can be found in Appendix D.

As mentioned in Subsection 3.4.1.2, T2G faults on the delta winding are capacitive in nature since the delta winding is ungrounded. The low-amplitude ground fault current circulates through the inherent transformer capacitances. For these faults at the power system frequency, the lumped capacitive effects of the Hybrid model are sufficient.

3.6.2 The Full Model for T2G Faults

The main distinction between the full model and the leakage model, described in Subsection 3.6.1, is the presence of the artificial $(N+1)^{th}$ coil located on the core leg surface in each phase. Similar to the full model for simulation of T2T faults described in Subsection 3.5.2, each per phase set of the mutually coupled inductances, representing the existing windings in each phase, has an additional mutually coupled inductance. This additional inductance is not associated with a physically present winding but is formulated mathematically to serve as an interface between the core model and the leakage model. This solution is adopted from the Hybrid model.

The core model, adopted for development and implementation of the coil volume internal fault full model for simulation of T2G faults, is the existing core representation implemented in the Hybrid model of ATPDraw starting from v.6.1p8. This core model is described in Subsection 2.4.2. The voltage rating and the number of turns of the artificial

$(N+1)^{th}$ coil are equal to those of the winding closest to the core, i.e., of the healthy delta winding.

3.6.2.1 T2G Faults on the Common Coil

Fig. 3.42 illustrates the full model for simulation of T2G faults on the common coil of phase A. The common coil is sectioned into two portions, and the bottom portion $C2$, in series with the X_{DNF} element, is shorted to ground to represent T2G faults. For this case, the number of windings is $N+1=5$ in the faulted phase and $N=4$ in the healthy phases. The elements combined by a dashed rectangular in each phase describe the phase leakage admittance submatrices $[A_A(N+1)]$, $[A_B(N+1)]$, and $[A_C(N+1)]$ of the total leakage admittance matrix $[A(N+1)]$. These submatrices are interconnected outside of the leakage model by including the zero-sequence flux paths in the core circuit which are represented as Z_{0A} , Z_{0B} , and Z_{0C} in Fig. 3.42.

Fig. 3.42. Simulation of T2G faults on the phase A common coil of the Mirage: the full model $[A(N)]$.

If the X_{DNF} element is removed from the circuit and the short is not applied, the model should produce the short-circuit and open-circuit behaviors of the healthy Mirage. This confirms that segmentation is performed correctly, meaning that both the leakage model and the core model behave as expected. Correctness of the attachment of the core circuit to the leakage representation $[A(N+1)]$ is verified by comparing the voltages across the

core legs and yokes with those of the healthy model (the XFMR Mirage). The corresponding voltages from these two models should be approximately equal if the X_{DNF} element is removed from the circuit and the short is not applied.

The coil volume internal fault full model based on Fig. 3.42 includes both the positive-sequence and zero-sequence behaviors.

3.6.2.1.1 Calculation of the Binary Reactances

Two sets of equations for estimation of the binary reactances, defined by the Cylindrical and Segmented approaches, for the coil volume internal fault full model are given below. The mmf distribution diagrams for each approach describe the binary leakage reactances from which the corresponding leakage admittance matrix $[A(N+I)]$ is calculated and neither matrix incorporates the effect of X_{DNF} . The fault condition is simulated with insertion of the X_{DNF} element in series with the circuit elements of the portion $C2$ directly in the implemented circuit and application of the short across all the circuit elements of the portion $C2$ outside of $[A(N+I)]$, as shown in Fig. 3.42. As in the case of the coil volume internal fault full model for simulation of T2T faults, it is expected that incorporation of the core effect should reduce the delta winding current amplitude, and consequently, reduce the fault current level compared to the air-core model, refer to Subsection 3.3.2.

The number of windings in the faulted phase $N+1=5$ defines the number of the elements on the main diagonal of $[A_A(N+I)]$. The number of the binary reactances in the faulted phase A which corresponds to the number of the off-diagonal elements m in $[A_A(N+I)]$ is calculated by (3.17), leading to 10 binary reactances for this case.

To keep track of the reactance reference, the binary reactances in this dissertation are always referred to the winding whose designation comes first, e.g., $X_{C2-Core}$ is the binary reactance between the portion $C2$ and the artificial $(N+I)^{th}$ coil and it is referred to the portion $C2$. The width of the $(N+I)^{th}$ coil is zero and, thus, all the equation components with this thickness vanish.

A. The Cylindrical approach

The related mmf distribution diagram of the faulted phase A, where the common coil is sectioned for simulation of T2G faults and the $(N+I)^{th}$ coil is present, is illustrated in Fig. 3.43. Most of the data used to calculate the binary short-circuit reactances are given in Fig. 3.1. The rest of the data, i.e., the width of each sub-coil $C1$ and $C2$, is defined by the number of turns involved.

Fig. 3.43. Mmf distribution diagram of the faulted phase A for simulation of T2G faults on the common coil: the full model $[A(N+1)]$, the Cylindrical approach.

The mutual coupling between each physically present winding in phase A and the artificial $(N+1)^{th}$ coil defines the difference between Figs. 3.43 and 3.39, and the rest is the same. Therefore, equations (3.61) – (3.66) are valid for Fig. 3.43, but the new $N=4$ equations, describing the binary reactances related to the $(N+1)^{th}$ coil, are given in (3.68) – (3.71). The Rogowski factor for cylindrical windings K_{cyl} , as per (2.25), is included. K_{cyl} can be omitted (if neglecting the radial component of the leakage flux) by substituting the modified coil height h_R for the actual coil height h , refer to Subsection 2.2.4.2.1 for more details.

$$X_{S-Core} = \omega\mu_0 \frac{N_S^2}{h_{R_S-Core}} \left(\frac{L_{mt,S} \cdot b_S}{3} + L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D + \frac{L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3}{3} \right), \quad (3.68)$$

$$X_{C1-Core} = \omega\mu_0 \frac{N_{C1}^2}{h_{R_C1-Core}} \left(\frac{L_{mt,C1} \cdot b_{C1}}{3} + L_{mt,C2} \cdot b_{C2} + L_{mt,D} \cdot b_D + \frac{L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2}{3} \right), \quad (3.69)$$

$$X_{C2-Core} = \omega\mu_0 \frac{N_{C2}^2}{h_{R_C2-Core}} \left(\frac{L_{mt,C2} \cdot b_{C2}}{3} + L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 \right), \quad (3.70)$$

$$X_{D-Core} = \omega\mu_0 \frac{N_D^2}{h_{R_C3-Core}} \left(\frac{L_{mt,D} \cdot b_D}{3} + L_{mt,a1} \cdot a_1 \right), \quad (3.71)$$

where $\omega=2\pi f$, f is the power frequency and the reactances have units of Ohms.

Equations (3.61) – (3.66) together with (3.68) – (3.71) form the set of the binary reactances estimated by the Cylindrical approach for the coil volume internal fault full model for simulation of T2G faults on the common coil. This model includes the core effect.

B. The Segmented approach

Refer to Subsections 3.5.2.1.1 and 3.6.1.1.1 for recommendations on developing formulas for the coil volume internal fault full model for simulation of T2G faults with application of the Segmented approach.

3.6.2.1.2 $[A(N+1)]$ Formulation

The leakage admittance representation of the faulted phase is formulated from the set of binary reactances estimated by either the Cylindrical or Segmented approach. The two sets of binary reactances produce different numbers. However, both lead to the same circuit as illustrated in Fig. 3.44. The additional $(N+1)^{th}$ node in Fig. 3.44 compared to Fig. 3.41 requires implementation of the additional row and column corresponding to the $(N+1)^{th}$ coil in $[A(N+1)]$. Therefore, (3.67) transforms to (3.72).

Fig. 3.44. Admittance formulation for the faulted phase A for simulation of T2G faults on the common coil: the full model $[A(N)]$.

The admittance formulation based on Fig. 3.44 is represented mathematically by (3.8) for the healthy phases. The healthy submatrices $[A_B(N+1)]$ and $[A_C(N+1)]$ are defined as shown in (3.8) and the faulted phase submatrix $[A_A(N+1)]$ is defined as

$$[A_A(N+1)] = \begin{bmatrix} a_{SS} & a_{SC1} & a_{SC2} & a_{SD} & a_{S-Core} \\ a_{SC1} & a_{C1C1} & a_{C1C2} & a_{C1D} & a_{C1-Core} \\ a_{SC2} & a_{C1C2} & a_{C2C2} & a_{C2D} & a_{C2-Core} \\ a_{SD} & a_{C1D} & a_{C2D} & a_{C3C3} & a_{D-Core} \\ a_{S-Core} & a_{C1-Core} & a_{C2-Core} & a_{D-Core} & a_{Core-Core} \end{bmatrix}. \quad (3.72)$$

The procedure for calculation of the leakage admittance matrix $[A(N+1)]$ from the binary reactances is given in Subsection 2.2.6.3. The resistance matrix $[R(N+1)]$ is a diagonal matrix similar to (2.64). $[R(N+1)]$ contains the resistances for all the physically present windings in each phase: 4 in the faulted phase and 3 in the healthy phases. The resistances corresponding to the $(N+1)^{th}$ coil are zero in each phase. The resistances of the portions $C1$ and $C2$ are calculated in proportion to the number of turns involved.

3.6.2.2 T2G Faults on the Series Coil

Discussions in Subsection 3.6.2.1 are valid here, as well, with the difference that T2G faults are applied on the phase A series coil. All the related figures and equations can be found in Appendix D.

3.6.2.3 T2G Faults on the Delta Coil

Discussions in Subsection 3.6.2.1 are valid here, as well, with the difference that T2G faults are applied on the phase A delta coil. All the related figures and equations can be found in Appendix D. The lumped capacitive effects of the Hybrid model (the XFMR Mirage) should be included.

4 MODEL IMPLEMENTATION AND BENCHMARKING

This chapter describes implementation of the coil volume internal fault model in ATP. The coil volume internal fault model undergoes two stages of benchmarking:

1. Stage 1. Before insertion of the X_{DNF} element and application of a short to simulate internal faults, the coil volume internal fault leakage and full models should be benchmarked against the healthy transformer models to justify that the base load, short-circuit and open-circuit representations of the coil volume internal fault model correspond to those of the healthy transformer. This benchmarking is performed against:
 - the Hybrid model of the healthy Mirage named the XFMR Mirage,
 - the healthy Mirage's leakage model, and
 - the healthy Mirage's full model.
2. Stage 2. After successful results are obtained in Stage 1, the short is applied and the results are compared against the data given by the transformer manufacturer. This data is a set of results obtained from the proprietary higher-order $R-L$ internal fault model. Other details about the model have not been shared. The data blocks 1, 4, 5, 6 from Table 3.1 have been given to the transformer manufacturer. Then, according to the benchmarking results, X_{DNF} of a specific value is inserted and the short is again applied across all the circuit elements of the shorted portion.

The terminology used in this chapter is given for clarity:

- Verification. Applicable when the fault measurements from a real transformer are available. These are not available for the project.
- Benchmarking. Applicable when results from the coil volume internal fault model are compared against results from other simulations. This term is adopted for Chapter 4.
- Validation. Applicable when simulations of an existing transformer are compared with its factory test report data. Since a nondisclosure agreement exists between a utility owing the real transformer and Michigan Technological University, this information is not shared for the existing transformer. Thus, this term cannot be used.

Implementation of the coil volume internal fault model described in this dissertation is carried out with the polarity marking of the coils and sub-coils always on the top. The results demonstrated in this chapter are based on a steady-state single T2T fault located on the phase A common coil at the 10% coil height from the bottom.

4.1 The Hybrid Model of the Healthy Mirage: the XFMR Mirage

The data from Table 3.1 is required to build the XFMR Mirage. Fig. 4.1 illustrates the ATPDraw circuits for the short-circuit tests of Stage 1 benchmarking. Appendix E contains

the calculations and corresponding voltage and current waveforms, both for the short-circuit and open-circuit tests, of the XFMR Mirage.

Fig. 4.1. Stage 1 benchmarking tests for the healthy Mirage (the XFMR Mirage).

4.2 Precision of Numbers

The precision of numbers used for calculations is as follows:

- Modern transformer test reports offer more detailed information on transformer measurements compared to earlier versions, e.g., resistance measurements provide up to 7 significant digits and impedance measurements up to 4 – 5 significant digits.
- MATLAB calculations can be set for single or double precision. The preference is given to double precision applied to all MATLAB calculations.
- ATPDraw uses 16 columns of precision for the $[A]$ and $[R]$ matrices in the input fields and 15 columns for the output fields related to the section “Sinusoidal steady-state phasor solution, branch by branch” in the .lis file.
- The results in this dissertation are shown with 3 – 6 significant digits depending on the size of a variable, e.g., $X_{HL} = 6.44\%$, $V_H = 144.9 \text{ kV}_{L-L}$, $I_F = 95,047 \text{ A}_{\text{peak}}$, $R = 0.0565 \Omega$, $R_{C2} = 0.37464 \text{ m}\Omega$.

4.3 Estimation of the Coil-to-Core Reactances

The Hybrid model of a transformer estimates its parameters based on the test report data, typical values, or design data. Mostly, users adopt the test report option to build the Hybrid model of a transformer since design data is not usually available. Estimation of the coil-to-core reactances, in this case, is approximated based on the empirically derived factor $K=0.5$ and equations (2.78) – (2.80) applied to healthy three-winding transformers, refer to Subsection 2.4.2 for more details.

Development of the coil volume internal fault model requires availability of the basic information on the core window design, which is available for the Mirage. Therefore, assumptions of the Hybrid model on the coil-to-core reactance estimation can be verified during development and implementation phases of the coil volume internal fault model.

To compare the coil-to-core reactances calculated from the design data of Fig. 3.1 (refer to Subsection 3.2.2.1) with those approximately estimated by the Hybrid model, both Ohms and p.u. values should be used. Table 4.1 shows the healthy Mirage's binary reactances between the coil pairs in Ohms (taken from Table 3.1) and their corresponding p.u. values, calculated according to the ratings of each coil and the 100 MVA base power.

Table 4.1. The Mirage's binary reactances in Ohms and p.u. values.

	Ω	Reference coil	$Z_{\text{base on 100 MVA}}$	p.u.
X_{SC}	22.534	series	52.42	0.430
X_{SD}	41.371	series	52.42	0.789
X_{CD}	12.185	common	52.46	0.232
		delta	5.71	

Table 4.2 shows the comparison of the coil-to-core reactances calculated from the Mirage's design data and those estimated by the XFMR Mirage. Table 4.2 also suggests how much the empirical factor K of the Hybrid model should be changed in order to make this result equal to that obtained from the design data.

As can be observed from Table 4.2, estimation of the coil-to-core reactances by the Hybrid model with application of the empirical factor $K=0.5$ and equations (2.78) – (2.80) produces a considerable discrepancy from the reactances calculated based on the core window design data. Further research is needed to come up with better values for the empirical factor K used by the test report option of the Hybrid model.

Table 4.2. Comparison of the coil-to-core reactances between calculated from the Mirage's design data and estimated by the XFMR Mirage.

	$X_{S-Core}, \Omega/\text{p.u.}$	$X_{C-Core}, \Omega/\text{p.u.}$	$X_{D-Core}, \Omega/\text{p.u.}$
Reference coil	series	common	delta
Design data	45.894 / 0.876	16.708 / 0.318	0.334 / 0.059
Hybrid model $K=0.5$	40.761 / 0.778	18.277 / 0.348	0.662 / 0.116
Ratio of reactances (design data vs. Hybrid model)	1.13	0.91	0.5
K used by the Hybrid model	$K+1=1.5$	$K+1=1.5$	$K=0.5$
Suggested K to correspond to design data	$K+1=1.922$	$K+1=1.371$	$K=0.254$
$K=0.5$ should be	increased by 28%	decreased by 8.6%	decreased by 51%

4.4 Implementation of the Healthy Transformer Model

In this section, implementation of the healthy three-phase three-winding autotransformer model is shown: healthy Mirage's leakage and full models. The corresponding model development is shown in Section 3.2.

4.4.1 Leakage Representation

In this case, the core circuit is omitted.

4.4.1.1 Implementation

Implementation of the leakage model consists of two parts: generation of the leakage admittance matrix $[A(N)]$ and building a .acp file.

The ATPDraw circuit of this simple model is shown in Fig. F.1, refer to Appendix F. The coil resistances are implemented as the ATPDraw's resistor components. Otherwise, they can be represented as the resistance matrix $[R(N)]$ in the same file where $[A(N)]$ is stored.

Formulation of the leakage admittance matrix $[A(N)]$ for this leakage model is described in Subsection 3.2.1. The sequence of the coils in the implemented $[A(N)]$ is as follows: series, common, delta; however, it can start with any coil. The same sequence of coils is applied in the resistance matrix $[R(N)]$.

The transformer leakage admittance matrix $[A(N)]$ undergoes the following manipulations during building the leakage model of a healthy transformer:

- calculation in MATLAB,
- conversion to the form readable by ATP, which is represented with the upper triangle of a specific ATP format – matrix formatting, and
- insertion in the user-specified library object of a corresponding .acp file.

Insertion of the formatted $[A(N)]$ in the user-specified library object located in a .acp file completes implementation of the model.

4.4.1.2 Leakage Admittance and Resistance Matrices

Three matrices corresponding to the healthy Mirage's leakage model are shown in Tables G.1 – G.3, refer to Appendix G:

- the leakage admittance matrix $[A(N)]$ in Si with K_{cyl} neglected – Table G.1,
- the leakage admittance matrix $[A(N)]$ in Si with K_{cyl} included – Table G.2, and
- the resistance matrix $[R(N)]$ in Ohms – Table G.3.

Table 4.3 shows the Rogowski factors K_{cyl} for the coil pairs of the leakage model of the healthy Mirage.

Table 4.3. The Rogowski factors K_{cyl} of coil pairs for the healthy Mirage: the leakage model $[A(N)]$.

	SC	SD	CD
K_{cyl}	0.9449	0.9321	0.9617

Usually, if the Rogowski factors are above 0.9, they are neglected [24], [52].

4.4.2 The Full Model

In this case, the core circuit is included.

4.4.2.1 Implementation

Implementation of the full model can be performed in a .acp file or a .atp file. The first option is helpful for visualization of the model, however, the second option takes less overall effort.

Subsection 3.2.2 describes formulation of the leakage admittance matrix $[A(N+I)]$ for this full model containing both the leakage and core representations. The sequence of the coils in the implemented $[A(N+I)]$ is as follows: series, common, delta, $(N+I)^{th}$ coil; however, it can start with any coil. The same sequence of coils is used in the resistance matrix $[R(N+I)]$ if working with a .atp file. In a .acp file, the resistance of each winding is implemented as the ATPDraw's resistor component. The core circuit is adopted from the automatically generated .atp file of the XMFR component built for the same transformer, i.e., the XFMR Mirage.

To build the full model consisting of the leakage admittance matrix $[A(N+I)]$ and the core circuit, two options are possible:

- Manual modification of the automatically generated .atp file by the XFMR component. For this, $[A(N+I)]$ of the ATP accepted form is inserted in the .atp file instead of the default matrix followed by the manual modification of the node names, if required. The core representation is left unchanged. This option assumes operation with the .atp file only.
- Formulation of a .acp file corresponding to the automatically generated .atp file of the XFMR component where $[A(N+I)]$ is inserted in the user-specified library object. In this .acp file, the core circuit, the coil resistances, and other system components are represented with the ATPDraw's standard components. This option assumes operation with the .acp file which is built on the basis of the automatically generated .atp file.

Both options show how an XFMR component can be transformed for user's purposes, e.g., future modifications of the leakage admittance matrix $[A(N+I)]$. This helps to understand implementation of the coil volume internal fault model described further in Chapter 4. The author prefers working with .atp files.

For a healthy transformer, both options are identical to the corresponding XFMR component if the coil-to-core reactances are estimated as per (2.78) – (2.80). However, if estimation of these reactances is based on design data (refer to Table 4.2), then the leakage admittance matrix of the full model is different from the one estimated by the XFMR component. Also, for the Mirage, being a made-up transformer, the resistance matrix estimated by the XFMR Mirage from the minimum test report data (without known values of coil resistances) differs from the one built with the coil resistances directly taken from the made-up test report given in Table 3.1. Therefore, it is necessary to keep track of the resistance values in the .atp files and make changes if needed.

Appendix H.1 contains the entire .atp file of the XFMR Mirage, where according to Subsection 2.4.2.2 the zero-sequence inductance is set to 750 mH.

The $[A(N+I)]$ and $[R(N+I)]$ matrices from the .atp file of the full model for the healthy Mirage are shown in Appendix H.2. $[A(N+I)]$ in this .atp file is built based on design data of Fig. 3.1, K_{cyI} is neglected. $[R(N+I)]$ is built based on resistance measurements available from the made-up test report given in Table 3.1. Substituting these matrices in the .atp file of the XFMR Mirage located in Appendix H.1, the entire .atp file for the full model based on design data is obtained for the healthy Mirage.

4.4.2.2 Leakage Admittance and Resistance Matrices

Five matrices corresponding to the healthy Mirage's full model are shown in Tables G.4 – G.8, refer to Appendix G:

- the leakage admittance matrix $[A(N+I)]$ in Si with K_{cyl} neglected – Table G.4,
- the leakage admittance matrix $[A(N+I)]$ in Si with K_{cyl} included – Table G.5,
- the leakage admittance matrix $[A(N+I)]$ in Si built by the XFMR Mirage – Table G.6,
- the resistance matrix $[R(N+I)]$ in Ohms from resistance measurements given in the test report from Table 3.1 – Table G.7, and
- the resistance matrix $[R(N+I)]$ in Ohms built by the XFMR Mirage – Table G.8.

Table 4.4 shows the Rogowski factors K_{cyl} for the coil pairs of the full model of the healthy Mirage.

Table 4.4. The Rogowski factors K_{cyl} of coil pairs for the healthy Mirage:
the full model $[A(N+I)]$.

	<i>SC</i>	<i>SD</i>	<i>CD</i>	<i>S-Core</i>	<i>C-Core</i>	<i>D-Core</i>
K_{cyl}	0.9449	0.9321	0.9617	0.9281	0.9577	0.99

Usually, if the Rogowski factors are above 0.9, they are neglected [24], [52].

4.5 Implementation of the Shorted Turn Self-Reactance

Implementation of the coil volume internal fault model and preliminary results offered in this chapter are shown for the case when a steady-state single T2T fault occurs on the phase A common coil. Referring to Fig. 3.12, it is located at the 10% coil height from the bottom. This is equivalent to the height of 10% of the total common coil turns counting from the coil bottom. This case is called the base case for the rest of this dissertation. Referring to Fig. 3.14, the sub-coil *C3* contains 10% of turns, i.e., 44 turns, the sub-coil *C3* contains 1 turn, and the sub-coil *C1* contains the rest of turns, i.e., 399 turns. As mentioned in Subsection 3.4.1, the coils are wound uniformly with one layer and the insulation thickness is neglected. The base case is represented both for the coil volume internal fault leakage and full models.

4.5.1 Estimation

As described in Section 3.3, the leakage self-reactance of a shorted turn (or a portion of a coil) is represented as a stand-alone inductor X_{DNF} whose initial value is estimated by (3.14). It is assumed that the faulted turn resistance is not affected by the increased temperature of the copper wire due to the high amplitude of the fault current. According to the simulation results for the base case, produced by the transformer manufacturer on their proprietary higher-order *R-L* internal fault model, the fault current circulating in the shorted turn is $I_F = 95,047 \text{ A}_{\text{peak}}$. Assuming the induced emf of the faulted turn is equal to the rated induced emf of a healthy turn:

$$e_{ind} \approx e_{rated} = \frac{V_X / \sqrt{3}}{N_{COMMON}} = \frac{72500 / \sqrt{3}}{444} = 94.3 \text{ V}_{RMS}, \quad (4.1)$$

where V_X is the rated voltage at the low-side terminals measured in V_{RMS} and N_{COMMON} is the number of turns of the common coil, both are given in Table 3.1. The resistance of a single turn of the common coil is calculated under the assumption that all the turns have equal resistance, where the resistance of the common coil R_{COMMON} is given in Table 3.1:

$$R_{turn} = \frac{R_{COMMON}}{N_{COMMON}} = \frac{0.166339}{444} = 0.37464 \text{ m}\Omega. \quad (4.2)$$

Substituting I_F , e_{rated} and R_{turn} into (3.14),

$$X_{DNF} = \frac{94.3 \cdot \sqrt{2} - 0.000375 \cdot 95047}{95047} = 1.029 \text{ m}\Omega \text{ or } L_{DNF} = 2.727 \text{ }\mu\text{H}. \quad (4.3)$$

However, this value is not final since the induced emf in the faulted turn, in reality, is slightly different than its rated value. Therefore, the value of the X_{DNF} element should be tuned and this can be done in two ways:

- Assumption 1: the fault current in the shorted turn of the implemented coil volume internal fault model should be as close as possible to the fault current from the proprietary model. This goal can be achieved by varying the X_{DNF} value in the implemented circuit.
- Assumption 2: the delta current of the implemented coil volume internal fault model should be as close as possible to the delta current from the proprietary model. This goal can also be achieved by varying the X_{DNF} value in the implemented circuit.

Assumption 1 and Assumption 2 generate different amounts of delta current. Assumption 2 produces more valuable results since, in this case, benchmarking of the currents from the H-, X-, Y-terminals of the coil volume internal fault model against those of the proprietary model should provide better results. Besides, this current set represents an input into a transformer protection relay and its successful prediction achieves the goal of any internal fault model – generation of the realistic terminal currents of a faulted transformer for relay performance testing.

The circulating delta current of any amplitude produces almost zero current in the Y-terminal in both assumptions. However, the correct balancing ATs are generated by Assumption 2.

Table 4.5 shows the values of X_{DNF} for both assumptions excluding and including the core effect for the base case. The corresponding currents produced by matching of the two models are also shown.

Table 4.5. Values of the X_{DNF} element for the two assumptions excluding and including the core effect: the base case.

	Core effect neglected		Core effect included	
	Assumption 1	Assumption 2	Assumption 1	Assumption 2
X_{DNF} , m Ω	1.293	2.9	1.293	2.9
L_{DNF} , μ H	3.438	7.692	3.438	7.692
Current, A_{peak} coil volume internal fault model	$I_F = 95,073.0$ $I_\Delta = 138.8$	$I_F = 44,688.7$ $I_\Delta = 63.5$	$I_F = 95,067.1$ $I_\Delta = 138.8$	$I_F = 44,685.9$ $I_\Delta = 63.5$
Current, A_{peak} proprietary model	N/A		$I_F = 95,047.0$ $I_\Delta = 63.5$	

As can be observed from Table 4.5, the fault current of Assumption 1 and the delta current of Assumption 2 of the two models demonstrate excellent agreement with those from the proprietary model.

4.5.2 Suppression of Possible Numerical Oscillations

In ATP, numerical oscillations may occur due to a linear inductor experiencing step changes in the inductor voltage. A damping resistor R_p in parallel with the linear inductor should be inserted to damp out these oscillations. In ATPDraw, this damping resistor in parallel with an inductor is activated in the linear inductor tab by choosing the damping coefficient K_p in the range between 5 and 10. The typical value of 7.5 is often used [77]. Calculation of the damping resistor value in ATPDraw is performed internally by applying equation (4.4) if inductances are entered in Ohms:

$$R_p = \frac{K_p \cdot X}{\Delta t \cdot \pi \cdot f}, \quad (4.4)$$

where X is the inductance value in Ohms, Δt is a simulation time step, and f is the power frequency. For more information refer to [67].

Therefore, the damping resistor in parallel with the X_{DNF} element should be activated. However, if the damping resistor in parallel with the X_{DNF} element is implemented directly in a .atp file, its value should be entered manually in the proper field keeping in mind the units in (4.4).

Table 4.6 shows the values of the X_{DNF} element and their corresponding parallel resistors R_p for two assumptions in Subsection 4.5.1.

Preliminary results show that insertion of the damping resistor creates insignificant changes in the current amplitudes.

Table 4.6. Damping resistor values: the base case.

	Core effect neglected		Core effect included	
	Assumption 1	Assumption 2	Assumption 1	Assumption 2
$X_{DNF}, \text{ m}\Omega$	1.293	2.9	1.293	2.9
$\Delta t, \text{ s}$	5e-6	5e-6	5e-6	5e-6
R_p, Ω	10.285	23.069	10.285	23.069

4.6 Implementation of the Coil Volume Internal Fault Model

Implementation of the coil volume internal fault model for T2T faults on the common coil based on the Cylindrical approach is presented in the following subsections. Due to the issues described in Subsection 3.4.3.3, implementation of the coil volume internal fault model based on the Segmented approach is reserved for future work.

4.6.1 Leakage Representation

This subsection describes implementation of the coil volume internal fault leakage model whose development is expressed in Subsection 3.5.1.1.

4.6.1.1 Implementation

Implementation of this model is similar to implementation of the healthy transformer from Subsection 4.4.1 with the difference that here common coil is sectioned into three sub-coils. The middle sub-coil with its resistance component is dedicated to T2T faults. The shorted turn resistance is implemented and the X_{DNF} element is omitted for now. This is necessary to analyze the results before the X_{DNF} element is implemented.

The sequence of the coils in the leakage admittance submatrix $[A_A(N)]$ of the faulted phase A is as follows: series, sub-coil C1, sub-coil C2, sub-coil C3, delta. The same sequence of coils should be applied in the resistance matrix $[R_A(N)]$. However, the resistance of each coil and sub-coil is implemented as a resistor component of ATPDraw. The leakage admittance submatrices $[A_B(N)]$ and $[A_C(N)]$ of the healthy phases are implemented as described in Subsection 4.4.1.

The ATPDraw circuit of this implemented model is illustrated in Fig. F.2, refer to Appendix F. The short is implemented as a time-controlled switch between two resistance components of $10^{-6} \Omega$. Building such a .acp file helps to visualize the coil volume internal fault leakage model for simulations of T2T faults on the common coil. Fig. F.2 also shows the node naming strategy, since in the coil volume internal fault model, the node names inside the transformer leakage representation do not correspond to those of the Hybrid model (refer to Appendix H.1). This appears because the faulted common coil, represented with three sub-coils, requires four additional nodes.

Insertion of the formatted $[A(N)]$ in the user-specified library object of a .acp file completes implementation of this model.

4.6.1.2 Leakage Admittance and Resistance Matrices

Three matrices corresponding to the coil volume internal fault leakage model for the base case are shown in Tables G.9 – G.11, refer to Appendix G:

- the leakage admittance matrix $[A(N)]$ in Si with K_{cyl} neglected – Table G.9,
- the leakage admittance matrix $[A(N)]$ in Si with K_{cyl} included – Table G.10, and
- the resistance matrix $[R(N)]$ in Ohms – Table G.11.

Table 4.7 shows the Rogowski factors K_{cyl} for the coil pairs of the coil volume internal fault leakage model – the base case.

Table 4.7. The Rogowski factors K_{cyl} of coil pairs: the leakage model $[A(N)]$, the base case.

	SC1	SC2	SC3	C1C2	C1C3	C1D
K_{cyl}	0.9475	0.9474	0.9449	0.9770	0.9745	0.9617
	C2C3	C2D	C3D	SC	SD	CD
K_{cyl}	0.9974	0.9846	0.9847	0.9449	0.9321	0.9617

Usually, if the Rogowski factors are above 0.9, they are neglected [24], [52].

4.6.1.3 Modified Leakage Representation

In this model, the leakage self-reactance of the faulted turn X_{DNF} is inserted in the middle portion *C2* dedicated for simulation of T2T faults. To suppress the possible numerical oscillations, a resistor is placed in parallel with the X_{DNF} element as described in Subsection 4.5.2. The suggested values for the X_{DNF} element are shown in Table 4.5.

These two modifications are made to the coil volume internal fault leakage model described in Subsection 4.6.1.1. Therefore, the modified model is named the coil volume internal fault modified leakage model for simulation of T2T faults on the common coil. The ATPDraw circuit of this implemented model is shown in Fig. F.3, refer to Appendix F.

4.6.2 The Full Model

This subsection describes implementation of the coil volume internal fault full model whose development is expressed in Subsection 3.5.2.1.

4.6.2.1 Implementation

Implementation of this model is similar to that for the healthy transformer from Subsection 4.4.2 with the difference that the common coil is sectioned into three sub-coils. The middle

sub-coil with its resistance component is dedicated to T2T faults. As in the case of the leakage model, the shorted turn resistance is implemented and the X_{DNF} element is omitted for now. This is necessary to analyze the results before the X_{DNF} element is implemented.

The sequence of coils in the leakage admittance submatrix $[A_A(N+1)]$ of the faulted phase A is as follows: series, sub-coil C1, sub-coil C2, sub-coil C3, delta, $(N+1)^{th}$ coil. The same sequence of coils should be applied in the resistance matrix $[R_A(N+1)]$ if working with a .atp file. In a .acp file, the resistance of each coil and sub-coil is implemented as a resistor component of ATPDraw. The leakage admittance submatrices $[A_B(N+1)]$ and $[A_C(N+1)]$ of the healthy phases are implemented as described in Subsection 4.4.2.

As mentioned at the beginning of Subsection 3.5.2, for the coil volume internal fault full model, the core circuit is adopted from the XMFR component built for the same transformer, i.e., the XFMR Mirage. However, in the future, the core leg of the faulted phase can be implemented as consisting of three magnetic regions. Each magnetic region should interact with the corresponding fraction of the leakage flux of the faulted coil. All three magnetic regions should interact as one piece with the leakage flux of the healthy coils.

In the coil volume internal fault full model for the base case, the node names inside the transformer leakage representation do not correspond to the ones of the coil volume internal fault leakage model for the base case (refer to Fig. F.2 in Appendix F). This appears because the $(N+1)^{th}$ coil is added in each phase which requires addition of the core nodes α - β - γ and α' - β' - γ' , i.e., two additional nodes per phase. However, the adopted node naming strategy is similar to the one of Fig. F.2. For easier visualization, Fig. 4.2 illustrates the faulted phase A of the coil volume internal fault full model in the format of a .acp file.

Fig. 4.2. The faulted phase A in the .acp file format: the full model $[A(N+1)]$.

The corresponding complete .atp file for the coil volume internal fault full model of the base case is shown in Appendix H.3. In this file, the data consists of three sets:

- the data preserved from the original .atp file of the XFMR Mirage (refer to Appendix H.1) – the core circuit with the corresponding changes in the core node names, the source, and the load,
- the data which is inserted instead of the original results – the leakage admittance matrix and the resistance matrix with the corresponding changes in node names, and
- the data additionally inserted to form the coil volume internal fault full model – the shorting components, the backbone structure, and connections between the coils.

The backbone structure is needed to avoid possible stability issues and is represented with independent resistance components of $10^8 \Omega$ inserted between the node pairs of the leakage representation, refer to Appendix H.3.

The following steps should be carried out to build a .atp file of the coil volume internal fault full model:

- generation of a .atp file for the same transformer by the Hybrid model, i.e., the XFMR Mirage,
- calculation of $[A(N+I)]$ for the coil volume internal fault full model in MATLAB,
- conversion of $[A(N+I)]$ to the form readable by ATP – matrix formatting,
- insertion of the formatted $[A(N+I)]$ in the .atp file of the XFMR component, i.e., the XFMR Mirage, instead of the original matrix and modification of the core node names,
- addition of the new nodes in $[R(N+I)]$ in the faulted phase – two for T2T faults and one for T2G faults,
- generation of the resistance values for each sub-coil of the faulted coil,
- insertion of the resistance values in the .atp file – for healthy coils from the test report and for sub-coils of the faulted phase as calculated above,
- insertion of the shorting components in the .atp file – a time controlled switch located between two resistance components of $10^{-6} \Omega$ as shown in Fig. F.2, and
- implementation of the backbone structure and connections between the coils in the .atp file.

4.6.2.2 Leakage Admittance and Resistance Matrices

Three matrices corresponding to the coil volume internal fault full model for the base case are shown in Tables G.12 – G.14, Appendix G:

- the leakage admittance matrix $[A(N+I)]$ in Si with K_{cyl} neglected – Table G.12,
- the leakage admittance matrix $[A(N+I)]$ in Si with K_{cyl} included Table G.13, and
- the resistance matrix $[R(N+I)]$ in Ohms Table G.14.

Table 4.8 shows the Rogowski factors K_{cyl} for the coil pairs of the coil volume internal fault full model – the base case.

Table 4.8. The Rogowski factors K_{cyl} of coil pairs: the full model $[A(N+1)]$, the base case.

	SC1	SC2	SC3	S-Core	C1C2	C1C3
K_{cyl}	0.9475	0.9474	0.9449	0.9281	0.9770	0.9745
	C1D	C1-Core	C2C3	C2D	C2-Core	C3D
K_{cyl}	0.9617	0.9577	0.9974	0.9846	0.9806	0.9847
	C3-Core	SC	SD	CD	C-core	D-core
K_{cyl}	0.9807	0.9449	0.9321	0.9617	0.9577	0.9900

Usually, if the Rogowski factors are above 0.9, they are neglected [24], [52].

4.6.2.3 Modified Full Model

In this model, the self-reactance of the faulted turn X_{DNF} is inserted in the middle portion C2 dedicated for simulation of T2T faults. To suppress possible numerical oscillations, a resistor is placed in parallel with the X_{DNF} element as described in Subsection 4.5.2. The suggested values for the X_{DNF} element are shown in Table 4.5.

These two modifications are made the coil volume internal fault full model described in Subsection 4.6.2.1. Therefore, the modified model is named the coil volume internal fault modified full model for simulation of T2T faults on the common coil. This is illustrated in Fig. 4.3.

Fig. 4.3. The faulted phase A in the .acp file format: the modified full model $[A(N+1)]$.

If these changes are reflected in the .atp file located in Appendix H.3 (the coil volume internal fault full model for the base case, X_{DNF} is omitted), then two new .atp files appear. The new files are the .atp file in Appendix H.4 (the coil volume internal fault modified full model for the base case, X_{DNF} is inserted to match the fault current) and the .atp file in Appendix H.5 (the coil volume internal fault modified full model for the base case, X_{DNF} is inserted to match the delta current).

4.7 Benchmarking of the Coil Volume Internal Fault Model

In this section, benchmarking Stages 1 and 2 are described and the results are given. Benchmarking of the coil volume internal fault leakage and full models is carried out against three independent models:

- the XFMR Mirage (Section 4.1) – Stage 1,
- the healthy Mirage’s leakage model (Subsection 4.4.1) – Stage 1,
- the healthy Mirage’s full model (Subsection 4.4.2) – Stage 1, and
- the proprietary internal fault model from the transformer manufacturer – Stage 2.

The difference between the first two models is defined by the method for calculating the coil-to-core reactances and the coil resistances, refer to Subsection 4.4.2.1. Estimation of the coil-to-core reactances in the coil volume internal fault full model (Subsection 4.6.2) is based on the same method as in the healthy Mirage’s full model (Subsection 4.4.2), i.e., calculated from the design data of Fig. 3.1. The resistance values for these two models are based on Table 3.1, whereas the XFMR Mirage calculates these resistances internally.

Since the first two models represent the healthy Mirage, the coil volume internal fault model can be benchmarked against them only when the fault is not applied and X_{DNF} is not inserted in the circuit.

The results available from the proprietary internal fault model are for the fault cases only. Therefore, the coil volume internal fault model should be benchmarked against them in two steps:

- the fault is applied and X_{DNF} is not inserted in the faulted turn,
- the fault is applied and X_{DNF} inserted to match either the fault current or the delta current (refer to Subsection 4.5.1).

For proper benchmarking, an identical system should be established for all the models.

4.7.1 System Description

The systems parameters used by the transformer manufacturer are as follows:

- 60 MVA three-phase base load on the X terminals of unity power factor, $R_{LOAD} = 87.24 \Omega$,

- the H terminals are connected to the 144.9 kV 60 Hz power source through the zero source impedance,
- the Y terminals are unloaded, and
- the neutral H0X0 is solidly grounded.

The available information about the proprietary model is as follows:

- the capacitive effects are neglected,
- the temperature effect in the shorted turn is not considered,
- the fault impedance is zero, i.e., the arc is not considered.

The Mirage autotransformer in the system is illustrated in Fig. 4.4.

Fig. 4.4. Benchmarking system for the Mirage autotransformer.

4.7.2 Benchmarking against the Healthy Models

As indicated at the beginning of Chapter 4, this is Stage 1 benchmarking. As mentioned above, for these cases neither the short is applied nor the X_{DNF} element is inserted.

4.7.2.1 The Coil Volume Internal Fault Leakage Model

Building the coil volume internal fault full model (Subsections 3.5.2.1 and 4.6.2) starts with building the coil volume internal fault leakage model (Subsections 3.5.1.1 and 4.6.1), which contains the transformer leakage representation only. Benchmarking of the coil volume internal fault leakage model should be carried out against the XFMR Mirage (Section 4.1) and the healthy Mirage's leakage model (Subsection 4.4.1). This is the first step where errors can be detected.

Success on the correct segmentation of the faulted coil in the leakage model is confirmed if the base load and short-circuit test results are close to those of the XFMR Mirage and the healthy Mirage's leakage model under the conditions: the fault is not applied and X_{DNF} is not inserted. Table I.1 in Appendix I.1 shows the current amplitudes from these tests for the three models. The data for the tests are shown in Fig. 4.1. The current phase shifts

should also be verified. The results correspond to those of the XFMR Mirage from Appendix E.1.

The results shown in Table I.1 require some explanation. In this table, the results for the healthy Mirage's leakage model and the coil volume internal fault leakage model are shown both with the Rogowski factor K_{cyl} neglected and included. Columns 2 and 3 correspond to the healthy Mirage's leakage model with K_{cyl} neglected and included, respectively, and columns 4 and 5 correspond to the internal fault leakage model, also with K_{cyl} neglected and included, respectively.

As expected, the results in columns 2 and 3 are slightly different since the binary reactances are slightly different; the same is true for columns 4 and 5. In columns 2 and 3, the currents are equal between the three phases. This is because the model is fully balanced without the core circuit. However, the currents in columns 4 and 5 vary in amplitude slightly between the phases with a maximum percent difference below 1.5 %. This appears because the phase A common coil only is subjected to segmentation and this leads to the insignificant current imbalance between the phases. As expected, the results in columns 2 and 4, 3 and 5 are very close confirming the phase A common coil is properly sectioned.

Questions might arise when comparing the results in column 1 (the XFMR Mirage) with the results in columns 2 and 3. It is logical to assume that the results of the XFMR Mirage should be closer to the results in column 3 where K_{cyl} is included instead of the results in column 2 where K_{cyl} is neglected. This would be true for a real transformer whose Hybrid model would be built from parameters measured during actual tests. However, the Mirage is a made-up transformer and its parameters have been created not from the physical measurements, but from the fact that the estimated binary reactances between the coil pairs have values as in Table 3.1 with the Rogowski factor K_{cyl} neglected. The same reactances have been given to the transformer manufacturer, and therefore, the results from the coil volume internal fault model with K_{cyl} neglected are expected to have better agreement with the results from the proprietary model. For that reason, the focus is on the results where the Rogowski factor K_{cyl} is neglected.

The Stage 1 benchmarking tests produce correct results for the coil volume internal fault leakage model. This confirms that the leakage representation is properly performed. Therefore, the coil volume internal fault leakage model is qualified for application of the short and insertion of the X_{DNF} element.

4.7.2.1.1 Kron Reduction Analysis

The Kron reduction calculations have been carried out by a graduate student, Hemanth Kumar Vemprala, a Ph.D. candidate in the ECE Department at Michigan Technological University. Details of the step-by-step transformations of the submatrices are shown in Appendix I.2.

Kron reduction is performed to compare the reduced submatrices between each other. This is done to verify that the submatrices of all the three phases in the coil volume internal fault leakage model produce correct results during simulations (when the short is not applied and X_{DNF} is not inserted). The original matrix $[A(N)]$ with K_{cyl} neglected is shown in Table G.9. The original submatrices of size 5×5 (the faulted phase A) and size 3×3 (the healthy phases B and C) of the original matrix $[A(N)]$ are all reduced to size 2×2 , i.e., the series and delta coils remain.

The differences between the corresponding elements of the 2×2 submatrices are calculated and shown in Si and p.u. values in Table 4.9. Since the healthy phases B and C are identical, the differences between the corresponding elements of their reduced submatrices are zero.

Table 4.9. Differences between the corresponding elements of the phase A and B (C) 2×2 submatrices, Si and p.u.: the leakage model $[A(N)]$.

	S		D	
	Si	p.u.	Si	p.u.
S	-5.555E-10	2.141E-06	5.787E-08	-1.997E-06
D	5.787E-08	-1.997E-06	1.380E-06	6.535E-07

As can be observed, the differences are almost zero indicating that the phase A common coil is properly sectioned for the base case T2T fault.

4.7.2.2 The Coil Volume Internal Fault Full Model

Since the coil volume internal fault full model (Subsections 3.5.2.1 and 4.6.2) includes both the leakage and core representations, benchmarking of each one should be carried out against the XFMR Mirage (Section 4.1) and the healthy Mirage's full model (Subsection 4.4.2).

For the coil volume internal fault full model, the leakage representation is benchmarked through the base load and short-circuit tests as described in Subsection 4.7.2.1. The core representation is benchmarked through the open-circuit tests. However, the core circuit benchmarking is not necessary since it is adopted from the XFMR component without topological changes. Correctness of the core circuit attachment to the leakage representation should be examined. This is performed by measuring the voltages across the core limbs (legs and yokes) for the base load and short-circuit tests. If the voltages are balanced and their values are as expected for these tests, then the core circuit is attached correctly. The data for the tests are shown in Fig. 4.1. Refer to Table I.8 in Appendix I.3 to observe the benchmarking results from the base load and short-circuit tests as well as the voltage measurements across the core limbs.

The results of the open-circuit test (voltage ratios, voltage phase shifts) for the healthy Mirage's full model and the coil volume internal fault full model correspond to those of the XFMR Mirage from Appendix E.2. The current phase shifts are also verified, and these results correspond to those of the XFMR Mirage from Appendices E.1. In the future, if the

core leg in the faulted phase is implemented in three magnetic regions, then more rigorous core circuit benchmarking should be performed.

Explanation for the results related to currents in Table I.8 is the same as given in Subsection 4.7.2.1 for Table I.1. Explanation for the results related to the voltages in Table I.8 is as follows:

- The difference between the voltage amplitudes in columns 2 and 3, and columns 4 and 5 is due to application of the Rogowski factor K_{cyl} .
- The differences between the voltage amplitudes in columns 1 and 2, and columns 1 and 3 are because the coil-to-core reactances are calculated differently. Estimation is based on the empirically derived factor $K = 0.5$ for column 1 and estimation is based on design data for columns 2, 3.
- The per-phase voltages in columns 4 and 5 vary in amplitude slightly with a maximum percent far below 1%. As in the case of the slight current imbalance, this appears because the phase A common coil only is subjected to segmentation and this also leads to insignificant voltage imbalance between the phases.
- As expected, the results in columns 2 and 4, 3 and 5 are very close, confirming the phase A common coil is properly sectioned and the core circuit is properly attached.

As explained in Subsection 4.7.2.1, the focus is on the results where the Rogowski factor K_{cyl} is neglected.

The Stage 1 benchmarking tests produce correct results for the coil volume internal fault full model. This confirms that the leakage representation and the core circuit attachment are properly performed. Therefore, the coil volume internal fault full model is qualified for application of the short and insertion of the X_{DNF} element.

4.7.2.2.1 Kron Reduction Analysis

The Kron reduction calculations, similar to those described in Subsection 4.7.2.1.1, have been carried out by Hemanth Kumar Vempala. Details of the step-by-step transformations of the submatrices are shown in Appendix I.4.

The original matrix $[A(N+I)]$ with K_{cyl} neglected is shown in Table G.12. The original submatrices of size 6×6 (the faulted phase A) and size 4×4 (the healthy phases B and C) of the original matrix $[A(N+I)]$ are all reduced to size 3×3 , i.e., the series, delta and $(N+I)^{th}$ coils remain.

The differences between the corresponding elements of the 3×3 submatrices are calculated and shown in Si and in p.u. values in Table 4.10. Since the healthy phases B and C are identical, the differences between the corresponding elements of their reduced submatrices are zero.

Table 4.10. Differences between the corresponding elements of the phase A and B (C) 3×3 submatrices, Si and p.u.: the full model $[A(N+1)]$.

	S		D		Core	
	Si	p.u.	Si	p.u.	Si	p.u.
S	2.233E-06	-3.094E-06	8.417E-06	-7.470E-06	-1.2519E-06	1.0058E-06
D	8.417E-06	-7.470E-06	-1.083E-05	-8.616E-06	1.0964E-06	6.3243E-07
Core	-1.2519E-06	1.0058E-06	1.0964E-06	6.3243E-07	-4.7628E-07	8.7103E-07

As can be observed, the differences are almost zero indicating the phase A common coil is properly sectioned for the base case T2T fault.

4.7.3 Benchmarking against the Proprietary Model for the Base Case T2T Fault

This is Stage 2 benchmarking as indicated at the beginning of Chapter 4. Since the coil volume internal fault leakage and full models successfully passed the Stage 1 benchmarking as documented in Subsection 4.7.2, they are qualified for application of the short and insertion of the X_{DNF} element for benchmarking with the base case T2T fault developed by the transformer manufacturer. The proprietary internal fault model has the core effect included but information on the core parameters have not been shared. The short is applied for the full length of each simulation to avoid transients due to switching.

From this point on, the term “modified” in the name of the coil volume internal fault model (as per Subsections 4.6.1.3 and 4.6.2.3) is omitted for simplification of terminology. Whether X_{DNF} is omitted or included, the developed model is referred to the coil volume internal fault model with or without X_{DNF} .

4.7.3.1 The Coil Volume Internal Fault Leakage Model

Table I.14 in Appendix I shows the comparison of current amplitudes between the four cases:

- The proprietary internal fault model – column 1.
- The coil volume internal fault leakage model:
 - X_{DNF} is not inserted – column 2 with K_{cyl} neglected and column 3 with K_{cyl} included. Refer to Fig. F2 in Appendix F for the ATPDraw circuit.
 - X_{DNF} is inserted to match the fault current in the shorted turn of the coil volume internal fault leakage model with that of the proprietary model – column 4 with K_{cyl} neglected and column 5 with K_{cyl} included. Refer to Fig. F3 in Appendix F for the ATPDraw circuit.
 - X_{DNF} is inserted to match the delta current of the coil volume internal fault leakage model with that of the proprietary model – column 6 with K_{cyl} neglected and column 7 with K_{cyl} included. Refer to the same circuit in Fig. F3.

The coil volume internal fault leakage model does not include the core effect. The proprietary model includes the core effect. However, details of the core parameters have not been shared. Setting their results to 100% for each current, Table I.15 shows the current percentages for all the cells of Table I.14 with respect to the corresponding currents from the proprietary model. As explained in Subsection 4.7.2.4, the focus is on the results where K_{cyl} is neglected.

The results, shown in Tables I.14 and I.15, require explanation:

- If X_{DNF} is not inserted, the fault current amplitude in the shorted turn is overestimated by 265% (refer to columns 2 and 3 versus column 1). Consequently, the H-terminal currents are off, especially the one in the faulted phase A is overestimated by 74%. Such overestimation is expected as explained in Subsection 3.4.1.3. The phase A common coil current reverses polarity and, thus, is underestimated. Due to the poor leakage field representation without the X_{DNF} element, the delta current is overestimated by 715%. At the same time, the current amplitudes in the healthy phases are not too far from those of the proprietary model. The maximum deviation in phase B is 12.4% (except the delta current), the maximum deviation in phase C is 15.8% (except the delta current). This is because the overestimated delta current transfers the overestimated ATs to the healthy phases. This leads to a guess that an additional parameter(s) of the shorted turn is missing. When the Rogowski factor K_{cyl} is included, the fault current increases by 0.16% forcing all other currents to change insignificantly.
- In this case, X_{DNF} is inserted to match the fault current of the two models (refer to columns 4 and 5 versus column 1), as per Assumption 1, in Subsection 4.5.1. The results from the coil volume internal fault leakage model show excellent agreement with the results from the proprietary model for the faulted phase A currents (less than $|-2\%|$ deviation in amplitudes) and the healthy phase B currents (less than $|\pm 3\%|$ deviation in amplitudes). However, the result for the healthy phase C is slightly less impressive:
 - the series coil current amplitude deviates by 3.7% and
 - the common coil current amplitude deviates by $|-4\%|$.

For this case, the maximum deviation among all the currents, except the delta current, from the results of the proprietary model is 4% (the common coil current in the healthy phase C). The delta current amplitude deviates by 119%. When the Rogowski factor K_{cyl} is included, the fault current amplitude increases by 0.17% forcing all other currents to change insignificantly.

- Matching the delta current, as per Assumption 2, seems more reasonable than matching the fault currents as explained in Subsection 4.5.1. If X_{DNF} is inserted to match the delta current of the two models (refer to columns 6 and 7 versus column 1), the results from the coil volume internal fault leakage model show perfect agreement with the results from the proprietary model for the healthy phases B and C. However, the result for the faulted phase A is slightly less impressive:
 - the series coil current amplitude deviates by $|-7\%|$,
 - the common coil current amplitude deviates by 2.4%, and

- the X-terminal current amplitude deviates by 0.7%.

For this case, the maximum deviation among all the currents, except the fault current in the shorted turn, from the results of the proprietary model is 7% (the series coil current in the faulted phase A). The fault current amplitude deviates by $|-47\%|$. When the Rogowski factor K_{cyl} is included, the fault current amplitude increases by 0.1% forcing all other currents to change insignificantly.

Overall, it seems that both options with the X_{DNF} element offer promising results:

- The pro for the option, where the fault current of both models is matched, is that the current amplitudes in the faulted phase A of the coil volume internal fault leakage model deviate from those of the proprietary model by less than 2% and slightly more for other phases (roughly, 3% for phase B and 4% for phase C).
- The pro for the option, where the delta current of both models is matched, is that the correct balancing ATs are transferred from the faulted phase to the healthy phases. This leads to the fact that the currents in the healthy phases B and C of the coil volume internal fault leakage model are identical to those of the proprietary model. The series coil current amplitude in the faulted phase A deviates by 7% while providing confidence that such an important parameter as the delta current is matched precisely between the two models.

4.7.3.2 The Coil Volume Internal Fault Full Model

Tables I.16 and I.17 repeat the structure of Tables I.14 and I.15 but show results for the coil volume internal fault full model, i.e., this model includes the core circuit with the zero-sequence inductance of $L_0=750$ mH set in the Advanced Core Settings of the XFMR Mirage. As explained in Section 3.3, attachment of the core circuit to the leakage model should lead to a reduction of the fault current and delta current considerably, thus reducing other currents as well. However, comparison of the results from Tables I.14 and I.16 shows only insignificant changes in the current amplitudes when the core circuit is added: the series and common coil currents show a maximum change of $0.1 A_{peak}$ or zero. Comparison of the fault and delta currents between the coil volume internal fault leakage and full models is given in Table I.18. This table shows the following:

- When X_{DNF} is not inserted, the core effect appears as a slight decrease in the fault current amplitudes by 0.007% not influencing the delta current.
- When X_{DNF} is inserted to match the fault current, the core effect appears as a slight decrease in the fault current amplitudes by 0.006% not influencing the delta current.
- When X_{DNF} is inserted to match the delta current, the core effect also appears as a slight decrease in the fault current amplitudes by 0.006% not influencing the delta current.

There might be two reasons why attachment of the core circuit demonstrates such a negligible effect on the currents:

1. The zero-sequence inductance value of 750 mH, which is defined in Subsection 2.4.2.2, is an approximate estimate for the Mirage autotransformer. Ideally, information on the transformer zero-sequence inductance value and other core circuit parameters should be available from an accurate source. Hopefully, for the remainder of this project, the transformer manufacturer would be willing to share their values for the core circuit parameters they applied in their proprietary model.
2. The core circuit automatically generated by the XFMR Mirage might not be the right version of the core circuit for the coil volume internal fault model. Perhaps, the leakage admittance matrix of the faulted Mirage does not interact with this core representation to a sufficient degree, and modification of the core circuit in the faulted phase is required. In the faulted phase, this modification should provide interaction of the individual portions of the faulted coil with the corresponding core leg regions, while the healthy coils should interact with these core leg regions as if they represent the one-piece core leg. This is also mentioned in Subsection 4.6.2.1.

Since the core effect is insignificant, an explanation of the results for Tables I.14 and I.15 is applicable to Tables I.16 and I.17.

As mentioned in Section 3.3, the iron core of the three-legged transformers partially offers a low reluctance path for the zero-sequence flux and thus increases the zero-sequence inductance compared to the air core. This, in turn, reduces the zero-sequence current circulating in the delta winding. This is the desired effect of the core circuit attachment.

Perhaps, the best results from the coil volume internal fault model based on the Cylindrical approach for the base case T2T fault would be obtained through the following three steps:

- Matching the fault current of the coil volume internal fault leakage model by inserting $X_{DNF} = 1.293 \text{ m}\Omega$. With this, the delta current is $138.8 \text{ A}_{\text{peak}}$, refer to Table I.14, columns 1 and 4.
- Attaching a core circuit with parameters such that the delta current decreases considerably, or in the best case, from $138.8 \text{ A}_{\text{peak}}$ down to near the value of $63.5 \text{ A}_{\text{peak}}$ from the proprietary model.
- Tuning the value of X_{DNF} to match the fault current value again since addition of the core circuit might decrease the fault current.

This three-step implementation with the modified core circuit should lead to more practical results instead of inserting $X_{DNF} = 2.9 \text{ m}\Omega$ for matching the delta current. This value compensates for the poor core representation during the base case T2T fault by matching the delta current of the coil volume internal fault model with that of the proprietary model. It also reduces the fault current amplitude in the shorted turn almost by half compared to that from the proprietary model ($95,047 \text{ A}_{\text{peak}}$):

- from $95,073 \text{ A}_{\text{peak}}$ down to $44,689 \text{ A}_{\text{peak}}$ – by changing the X_{DNF} value in the coil volume internal fault leakage model, refer to Table I.14, column 6 versus columns 4 and 1,

- from 95,067 A_{peak} down to 44,686 A_{peak} – by changing the X_{DNF} value in the coil volume internal fault full model, refer to Table I.16, column 6 versus columns 4 and 1.

Therefore, derivation of the proper core circuit in the faulted phase which reduces the delta current amplitude considerably is the main step in advancing the coil volume internal fault full model based on the Cylindrical approach. Expectations of using such a core model are well defined here, however, finding the proper combination and values of the core circuit elements in the faulted phase requires some creativity.

The core circuit of the XFMR Mirage has the zero-sequence inductance value of 750 mH set in the Advanced Core settings. Since presence of this core circuit in the coil volume internal fault full model does not reduce the delta current amplitude, it is important to carry out sensitivity analysis for the zero-sequence inductance value. This will help to verify its effect on the delta current amplitude.

4.7.3.2.1 Sensitivity Analysis for the Zero-Sequence Inductance

The parameters affecting the delta current are

- the leakage self-inductance of the faulted turn X_{DNF} ,
- the transformer zero-sequence inductance, and
- the resistance value of the delta coils.

The latter is strictly defined by the transformer design and can be easily measured. Variation of this parameter for a manufactured transformer does not make sense. As observed in Tables I.14 and I.16, X_{DNF} has a direct effect on the delta current amplitude.

Table I.19 shows the effects of the zero-sequence inductance variation on the delta current amplitude. As can be observed, variation of the 750 mH value by $\pm 50\%$ does not affect the delta current amplitude (columns 3, 4, 5). For each case of Table I.19, the only change in the delta current is observed, when the zero-sequence inductance in the Advanced Core Settings is omitted (column 6), and the results for this are as follows:

- When X_{DNF} is not inserted, the delta current amplitude increases by 0.16% and the fault current amplitude decreases by 0.03%.
- When X_{DNF} is inserted to match the fault current, the delta current amplitude increases by 0.16% and the fault current amplitude does not change compared to the cases where $L_0=750 \text{ mH} \pm 50\%$.
- When X_{DNF} is inserted to match the delta current, the delta current amplitude increases by 0.27% and the fault current amplitude does not change compared to the cases where $L_0=750 \text{ mH} \pm 50\%$.

The cases without the core circuit and with the core circuit attached when $L_0=750 \text{ mH} \pm 50\%$ do not show any changes in the delta current amplitudes (columns 2 – 5). However, they

do show insignificant changes in the fault current amplitudes (refer to Subsection 4.7.3.2). Therefore, the zero-sequence inductance might not be properly represented in the adopted core circuit.

Thus, based on Tables I.18 and I.19, the cases without the core attached (the coil volume internal fault leakage model) and with the core attached (the coil volume internal fault full model) are equivalent and the expected reduction in the delta current amplitude does not occur. From these results, it is concluded again, that the core circuit in the faulted phase should be upgraded.

4.8 Induced Volts per Turn

In this section, the induced volts per turn are compared between

- The XFMR Mirage,
- the healthy Mirage's leakage and full models,
- the coil volume internal fault leakage and full models structured for the base case, but the fault is not applied, and
- the coil volume internal fault leakage and full models structured for the base case and the fault is applied.

The goal is to observe how the induced voltage of a single turn changes when it becomes shorted.

4.8.1 The Healthy Model

Since the focus is on the cases with K_{cyl} neglected, as explained in Subsection 4.7.3.1, the cases with K_{cyl} included are not considered past Section 4.7. Fig. 4.5 illustrates e_{ind} and V_{drop} for a healthy common coil: a) coil lumped parameters, b) turn lumped parameters. As described in Subsection 3.3.1, each turn contains the induced voltage e_{ind} and the voltage drop across the turn V_{drop} . In a healthy turn, the voltage drop is defined mainly by the turn resistance and is negligible compared to the induced voltage.

Fig. 4.5. Volts per turn estimation for the healthy common coil:
 a) coil lumped parameters, b) turn lumped parameters.

The turn lumped parameters are obtained through division by the coil's number of turns:

$$V_{out_turn} = \frac{V_{COMMON}}{N_{COMMON}}, \quad (4.5)$$

$$\frac{e_{ind}}{turn} = \frac{e_{ind_COMMON}}{N_{COMMON}}, \quad (4.6)$$

$$\frac{V_{drop}}{turn} = \frac{V_{drop_COMMON}}{N_{COMMON}}, \quad (4.7)$$

$$V_{out_turn} = \frac{e_{ind}}{turn} - \frac{V_{drop}}{turn}. \quad (4.8)$$

Referring to Figs. 3.2 and 3.5, it should be noted that the induced voltages are not the voltages across the independent inductor component X_{DNF} . These are the voltages between each two nodes corresponding to the coupled inductances.

Table J.1 in Appendix J shows a comparison of the measured coil lumped parameters and the calculated parameters corresponding to a single turn for the healthy Mirage:

- the XFMR Mirage,
- the healthy Mirage's leakage model, and
- the healthy Mirage's full model.

Table 4.11 below shows a part of Table J.1, where the calculated data for a single turn is in focus.

Table 4.11. Calculated parameters of a single turn of the healthy common coil (Part of Table J.1).

Phase A voltages, V_{peak}		XFMR Mirage $L_0 = 750$ mH (Section 4.1)	Healthy Mirage's leakage model (Subsection 4.4.1)	Healthy Mirage's full model $L_0 = 750$ mH (Subsection 4.4.2)
		1	2	3
		K_{cyl} neglected		
Per turn: calculated	$e_{ind} : \text{eq. (4.6)}$	132.94	133.0	133.0
	$V_{drop} = V_R : \text{eq. (4.7)}$	0.11	0.13	0.13
	$V_{out} : \text{eq. (4.8)}$	132.83	132.87	132.87

As can be observed from these tables, the following is as expected:

- The induced voltage and the voltage drop are 180 degrees out phase.

- The voltage drop is negligible compared to the induced voltage, and therefore, the output voltage of any healthy turn is almost equal to the induced voltage.
- For the healthy transformer, the studied parameters are not affected by addition of the core circuit. Refer columns 2 (the core circuit is omitted) and 1, 3 (the core circuit is attached), in Tables J.1 and 4.11.

4.8.2 Sectioned Common Coil but the Fault Is Not Applied

The phase A common coil is sectioned into three sub-coils for the base case but the short is not applied. Therefore, the portion C2 represents a discrete healthy turn, and its induced voltage, voltage drop, and output voltage can be measured directly. This circuit in the .acp file format is shown in Fig. 4.2 but the switch should be open. Resistance of the single turn is estimated by (4.2), and thus, $R_{C2} = 0.37464 \text{ m}\Omega$.

Table 4.12 below shows a comparison of these voltages in phase A for the coil volume internal fault leakage and full models structured for the base case when the short is not applied. For these cases, X_{DNF} is not relevant, since both models still represent the healthy transformer.

Table 4.12. Measured parameters of the single turn
(the sub-coil C2) in the healthy common coil.

Phase A voltages, V_{peak}		Base case structure but the short is not applied			
		Coil volume internal fault leakage model (Subsection 4.6.1)		Coil volume internal fault full model $L_0 = 750 \text{ mH}$ (Subsection 4.6.2)	
		1		2	
		K_{cyl} neglected			
$V_{\text{COMMON}} = V_{X_LG} : \text{measured}$		58994.5		58992.4	
$V_{\text{out}} : \text{eq. (4.5)}$		132.87		132.87	
		Value at $e_{\text{ind}} = \text{max}$	amplitude	Value at $e_{\text{ind}} = \text{max}$	amplitude
Single turn: measured	e_{ind}	133.02	133.02	133.01	133.01
	$V_{\text{drop1}} = V_{R_C2}$	-0.13	0.13	-0.13	0.13
	$V_{\text{drop2}} = V_{\text{DNF}}$	N/A		N/A	
	$V_{\text{drop}} = V_{\text{drop1}} + V_{\text{drop2}}$	-0.13	0.13	-0.13	0.13
	V_{out}	132.89		132.88	

From this table, it is observed:

- The voltages of interest are practically the same as in Table 4.11 (J.1). This is the expected result, also proving that segmentation of the phase A common coil is performed correctly, i.e., the corresponding leakage admittance matrix is appropriate.

- The induced voltage and the output voltage of the single turn are almost equal in the healthy turn.
- For the healthy transformer, the studied parameters are not affected by addition of the core circuit.

4.8.3 Sectioned Common Coil and the Fault Is Applied

Table J.2 in Appendix J shows a comparison of the measured parameters. They are the induced voltage, voltage drop, output voltage, as well as other related parameters of the faulted turn in the coil volume internal fault leakage and full models (the base case). Each shows the three cases:

- X_{DNF} is not inserted,
- $X_{DNF} = 1.293 \text{ m}\Omega$ to match the fault current, and
- $X_{DNF} = 2.9 \text{ m}\Omega$ to match the delta current.

Refer to Figs. 4.2 and 4.3 for the .acp file format. Resistance of the shorted turn is $R_{C2} = 0.37464 \text{ m}\Omega$.

Table J.2 helps to observe the following changes in phase A during the base case T2T fault:

- The induced voltage in the studied turn e_{ind} reduces by a maximum of 3.8% among all the cases compared to Table 4.12. Averaging the six cases of Table J.2 yields a change from $133 \text{ V}_{\text{peak}}$ (healthy) to $130 \text{ V}_{\text{peak}}$ (faulted). Such a low percentage of induced voltage reduction, due to the base case T2T fault, leads to the assumption that the induced voltage in the shorted turn is equal to the output voltage of the healthy turn.
- The output voltage of the shorted turn is close to zero, as expected.
- The theoretical assumption that the induced voltage in the shorted turn is expended between the turn resistance and the “do-nothing” flux reactance of the turn seems correct.
- As expected, the voltage drop in the faulted turn increases from an insignificant value, before the fault, up to the value almost equal the induced voltage during the fault and is 180 degrees out of phase with the induced voltage. When the “do-nothing” flux is considered in the faulted turn, the voltage drop occurs mainly over the X_{DNF} element. This is because its Ohms value is
 - 3.5 times more than the turn resistance R_{C2} when $X_{DNF} = 1.293 \text{ m}\Omega$ for matching the fault current as per Assumption 1 and
 - 7.7 times more than the turn resistance R_{C2} when $X_{DNF} = 2.9 \text{ m}\Omega$ for matching the delta current as per Assumption 2.
- The common coil voltage, being also the X-terminal voltage, drops by a maximum of 2% among all the cases.
- Again, the parameters of interest almost do not change when comparing the coil volume internal fault leakage and the full models (columns 1 – 3 vs. columns 4 –

- 6). When a fault is applied, the core effect should be prominent and, as mentioned in Subsection 4.7.3.2, another core representation should be derived for the faulted phase.
- The fault current circulating in the shorted turn can be approximately estimated by (3.12), or more specifically, by (4.9) below based on two assumptions:
 - the induced voltage in the shorted turn is approximately equal to the output voltage of the healthy turn and
 - the X_{DNF} value of the case is approximately known.

$$I_{fault_estimated} = \frac{e_{induced_faulted}}{R_{C2} + jX_{DNF}} \approx \frac{V_{out_healthy}}{R_{C2} + jX_{DNF}}. \quad (4.9)$$

The results from application of this formula are shown in the last row of Table J.2. In the future, a study should be performed on how to quickly estimate the X_{DNF} parameter value for a wide range of transformers. Perhaps, this value is affected by the transformer MVA capacity, coil voltage, coil resistance, and transformer short-circuit reactance. For now, based on the observations above for the 60 MVA autotransformer, its value initially can be estimated through multiplication of the single turn resistance value R_{C2} by a factor in the range [3.5, 7.7], i.e.,

$$X_{DNF} = [3.5, 7.7] \times R_{C2}. \quad (4.10)$$

It is useful to remember that two X_{DNF} values corresponding to these two factors from (4.10) are taken from Table 4.5. Factor 3.5 corresponds to the case when X_{DNF} is set to match the fault current and the factor 7.7 to match the delta current. The values between the two boundaries have not been tested.

4.9 Leakage Admittance Matrices: Properties and Sensitivity of Elements

This section explores the condition number, rank and sensitivity of elements of the transformer matrices calculated for the healthy and faulted Mirage (the base case).

4.9.1 Properties of Leakage Admittance Matrices

As explained in Subsection 2.2.6.3, the leakage admittance matrix of a transformer by itself is singular since it is formed based on the singularity property: the sum of the row/column elements in a matrix, described in p.u. values, is zero. However, when it is interconnected with the grounded core, like in the Hybrid model, the overall matrix is not singular anymore.

By definition, the condition number of a singular matrix is infinity. Computers cannot operate with such an undefined value, and therefore for computers, infinity should be

defined as a very large number based on their precision. Due to roundoff errors, for computations with double precision in MATLAB, a value of the order 10^{16} represents infinity and a value of the order 10^{-15} represents zero.

The rank of a singular matrix is less than the number of rows (columns). And since a singular matrix is rank deficient, it is noninvertible. The leakage admittance matrix $[A]$ is singular and, therefore, cannot be inverted to obtain a corresponding inductance matrix $[X]$. The leakage admittance matrices in this project are rank deficient and their condition numbers are of the order 10^{16} due to roundoff errors.

In this subsection, the following leakage admittance matrices are inspected for the condition number and rank:

- the healthy Mirage's leakage model – Table G.1,
- the healthy Mirage's full model – Table G.4,
- the coil volume internal fault leakage model: the base case – Table G.9, and
- the coil volume internal fault full model: the base case – Table G.12.

The condition number and rank of each matrix, represented in p.u. and Si values, are calculated with the MATLAB commands `cond()` and `rank()`. The studied matrices in Si are given in Appendix G. The results are shown in Tables 4.13 and 4.14 below.

Tables 4.13 and 4.14 confirm the rank deficiency and infinite condition number of the leakage admittance matrices for the healthy and faulted Mirage. Other cases for T2T faults as well as the cases for T2G faults will show similar results. However, if the overall transformer matrix (the one which includes the grounded core circuit) is extracted from a .atp file, it can be proved mathematically that the matrix is not singular. How to do this is a topic for future research.

Table 4.13. Condition number and rank for matrices of the healthy Mirage.

	healthy Mirage's leakage model: [A(9×9)] (Subsection 4.4.1)		healthy Mirage's full model: [A(12×12)] (Subsection 4.4.2)	
values	p.u.	Si	p.u.	Si
Matrix location	-	Table G.1	-	Table G.4
Condition number	2.849E16	7.198E16	5.285E16	3.855E17
Rank	6	6	9	9

Table 4.14. Condition number and rank for matrices of the faulted Mirage: the base case.

	coil volume internal fault leakage model: [A(11×11)] (Subsection 4.6.1)		coil volume internal fault full model: [A(14×14)] (Subsection 4.6.3)	
values	p.u.	Si	p.u.	Si
Matrix location	-	Table G.9	-	Table G.12
Condition number	4.526E17	2.997E22	2.649E17	2.110E22
Rank	8	8	11	11

4.9.2 Sensitivity of Elements of Leakage Admittance Matrices

First, this subsection explains the approach applied to identify the sensitivity of the matrix elements, and second, it offers the results along with a discussion.

4.9.2.1 Approach

Sensitivity of the matrix elements is studied by applying a perturbation of the off-diagonal elements by $\pm 10\%$, one at a time. This is performed on the leakage admittance matrices represented in p.u. values because the singularity property – summation of the row/column elements is zero – is valid for the matrices calculated in p.u. values. It is important to keep in mind:

- after perturbing an off-diagonal element, the sum of the row/column elements should still be preserved as zero and
- the leakage admittance matrix is symmetric.

Therefore, perturbation of one off-diagonal element in the leakage admittance matrix of any transformer leads to the corresponding perturbations of four elements in total:

- the off-diagonal element of interest in any row,
- the element located on the main diagonal in the same row – to preserve the singularity property in this row,
- the symmetric off-diagonal element in the corresponding column, and
- the element located on the main diagonal in the corresponding row – to also preserve the singularity property.

Perturbation of the off-diagonal elements is performed on phase A in the leakage admittance matrices of the following transformer models:

- the healthy Mirage's leakage model,
- the healthy Mirage's full model,
- the coil volume internal fault leakage model: the base case, and
- the coil volume internal fault full model: the base case.

The sensitivity analysis consists of two parts:

Part 1. Perturbation by $\pm 10\%$ helps to identify the most sensitive matrix elements. They are identified based on changes in the current amplitudes for the base load and short-circuit tests by at least $\pm 5\%$. These tests are required to confirm the sensitivity because changes in the leakage admittance matrix lead to changes in the short-circuit behavior of a transformer. The changes can be observed through the base load and short-circuit tests. For this part, the base load and short-circuit test currents from the new matrices are compared with the corresponding currents from the original matrix. Part 1 of the sensitivity analysis is performed on the matrices of the healthy Mirage and the matrices of the coil volume internal fault leakage and full models. The two models of the faulted Mirage are structured for the base case but the short is not applied and X_{DNF} is not inserted.

Part 2. After the most sensitive elements have been identified in Part 1, they are perturbed further by $\pm 40\%$ and $\pm 80\%$ in Part 2 to observe the tendencies of these sensitive elements to affect the certain currents. Part 2 of the sensitivity analysis is performed on the matrices of the coil volume internal fault leakage and full models for the base case T2T fault with X_{DNF} omitted. The analysis of Part 2 helps to understand if the X_{DNF} element can be avoided and only perturbations of the matrix elements can bring the results from the coil volume internal fault model closer to the results from the proprietary model. For simulations of Part 2, the short is applied while the Mirage autotransformer is supplying the base load as per Subsection 4.7.1. The currents from the coil volume internal fault leakage and full models are compared with the corresponding currents from the proprietary internal fault model.

Since for the sensitivity analysis, all the perturbations are performed in phase A only, the increased delta current is an expected outcome. X_{DNF} does not participate in both parts.

4.9.2.2 Results

Tables with the sensitivity analysis results are given in Appendix K. The analysis is performed for the healthy Mirage's leakage and full models, and for the coil volume internal fault leakage and full models, structured for the base case.

4.9.2.2.1 The Healthy Mirage

Tables K.1 – K.4 are related to the healthy Mirage's leakage model. Tables K.5 – K.8 are related to the healthy Mirage's full model. The healthy Mirage is subjected only to Part 1 of the sensitivity analysis.

Table K.1 shows the currents from the base load and short-circuit tests for the original leakage admittance matrix of the healthy Mirage's leakage model (this matrix is in Table G.1) and the cases when the three off-diagonal elements a_{12} , a_{13} , and a_{23} are perturbed by $+10\%$, one at a time. Table K.2 represents the values of Table K.1 as percentages with respect to the corresponding currents from the original matrix. For this, each current of the

original matrix is set as 100%. The sensitivity of the perturbed matrix elements is judged high when the currents deviate from the corresponding original ones by at least $\pm 5\%$. At that, if the current deviation is below $\pm 5\%$, but can be rounded to $\pm 5\%$, then the matrix element is still considered to be sensitive to the $+10\%$ perturbation.

The current percentages with deviations of about $\pm 5\%$ and more are highlighted in Table K.2. As mentioned in Subsection 4.9.2.1, since all the perturbations are performed in phase A only, the delta current rises, but it is not a good indicator of sensitivity. This is because the delta current rises from almost zero to some relatively small value, but the percentage change is very high. Therefore, for sensitivity of the matrix elements, the delta current rise and its percent change are not examined.

According to the results from Tables K.1 and K.2, the elements most sensitive to the $+10\%$ perturbation are those, located closer to the main diagonal, i.e., the elements a_{12} and a_{23} . The element a_{13} is the least sensitive because the corresponding currents are a little below the $\pm 5\%$ margin for the sensitivity threshold. In terms of the coil names, the most sensitive are the elements related to the binary reactances between the coil pairs series – common and common – delta. The element a_{13} related to the binary reactance of the coil pair series – delta is the least sensitive.

The description of Tables K.3 and K.4 is similar to that for Tables K.1 and K.2 but is related to the -10% perturbation (the original matrix is also in Table G.1). The results from Tables K.3 and K.4 identify the same elements as the most and least sensitive.

Table K.5 shows the currents from the base load and short-circuit tests for the original leakage admittance matrix of the healthy Mirage's full model (this matrix is in Table G.4). The cases when the six off-diagonal elements a_{12} , a_{13} , a_{14} , a_{23} , a_{24} , and a_{34} are perturbed by $+10\%$, one at a time, are also included. Table K.6 describes the values of Table K.5 as percentages with respect to the corresponding currents from the original matrix. Tables K.7 and K.8 are similar to Tables K.5 and K.6 but are related to the -10% perturbation (the original matrix is also in Table G.4).

The results from these four tables for the healthy Mirage's full model show that the elements a_{12} , a_{13} , and a_{23} are the most sensitive, but now the element a_{13} shows more sensitivity to the perturbations than in the healthy Mirage's leakage model. With the core circuit attached, the currents related to the $\pm 10\%$ perturbations of the element a_{13} exceed the $\pm 5\%$ sensitivity threshold.

4.9.2.2.2 Sectioned Common Coil but the Fault Is Not Applied

This subsection describes the sensitivity analysis results for the matrices of the coil volume internal fault model (the base case) corresponding to Part 1, i.e., the short is not applied.

Tables K.9 – K.12 are related to the coil volume internal fault leakage model (the base case). Each of the tables consists of two pieces. Tables K.13 – K.16 are related to the coil

volume internal fault full model (the base case). Each of the tables consists of three pieces. For this part of the sensitivity analysis, the short is not applied.

Tables K.9 and K.11 describe variations of the currents during the base load and short-circuit tests for perturbation of the original matrix elements of the coil volume internal fault leakage model (this matrix is in Table G.9) by +10% and -10%, respectively. The following 10 off-diagonal elements, one at a time, are perturbed: a_{12} , a_{13} , a_{14} , a_{15} , a_{23} , a_{24} , a_{25} , a_{34} , a_{35} , and a_{45} . Tables K.10 and K.12 describe the currents of Tables K.9 and K.11, respectively, as percentages with respect to the corresponding currents from the original matrix (Table G.9).

The results from Tables K.9 – K.12 show that the elements a_{12} , a_{13} , a_{23} , and a_{45} are the most sensitive based on the $\pm 5\%$ sensitivity threshold, i.e., the changes in currents from the base load and short-circuit tests exceed the $\pm 5\%$ margin. In terms of the coil names, these elements are related to the binary reactances between the coil pairs, respectively: series – sub-coil $C1$, series – sub-coil $C2$, sub-coil $C1$ – sub-coil $C2$, and sub-coil $C3$ – delta. The sub-coil $C2$ corresponds to the single turn. The elements a_{12} , a_{23} , and a_{45} are located near the main diagonal. The element a_{34} related to the binary reactance of the coil pair sub-coil $C2$ – sub-coil $C3$ and also located near the main diagonal is not sensitive to the $\pm 10\%$ perturbations. The element a_{13} is sensitive but is not close to the main diagonal.

Tables K.13 and K.15 describe variations of the currents during the base load and short-circuit tests for perturbation of the original matrix elements of the coil volume internal fault full model (this matrix is in Table G.12) by +10% and -10%, respectively. The following 15 off-diagonal elements, one at a time, are perturbed: a_{12} , a_{13} , a_{14} , a_{15} , a_{16} , a_{23} , a_{24} , a_{25} , a_{26} , a_{34} , a_{35} , a_{36} , a_{45} , a_{46} , a_{56} . Tables K.14 and K.16 describe the currents of Tables K.13 and K.15, respectively, as percentages with respect to the currents from the original matrix (Table G.12).

Results from Tables K.13 – K.16 show that the elements a_{12} , a_{13} , a_{23} , a_{35} , and a_{45} are the most sensitive based on the $\pm 5\%$ sensitivity threshold. This result coincides with the result from the coil volume internal fault leakage model, except the element a_{35} . With the core circuit added, the element a_{35} , related to the binary reactance of the coil pair sub-coil $C2$ – delta, also becomes sensitive.

It is important to keep in mind:

- This sensitivity analysis for the matrices of the coil volume internal fault model is performed for the base case, i.e., when the single T2T fault is located on the common coil at the height of 10% from the coil bottom. When the fault location changes, the widths of the sub-coils $C1$, $C2$, and $C3$ in the Cylindrical approach change, and consequently, the leakage admittance matrix changes. Due to this change, other elements might show higher sensitivity. However, the sensitivity should be similar since the sub-coil $C2$ always contains one turn.

- If the number of turns involved in the sub-coil $C2$ is increased, a similar sensitivity analysis should be performed.
- T2T faults on the other coils, i.e., series and delta, most probably should also lead to the higher sensitivity of the matrix elements located closer to the main diagonal.
- A similar analysis should be carried out to study sensitivity of the matrix elements for T2G faults. The severity of a T2G fault is defined by the number of turns involved counting from the neutral point. In this case, sensitivity of the matrix elements is expected to be slightly different for each step in severity.

4.9.2.2.3 Sectioned Common Coil and the Fault Is Applied

This subsection describes the sensitivity analysis results for the matrices of the coil volume internal fault model (the base case) corresponding to Part 2, i.e., the short is applied while the Mirage is supplying the base load as per Subsection 4.7.1.

The most sensitive elements of the coil volume internal fault leakage model from Part 1 (refer to Subsection 4.9.2.2.2) are perturbed by $\pm 40\%$ and $\pm 80\%$: a_{12} , a_{13} , a_{23} , and a_{45} . For the coil volume internal fault full model, the same elements and the element a_{35} are tested. The results are shown in Tables K.17 – K.20 for the coil volume internal fault leakage model, and in Tables K.21 – K.24 for the coil volume internal fault full model. The first two tables in each set, i.e., Tables K.17, K.18 and Tables K.21, K.22 show results for the $+40\%$ and -40% perturbations, respectively. The second two tables in each set, i.e., Tables K.19, K.20 and Tables K.23, K.24 show results for the $+80\%$ and -80% perturbations, respectively.

The goal is to observe how the H-terminal current I_H , the common coil current I_C , and the fault current I_F change with perturbations and how these results shift towards the results from the proprietary model. The delta current I_Δ is not a useful indicator here since it is present in excess for these cases. This is because perturbations are only performed in phase A.

It is important to remember that for the base case without the X_{DNF} element both in the coil volume internal fault leakage and full models, the common coil current I_C reverses its polarity and drops to a very low amplitude, refer to Tables I.14 and I.16 in Appendix I. Therefore, the desired effect of these perturbations is to increase I_C during the base case T2T fault making it closer to the value obtained from the proprietary model. The other desired effects are to decrease I_H and I_F .

The results in Tables K.17 – K.20 for the coil volume internal fault leakage model show that:

- Increase in the value of the element a_{13} (the coil pair series – sub-coil $C2$) tends to lead to the desired effect: $I_H\downarrow$, $I_C\uparrow$, and $I_F\downarrow$, both for the $+40\%$ and $+80\%$ perturbations.

- Decrease in the value of the element a_{12} (the coil pair series – sub-coil $C1$) tends to also lead to the desired effect: $I_H\downarrow$, $I_C\uparrow$, and $I_F\downarrow$, for the +40% perturbations. However, at the –80% perturbations, I_C grows too large, leading to the excess increase in I_H .
- Decrease in the value of the element a_{23} (the coil pair sub-coil $C1$ – sub-coil $C2$) tends to also lead to the desired effect: $I_H\downarrow$, $I_C\uparrow$, and $I_F\downarrow$, both for the +40% and +80% perturbations.
- Changes in the value of the element a_{45} (the coil pair sub-coil $C3$ – delta) does not lead towards the desired effect and its –80% perturbation produces 0.1E20 values in the .lis file. This indicates that the value of the element a_{45} cannot be perturbed that much but the –70% perturbation does not produce this issue.
- The $\pm 40\%$ and $\pm 80\%$ perturbations of the other elements tested in Part 2 do not lead towards the desired effect, therefore only the elements a_{12} , a_{13} , and a_{23} can be considered sensitive and at the same time lead towards the desired effects.

The results in Tables K.21 – K.24 for the coil volume internal fault full model show that:

- The current values corresponding to perturbations of the elements a_{12} , a_{13} , a_{23} are close to the ones from the coil volume internal fault leakage model, refer to Tables K.17 – K.20.
- The element a_{35} (the coil pair sub-coil $C2$ – delta), which shows its sensitivity in Part 1 due to addition of the core circuit, does not show the desired tendencies in Part 2 for the $\pm 40\%$ and $\pm 80\%$ perturbations.
- As in the coil volume internal fault leakage model, changes in the value of the element a_{45} (the coil pair sub-coil $C3$ – delta) does not lead towards the desired effect and its –80% perturbation produces an error message KILL=212 in the .lis file related to the iteration limit. This also indicates that the value of the element a_{45} cannot be perturbed that much. However, the –60% perturbation does not produce this issue, while the –70% perturbation still does.
- The –60% perturbation of the element a_{45} show that the coil volume internal fault leakage and full models produce very different amplitudes of the corresponding currents (refer to Table K.25). Perturbations of all other elements produce similar results for both models. Additional work has been carried out to verify that there is no error in the files related to the sensitivity check of the element a_{45} both for the coil volume internal fault leakage and full models.

Perturbations of single elements in the studied matrices of Part 2 do not produce the results which shift closer to those from the proprietary model. However, the results obtained show that perturbation of certain matrix elements leads to the desired changes in the faulted phase A current amplitudes: $\downarrow I_H$, $\uparrow I_C$, and $\downarrow I_F$.

The results from Part 2 of the sensitivity studies provide additional justification that the X_{DNF} element is needed. In general, it seems that simultaneous perturbation of several original matrix elements should be performed to bring the results from the coil volume internal fault model without X_{DNF} closer to the results from the proprietary model. For this,

an optimization algorithm should be developed to identify which elements to perturb and by how much.

It is hard to state firmly whether the best solution from this algorithm will be successful or not. However, the author of this dissertation is leaning towards “no”. This is because the leakage admittance matrix represents the coupled inductances and perturbations of the matrix elements do not seem to reduce the fault current effectively and, besides, such perturbations in one phase only increase the delta current. The X_{DNF} element, which is uncoupled from the matrix elements, effectively reduces the fault current and consequently the delta current.

This guess is also confirmed by the results from Table I.14 (refer to Subsection 4.7.3.1), which shows that without X_{DNF} the currents in the healthy phases are not too far from those of the proprietary model. The reason why they do not match exactly is that the overestimated delta current in the faulted phase A produces incorrect balancing ATs for all the phases.

Perturbations of the elements in the healthy phases are not needed. Perturbations of the elements in one phase only will lead to improper delta current amplitude. Therefore, only the faulted phase A should be manipulated to improve the current prediction, i.e., insertion of the self-reactance X_{DNF} in the shorted turn.

4.10 Net MMF per Leg and Mean Net MMF per Leg

This section describes calculation of the net mmf per core leg for the following options of the models for the healthy Mirage:

- the XFMR Mirage,
- the healthy Mirage’s leakage model,
- the healthy Mirage’s full model,
- the coil volume internal fault leakage model: the common coil is sectioned for the base case and the short is not applied,
- the coil volume internal fault full model: the common coil is sectioned for the base case and the short is not applied,

as well as for the following options of the models for the faulted Mirage:

- the proprietary internal fault model,
- the coil volume internal fault leakage model: the common coil is sectioned for the base case and the short is applied,
- the coil volume internal fault full model: the common coil is sectioned for the base case and the short is applied.

4.10.1 Approach

For calculation of the net mmf per core leg in each model, the current amplitudes and their phase angles from the corresponding .lis files are used. Since each core leg accommodates series, common (healthy or sectioned) and delta coils, the phasor values of their ampere-turns (ATs) are summed in each phase to obtain the net mmf per leg phasors. It is important to keep in mind the polarity marking of the coils and sub-coils. As mentioned at the beginning of Chapter 4, the polarity marking of the coils and sub-coils is always on the top, refer also to Figs. 4.2, 4.3 and F.1, F.2, F.3 in Appendix F.

4.10.2 Results

Appendix L contains all the tables with the results for each case. Analysis of these results is given below.

4.10.2.1 *The Healthy Mirage*

The set of Tables L.1 – L.3 is related to calculation of the net mmf per leg and the mean net mmf per leg for the XFMR Mirage. Table L.1 shows phasor values of the coil currents in each phase and the corresponding ATs. Table L.2 shows phasors of the net mmf per leg and amplitude of the mean mmf per leg. Table L.3 shows differences in the net mmf per leg between the leg pairs and the same differences in percent with respect to the amplitude of the mean net mmf per leg. All other table sets in Subsection 4.10.2 also contain three tables displaying the same type of data.

It is useful to remember that the Hybrid model (XFMR) is the built-in model in ATPDraw, and therefore, the XFMR Mirage is used for benchmarking purposes.

Tables L.2 and L.3 show that there is a little net mmf imbalance in amplitudes between the core legs although they are symmetrically displaced by 120° . Probably, this imbalance appears due to the geometrical unsymmetry of the core structure. This result appears to contradict reference [42], which states that fluxes in each core leg are balanced since, in three-legged transformers, ATs on any pair of core legs are always equal. However, the results of Tables L.2 and L.3 are obtained through computer simulations. Results from transformer modeling will always contain some roundoff and other errors. But, operation of a real transformer should not contradict the statement of [42].

Tables L.4 – L.6 are related to calculation of the net mmf per leg and the mean net mmf per leg of the healthy Mirage's leakage model. In this case, since the core circuit representing the unsymmetrical magnetic structure is not included, the phase net mmf's are perfectly balanced in Table L.5. Compared to the XFMR Mirage, the mean net mmf decreases by 28.6%. The current amplitudes in Table L.4 differ from those of the XFMR Mirage in Table L.1 by insignificant amounts. This provides the perfect balance of the phase net mmf's in Table L.5.

Tables L.7 – L.9 are related to calculation of the net mmf per leg and the mean net mmf per leg of the healthy Mirage’s full model. Since the core circuit is present in this model, the results from these tables are fully in agreement with the ones of the XFMR Mirage. Comparing the results from Tables L.7 – L.9 with those from Tables L.4 – L.6, the influence of the core circuit is observed through changes in the net mmf per leg and the mean net mmf per leg.

4.10.2.2 Sectioned Common Coil but the Fault Is Not Applied

Although the phase A common coil is sectioned for the base case, the transformer remains healthy since the short is not applied and X_{DNF} is not inserted.

Tables L.10 – L.12 are related to calculation of the net mmf per leg and the mean net mmf per leg of the coil volume internal fault leakage model. In this case, even if the core circuit is not included, the phase net mmf’s are not balanced: neither the amplitudes nor the phase angles. The balance should remain if the common coil is sectioned in all the phases. If it is sectioned only in one phase, the delta current of $1.3 A_{peak}$ appears. Due to this, the mean mmf per leg (Table L.11) rises by 3.5% compared to that of the healthy Mirage’s leakage model (Table L.5). However, comparison of the series and common coil currents (Table L.10) with those of the healthy Mirage’s leakage model (Table L.4) or the healthy Mirage’s full model (Table L.7) shows insignificant differences. Therefore, segmentation of the common coil in the faulted phase only is possible since it leads only to a minor rise of the delta current amplitude.

Tables L.13 – L.15 are related to calculation of the net mmf per leg and the mean net mmf per leg of the coil volume internal fault full model. In this case, the imbalance due to segmentation of the phase A common coil increases since the core circuit is present in the model. The delta current is also $1.3 A_{peak}$, the mean net mmf per leg increases by 40.3% compared to the coil volume internal fault leakage model (Table L.11) and by 3.6% compared to the healthy Mirage’s full model (Table L.8). However, comparison of the series and common coil currents (Table L.13) with those of the healthy Mirage’s full model (Table L.7) or the coil volume internal fault leakage model (Table L.10) shows insignificant differences.

Table 4.14 below shows a comparison of the phase net mmf’s and the mean phase net mmf’s for the models from Subsections 4.10.2.1 and 4.10.2.2. The last column describes percentage of the mean phase net mmf’s with respect to the mean phase net mmf of the XFMR Mirage. Imbalances of the phase net mmf’s observed in Table 4.14 are due to the core circuit (possible imperfections of the core modeling) and segmentation of the phase A common coil.

It is concluded that the faulted common coil can be sectioned only while leaving the common coils of the healthy phases intact since this adds a negligible amount of imbalance to the coil currents and related mmf’s. Also, as stated above, the leakage models have lower

mean net mmf per leg because they do not contain the core circuit which represents an unsymmetrical magnetic structure.

Table 4.14. Comparison of the phase net mmf's and the mean phase net mmf's for the models from Subsections 4.10.2.1 and 4.10.2.2.

	Net mmf per leg, Ampere-turns _{peak}			Mean net mmf per leg	Percentage for mean net mmf per leg
	A	B	C		
XFMR Mirage	310 @-3.7°	265.6 @-123.7°	354.4 @116.3°	310	100%
Healthy Mirage's leakage model	221.2 @-3.7°	221.2 @-123.7°	221.2 @116.3°	221.2	71.4%
Healthy Mirage's full model	310 @-3.7°	265.6 @-123.7°	354.4 @116.3°	310	100%
Coil volume internal fault leakage model without X_{DNF} and short	182 @-3.7°	235.2 @-81.7°	269.5 @80.6°	228.9	73.8%
Coil volume internal fault full model without X_{DNF} and short	270.8 @-3.7°	306.7 @-92.9°	386 @92.2°	321.2	103.6%

4.10.2.3 Sectioned Common Coil and the Fault Is Applied

Tables L.16 – L.18 are related to calculation of the net mmf per leg and the mean net mmf per leg for the base case T2T fault simulated by the proprietary model where the core representation is included. All other sets of tables in Subsection 4.10.2.3 also correspond to the base case T2T fault and are compared with the results from the proprietary model.

As can be observed from Tables L.17 and L.18, the phase net mmf's are not balanced, which likewise can be explained by imperfections of the core modeling. When comparing the mean net mmf per leg produced by the proprietary model (Table L.17) with that produced by the healthy Mirage's full model (Table L.8), it is observed that during the base case T2T fault, the mean net mmf per leg increases by 34.3%. Table 4.15 compares the phase net mmf's and the mean phase net mmf's of the proprietary model (the base case T2T fault) with those of the XFMR Mirage (the healthy transformer).

Table 4.15. Comparison of the phase net mmf's and the mean phase net mmf's for the healthy and faulted Mirage.

Core effect is included in both models	Net mmf per leg, Ampere-turns _{peak}			Mean net mmf per leg	Percentage for mean net mmf per leg
	A	B	C		
XFMR Mirage: the healthy transformer	310 @-3.7°	265.6 @-123.7°	354.4 @116.3°	310	100%
Proprietary model: The base case T2T fault	414.5 @-143.8°	484.8 @-40.3°	349.7 @-155.3°	416.3	134.3%

Since no information about the core circuit of the proprietary model is available, the comparison in Table 4.15 is approximate. As it is stated above, the mean net mmf per leg increases during the base case T2T fault by 34.3%. The net mmf in the faulted phase A increases by 33.7%. However, a more considerable rise of the net mmf occurs in the healthy phase B – by 82.5%, while in the healthy phase C there is a little reduction in the net mmf – by 1.3%.

As described in Subsection 4.7.3, the results of simulation of the base case T2T fault show that the coil volume internal fault model requires insertion of X_{DNF} in the shorted turn to obtain results similar to those of the proprietary model. Since it is preferable to match the delta current of the coil volume internal fault model with the one of the proprietary model, instead of the fault current, the value of X_{DNF} should be 2.9 m Ω (refer to Table 4.5). Again, this is because the core circuit adopted from the XFMR Mirage does not provide the desired effect of reducing the delta current. To compensate for the insufficient influence of the core circuit, the delta current is reduced by adjusting X_{DNF} .

In this case, according to Subsection 4.7.3.1, the maximum deviation among all the currents, except the fault current, from those of the proprietary model is 7% – for the phase A series coil current, and it is much lower for other currents in the faulted phase A. The currents of the healthy phases are matching perfectly between the two models.

The results offered in Subsection 4.7.3 contain only the current amplitudes. The phase angles are omitted to avoid too much detail in the preliminary results. In the following two Subsections 4.10.2.3.1 and 4.10.2.3.2, calculation of the phase net mmf's and the mean phase net mmf's for the base case T2T fault requires using the phasor values of the currents. Therefore, a more detailed comparison of results for the base case T2T fault is available here.

Further in Subsection 4.10.2.3, the mmf results for the base case T2T fault are shown for the coil volume internal fault leakage and full models with the following pattern:

- the fault is applied, X_{DNF} is not inserted,
- the fault is applied, $X_{DNF}=1.293$ m Ω to match the fault current with that of the proprietary model,
- the fault is applied, $X_{DNF} = 2.9$ m Ω to match the delta current with that of the proprietary model.

4.10.2.3.1 The Coil Volume Internal Fault Leakage Model

For this model, the core circuit is omitted. The table sets mentioned below are related to calculation of the net mmf per leg and the mean net mmf per leg for the base case T2T fault simulated by the coil volume internal fault leakage model:

- Tables L.19 – L.21, where X_{DNF} is not inserted,
- Tables L.22 – L.24, where $X_{DNF} = 1.293$ m Ω is inserted in the faulted turn, and

- Tables L.25 – L.27, where $X_{DNF} = 2.9 \text{ m}\Omega$ is inserted in the faulted turn.

Comparison of the current phasors for these three options with those from the proprietary model is given in Table L.28. A detailed description of the comparison of the current amplitudes for these options is given in Subsection 4.7.3.1. Below, the comparison of the current phase angles is discussed. From Table L.28 it is observed:

- The option without X_{DNF} produces both the current amplitudes and phase angles which are far from the ones of the proprietary model.
- The option where $X_{DNF} = 1.293 \text{ m}\Omega$ shows that the phase angles are very close to those of the proprietary model. The current amplitudes and their phase angles of the series and common (the healthy part) coils in the faulted phase A as well as of the fault current are very close to those of the proprietary model (the maximum deviation in amplitudes is 2%, refer to Subsection 4.7.3.1). However, the delta current amplitude is roughly twice that of the proprietary model, although their phase angles are relatively close. Therefore, this affects the closeness of the current amplitudes in the healthy phases of the two compared models (the maximum deviation is 4%, see Subsection 4.7.3.1). But, the phase angles are very close. This result demonstrates that a core circuit should have such parameters to reduce the delta current amplitude considerably, ideally by half. However, the phase angle should not change much. This should reduce the maximum deviation in the healthy phases.
- The option where $X_{DNF} = 2.9 \text{ m}\Omega$ shows that the phase angles of the series and common (the healthy part) coil currents in the faulted phase A are further from those of the proprietary model when compared to the option where $X_{DNF} = 1.293 \text{ m}\Omega$. Although, the amplitude and phase angle of the delta current are well matched with those of the proprietary model. The maximum deviation of the series and common (the healthy part) coil current amplitudes in the faulted phase A from those of the proprietary model is 7%, refer to Subsection 4.7.3.1. However, in this case, the current phasors of the healthy phases are matched almost perfectly with those of the proprietary model. This result demonstrates the importance of matching the delta current phasor of the coil volume internal fault model with that of the proprietary model since this leads to the same balancing ATs in the two models.

Table 4.16 below shows a comparison of the phase net mmf's and the mean phase net mmf's for three options of the coil volume internal fault leakage model versus those of the proprietary model. The last column describes percentage of the mean phase net mmf's with respect to the mean phase net mmf of the proprietary model.

As observed from Table 4.16, the option where $X_{DNF} = 2.9 \text{ m}\Omega$ produces the net mmf per leg and the mean net mmf per leg which are the closest to those of the proprietary model.

Table 4.16. Comparison of the phase net mmf's and the mean phase net mmf's for the options from Subsection 4.10.2.3.1: the coil volume internal fault leakage model vs. the proprietary model, the base case.

	Net mmf per leg, Ampere-turns _{peak}			Mean net mmf per leg	Percentage for mean net mmf per leg
	A	B	C		
	Proprietary model				
	414.5 @-143.8°	484.8 @-40.3°	349.7 @-155.3°	416.3	100%
	Coil volume internal fault leakage model: options				
X_{DNF} omitted	2897.1 @166.1°	3336.7 @-170.5°	3519.8 @164.2°	3251.2	781%
$X_{DNF} = 1.293 \text{ m}\Omega$	825 @102.2°	807.5 @111.8°	1089.8 @101.9°	907.4	218%
$X_{DNF} = 2.9 \text{ m}\Omega$	497.1 @71.7°	469.4 @127.4°	595.6 @99°	520.7	125.1%

4.10.2.3.2 The Coil Volume Internal Fault Full Model

For this model, the core circuit adopted from the XFMR Mirage is attached to the corresponding leakage model.

The table sets mentioned below are related to calculation of the net mmf per leg and the mean net mmf per leg for the base case T2T fault simulated by the coil volume internal fault full model:

- Tables L.29 – L.31, where X_{DNF} is not inserted,
- Tables L.32 – L.34, where $X_{DNF} = 1.293 \text{ m}\Omega$ is inserted in the faulted turn, and
- Tables L.35 – L.37, where $X_{DNF} = 2.9 \text{ m}\Omega$ is inserted in the faulted turn

Comparison of the current phasors for these three options with those from the proprietary model is given in Table L.38. A detailed description of the comparison of the current amplitudes for these options is given in Subsection 4.7.3.2. Attachment of the core circuit adopted from the XFMR Mirage yields insignificant changes in the current amplitudes (by 0.1 and 0.2 A_{peak}) compared to those from Table L.28 while keeps the phase angles the same. Therefore, the observations derived in Subsection 4.10.2.3.1 for the results from Table L.28 are valid for Table L.38 as well.

Table 4.17 below shows a comparison of the phase net mmf's and the mean phase net mmf's for three options of the coil volume internal fault full model versus those of the proprietary model. The last column describes percentage of the mean phase net mmf's with respect to the mean phase net mmf of the proprietary model.

Table 4.17. Comparison of the phase net mmf's and the mean phase net mmf's for the options from Subsection 4.10.2.3.1: the coil volume internal fault full model vs. the proprietary model, the base case.

	Net mmf per leg, Ampere-turns _{peak}			Mean net mmf per leg	Percentage for mean net mmf per leg
	A	B	C		
	Proprietary model				
	414.5 @-143.8°	484.8 @-40.3°	349.7 @-155.3°	416.3	100%
	Coil volume internal fault full model: options				
X_{DNF} omitted	2811.9 @167.5°	3601.9 @-169°	3597.8 @162.5°	3337.2	801.6%
X_{DNF} = 1.293 mΩ	521.2 @80.4°	760.5 @117.3°	1262.7 @103.9°	848.1	203.7%
X_{DNF} = 2.9 mΩ	330.8 @38.7°	448.5 @138.2°	767 @102.9°	515.4	123.8%

As observed from Table 4.17, the option where $X_{DNF} = 2.9 \text{ m}\Omega$ produces the net mmf per leg and the mean net mmf per leg which are the closest to those of the proprietary model, likewise in Table 4.16. Attachment of the core circuit brings more imbalance in the phasors of the net mmf per leg, as expected, refer to Table 4.17 versus Table 4.16.

Table 4.18 below shows the overall comparison of the mean net mmf per leg for the coil volume internal fault leakage and full models versus the one of the proprietary model.

Table 4.18. Comparison of the mean phase net mmf's for the options from Subsections 4.10.2.3.1 and 4.10.2.3.2.

	Proprietary model: core is included	Coil volume internal fault leakage model: options; core is omitted			Coil volume internal fault full model: options; core is included		
		X_{DNF} omitted	X_{DNF} 1.293 mΩ	X_{DNF} 2.9 mΩ	X_{DNF} omitted	X_{DNF} 1.293 mΩ	X_{DNF} 2.9 mΩ
		1	2	3	4	5	6
Mean net mmf per leg, Ampere-turns _{peak}	416.3	3251.2	907.4	520.7	3337.2	848.1	515.4

As observed from Table 4.18, when X_{DNF} is present in the shorted turn and the core circuit is attached, the mean net mmf per leg reduces insignificantly, refer to columns 4 and 7. When the upgraded core circuit, suitable for simulation of internal faults, will be attached, then more mmf reduction is expected in columns 5, 6, 7 compared to columns 1, 2, 3 because the delta current should reduce considerably. Since the existing core model does not take care of the delta current reduction during faults, this is achieved by adjusting the value of X_{DNF} , refer to columns 6 and 7.

Among all the options of the coil volume internal fault leakage and full models explored in Subsections 4.10.2.3.1 and 4.10.2.3.2, the results closest to those of the proprietary model for the base case T2T fault are observed for the options, where $X_{DNF} = 2.9 \text{ m}\Omega$ is inserted in the faulted turn, i.e., the delta current is matched according to Assumption 2 of Subsection 4.5.1:

- In terms of the current phasors – the corresponding currents are almost equal in the coil volume internal fault leakage and full models, refer to Tables L.28 and L.38.
- In terms of the mean net mmf per leg – the coil volume internal fault full model shows a slightly better result over the coil volume internal fault leakage model, refer to Table 4.18. Therefore, even the adopted core circuit offers some effect of mmf reduction which, of course, should be much more prominent with the upgraded core circuit.

4.11 Model Implementation Discussion

Results from the coil volume internal fault model were not satisfactory for a long time. Attachment of the core circuit adopted from the XFMR Mirage did not help to improve the results. It took quite a long time to realize that the fault current in the shorted turn should be limited by an uncoupled inductance inserted in this turn. It represents the self-inductance of the shorted turn related to the considerable amount of the leakage flux which is not linked with the healthy portions of the faulted coil and with the healthy coils. It is named the “do-nothing” flux and designated as X_{DNF} in this dissertation.

With this realization, the understanding came that successful benchmarking of the coil volume internal fault model with the proprietary model can be achieved through matching the fault current phasor of the two models by setting $X_{DNF} = 1.293 \text{ m}\Omega$. Thus, nearly perfect agreement of the currents in the faulted phase A with those from the proprietary model was achieved. Simultaneously, the delta current amplitude was twice that from the proprietary model. Since the balancing ATs were not equal in these two models, this led to the less accurate agreement of the current phasors in the healthy phases. The maximum current amplitude deviation was 4%. Therefore, the question about the high delta current amplitude still was remaining – how to reduce it by half?

As mentioned in Subsection 3.3.2, the iron core of three-legged transformers partially offers a low reluctance path for the zero-sequence flux. Thus, the zero-sequence inductance is increased compared to the air core. This, in turn, reduces the zero-sequence current circulating in the delta winding. This is the desired effect of the core circuit attachment.

However, attachment of the existing core circuit did not affect the delta current and since then it was clear that this core representation was not suitable for simulation of internal faults. A clear understanding of how to develop the new core circuit was not forthcoming: what are all the expectations from attachment of the proper core circuit? Later, it became clear that the core circuit should affect the delta current directly. In other words, the proper core circuit should remove the previously obtained 4% current deviation in the healthy

phases. Since the delta current is a consequence of an imbalance – due to the fault current – it is also expected that the upgraded core circuit might affect the fault current indirectly. Also, it seems that the core circuit needs to be re-derived for the faulted phase only.

Next, the understanding came that even if the existing core circuit did not reduce the delta current amplitude, this current can be reduced by setting $X_{DNF} = 2.9 \text{ m}\Omega$ to match the delta current of the two models. In this case, as expected, nearly perfect agreement of the current phasors in the healthy phases was achieved between the two models. With the delta current phasors matched between the two models, the fault current amplitude became half of that from the proprietary model. This, in turn, affected the almost perfect agreement of the series and common coil (the healthy part) current phasors in the faulted phase A leading to a maximum current amplitude deviation of 7% between the two models.

The best results from the coil volume internal fault full model based on the Cylindrical approach for the base case T2T fault would be obtained through the following three steps:

- Matching the fault current of the coil volume internal fault leakage model by inserting $X_{DNF} = 1.293 \text{ m}\Omega$. With this, the delta current is $138.8 \text{ A}_{\text{peak}}$, refer to Table I.14, columns 1 and 4.
- Attaching a core circuit with the parameters such that the delta current decreases considerably, or ideally, from $138.8 \text{ A}_{\text{peak}}$ down to near the value of $63.5 \text{ A}_{\text{peak}}$ from the proprietary model.
- Tuning the value of X_{DNF} to match the fault current value again since attachment of the core circuit might decrease the fault current.

This three-step implementation should lead to more practical results instead of inserting $X_{DNF} = 2.9 \text{ m}\Omega$ for matching the delta current. Although this value compensates for the poor core representation during the base case T2T fault, it reduces the fault current amplitude in the shorted turn almost in half compared to the one from the proprietary model. The same three-step scheme should be applied to obtain results for other internal faults and the values of X_{DNF} should be recalculated. The core parameters in the faulted phase, related to the three magnetic regions, should also be recalculated for each internal fault. Also, it is desired that the transformer manufacturer would share their details on the core model, at least the zero-sequence inductance value.

This detailed understanding, appearing simple now, occurred too late in order to develop the appropriate core circuit to be a part of this dissertation. However, expectations for the upgraded core model have been well documented here, which should assist future researchers in developing the appropriate core circuit. Therefore, derivation of the proper core circuit for the faulted phase, which affects the delta current in an expected way, is the main step in advancing the coil volume internal fault full model based on the Cylindrical approach.

The description above is related to “how it should be”, but the results in Chapter 5 are based on “how it is for now”. To summarize, the present state of the coil volume internal fault model is shown in Table 4.19 on the basis of the base case T2T fault.

Since even the existing core representation offers some mmf reduction compared to the air core (refer to Table 4.18), the highlighted option of the coil volume internal fault full model in Table 4.19 is considered the best so far. The coil volume internal fault full model from Table 4.19, developed on the basis of the base case T2T fault, is adopted for generation of results in Chapter 5.

Table 4.19. The present state of the coil volume internal fault leakage and full models: based on the base case T2T fault.

Coil volume internal fault model	Leakage model: core is omitted	X_{DNF} omitted	Unsatisfactory results
		X_{DNF} 1.293 m Ω	Currents in the series and common (the healthy part) coils of the faulted phase A almost perfectly match with the ones of the proprietary model with the max deviation of 2%. The fault current is matched. The delta current is twice high. The max deviation of the series and common coil currents in the healthy phases is 4%.
			Mean net mmf per leg is 907.4 Ampere-turns _{peak}
		X_{DNF} 2.9 m Ω	Currents in the healthy phases perfectly match with the ones of the proprietary model. The fault current reduces by half. The delta current is matched. The max deviation of the series and common coil currents in the faulted phase A is 7%.
			Mean net mmf per leg is 520.7 Ampere-turns _{peak}
	Full model: core is adopted from XFMR Mirage	X_{DNF} omitted	Unsatisfactory results
		X_{DNF} 1.293 m Ω	Currents in the series and common (the healthy part) coils of the faulted phase A almost perfectly match with the ones of the proprietary model with the max deviation of 2%. The fault current is matched. The delta current is twice high. The max deviation of the series and common coil currents in the healthy phases is 4%.
			Mean net mmf per leg is 848.1 Ampere-turns _{peak}
		X_{DNF} 2.9 m Ω	Currents in the healthy phases perfectly match with the ones of the proprietary model. The fault current reduces by half. The delta current is matched. The max deviation of the series and common coil currents in the faulted phase A is 7%.
			Mean net mmf per leg is 515.4 Ampere-turns _{peak}

5 RESULTS AND OBSERVATIONS

Chapter 5 describes the results of simulations of steady-state single T2T faults located at different heights on the phase A common coil of the Mirage autotransformer. These results are compared with the data obtained from the coil volume internal fault model and from the proprietary internal fault model. The leakage admittance matrix of for each case is built using the Cylindrical approach. As mentioned in Subsection 4.7.2.1, the Rogowski factor for cylindrical windings K_{cyl} is neglected. The core representation is adopted from the XFMR Mirage.

5.1 Overview

Refer to Fig. 3.17 for the assumptions of the T2T fault representation by the Cylindrical approach. Fig. 5.1 below shows a representation of the single T2T fault in the middle of the common coil.

Fig. 5.1. A single T2T fault in the middle of the phase A common coil:
a) simplified reality and representation by the Segmented approach,
b) representation by the Cylindrical approach.

Single T2T faults modeled using the Cylindrical approach are located at the following heights of the phase A common coil of the Mirage autotransformer:

- 10% of the common coil height counting from its bottom. This is equivalent to the height of 10% of the total common coil turns, refer to Section 4.5. The sub-coil C3 contains 10% of the common coil turns. This case is named “bottom 10%”, or the base case according to Chapter 4.
- 30% of the common coil height counting from its bottom. The sub-coil C3 contains 30% of the common coil turns. This case is named “bottom 30%”.
- 50% of the common coil height counting from its bottom. The sub-coil C3 contains 50% of the common coil turns. This case is named “bottom 50%” and corresponds to the T2T fault in the coil middle.

- 30% of the common coil height counting from its top. The sub-coil *CI* contains 30% of the common coil turns. This case is named “top 30%”.
- 10% of the common coil height counting from its top. The sub-coil *CI* contains 10% of the common coil turns. This case is named “top 10%”.

The cases “bottom 10%” and “top 10%”, and “bottom 30%” and “top 30%” represent two pairs of symmetrical locations. Appendix M contains Tables M.1 – M.10 with the results for the five cases mentioned above. Each case is represented with two tables:

- The first table compares the current phasors from the proprietary model with those from the coil volume internal fault leakage and full models with three X_{DNF} values:
 - X_{DNF} is not inserted,
 - $X_{DNF} = 1.293 \text{ m}\Omega$ to match the fault current, and
 - $X_{DNF} = 2.9 \text{ m}\Omega$ to match the delta current.
- The second table has the same structure as the first one, but offers the results of the first table as deviations with respect to the results from the proprietary model:
 - each current phasor of the proprietary model is set as 100% amplitude @ 0° phase shift,
 - the current amplitudes from the coil volume internal fault leakage and full models are calculated as percentages with respect to the corresponding ones from the proprietary model,
 - the current phase angles from the coil volume internal fault leakage and full models are calculated as phase shifts in degrees with respect to the corresponding ones from the proprietary model.

The highlighting in the second table requires explanation. The cells highlighted in light brown are not used when comparing the models because:

- In the cases where X_{DNF} is omitted the results are poor.
- When the fault current is matched, the delta current is poorly represented, which is acceptable as an intermediate result and unacceptable as the ultimate result obtained from the coil volume internal fault model.
- When the delta current is matched, the fault current is poorly represented, but it is still high enough to be considered as the fault current. This is an acceptable result since, in this case, the transformer H-, X-, and Y-terminal currents generated by the coil volume internal fault model based on the Cylindrical approach show good agreement with those from the proprietary model. These currents are of primary importance since they represent an input set into a transformer protection relay.

Matching the delta current means that all the phases in the two models have the same amount of balancing ATs and the fault impact on the currents in the healthy phases is the same for both models. Sometimes, transformers have built-in CTs in the delta windings. However, the fault current cannot be measured, yet its value can be estimated by transformer manufacturers if they have an appropriate transformer model. Also, the fault current can be roughly approximated based on the fact that during internal faults, the copper

conductors quite often melt. The current is estimated to be that required to raise the copper conductor temperature up to the melting point of 1,083 °C.

The red-colored numbers in the second table correspond to the maximum values in each phase for each option. The cells highlighted in blue show the maximum deviation of the current amplitude in percent and phase angle in degrees, also for each option and each phase.

5.2 Criteria of Good/Acceptable Results

To identify results as good/acceptable or unacceptable, two criteria are developed.

5.2.1 Criterion 1

This criterion is applied to the proposed model without the core circuit, i.e., the coil volume internal fault leakage model.

According to transformer physics, the delta current in an air-core transformer is higher than that in the same transformer with an iron-core because of the high reluctance path for the leakage flux in the former, refer to Subsection 3.3.2. The coil volume internal fault leakage model does not include the core effect, and the proprietary model includes the core effect. Therefore, if the delta current amplitude predicted by the coil volume internal fault leakage model is higher than that from the proprietary model, then development and implementation of the proposed model are on the correct path.

5.2.2 Criterion 2

Of primary interest is the coil volume internal fault full model, where the delta current is matched with that of the proprietary model.

In this chapter, the results of the single T2T fault simulations produced by the coil volume internal fault full model based on the Cylindrical approach are considered good/acceptable if they fall within the following deviation criterion with respect to the results from the proprietary model:

- $\pm 10\%$ for current amplitudes and
- $\pm 18^\circ$ (5% of the full 360° circle) for current phase shifts.

Since the presence of the adopted core circuit in the coil volume internal fault full model shows almost no effect on the current amplitudes (less than 0.5% change) and zero effect on the current phase angles compared to those of the coil volume internal fault leakage model, it should be understood that when the results as per Criterion 2 are described for one model, they are valid for the other.

5.3 Results

When a single T2T fault changes its location along the common coil height, two major changes of the model parameters are required:

- The leakage admittance matrix should be recalculated.
- Resistances of the sub-coils $C1$ and $C3$ of the faulted coil should be recalculated. Since this is always a single T2T fault, the sub-coil $C2$ always contains just one turn and its resistance is constant for all the fault locations, $R_{C2} = 0.37464 \text{ m}\Omega$.

Refer to Tables M.11 and M.12 for the sets of the binary reactances and coil resistances.

The option, where the fault current is matched, is an important intermediate step where it is observed by how much the delta current should be changed, refer to Subsection 4.7.3.2. Since according to the results presented in Subsections 4.7.3 and 4.10.2.3, attachment of the existing core circuit does not impact the delta current, an appropriate value of X_{DNF} is chosen for each fault location to compensate for the weakness of the core model and to obtain the proper delta current amplitude. Without X_{DNF} , results would show overestimated currents, refer to Subsection 3.4.2.

5.3.1 The Case “Bottom 10%” – the Base Case

Refer to Fig. 3.17 again for visualization of this case. The corresponding tables with results in Appendix M are Tables M.1 and M.2. The detailed description of the results for the case “bottom 10%”, i.e., the base case, is given in Subsections 4.7.3 and 4.10.2.3.

Inspecting the delta current prediction by the coil volume internal fault leakage model, the initial focus is on the option where the fault current is matched. This is the intermediate step. It is marked with red in Table 5.1 below. Table 5.1 is a snapshot of this data from Table M.1.

Table 5.1. Delta current prediction:
the coil volume internal fault leakage model, case “bottom 10%” (the base case).

I, A_{peak}	Proprietary model: core is included	Coil volume internal fault leakage model: Cylindrical approach, K_{cy} neglected		
		X_{DNF} omitted	$X_{DNF} = 1.293 \text{ m}\Omega$ for matching I_F	$X_{DNF} = 2.9 \text{ m}\Omega$ for matching I_Δ
Delta current	63.5@-83°	517.7@-13°	138.8@-79° $I_\Delta > I_{\Delta_proprietary}$	63.5@-87°

Table 5.1 shows that the delta current, marked with red, is higher than that of the proprietary model, which agrees with Criterion 1. This result also suggests that the delta current amplitude, marked with red, should be reduced by 2.2 times to match that from the proprietary model, where the core is included. Ideally, this reduction should be handled by attachment of the re-derived core circuit with the carefully determined parameters and then

the X_{DNF} value should be fine-tuned as described in Subsection 4.7.3.2. Otherwise, at the present state of the research, it is achieved by setting $X_{DNF} = 2.9 \text{ m}\Omega$ since attachment of the core circuit adopted from the XFMR Mirage produces only insignificant changes in currents, refer Table M.1 and Subsections 4.7.3, 4.10.2.3.

Based on Tables M.1 and M.2 and to summarize the results given in Subsections 4.7.3 and 4.10.2.3, the data for the case “bottom 10%” are given in Table 5.2 below. It shows the maximum per phase deviations of the current amplitudes (in percent) and phase angles (in degrees): the coil volume internal fault full model with respect to the proprietary model. The column of primary interest, where the delta current is matched, is highlighted in green, with the maximum deviations marked with red. The column where the fault current is matched is the intermediate step and its results are discussed further in Subsection 5.3.6.2.

For the same case produced by the coil volume internal fault leakage model, these deviations are insignificantly different, refer to Table M.2.

Table 5.2. Maximum per phase deviations of results: the coil volume internal fault full model with respect to the proprietary model, case “bottom 10%” (the base case).

			Proprietary model: core is included	Coil volume internal fault full model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750 \text{ mH}$	
				$X_{DNF} = 1.293 \text{ m}\Omega$ for matching I_F	$X_{DNF} = 2.9 \text{ m}\Omega$ for matching I_Δ
Phase A: faulted	Max deviation	amplitude	0	-1.54%	-6.86% - series
		angle		2°	10° - common
Phase B: healthy	Max deviation	amplitude	0	-2.67%	-0.15%
		angle		2°	4°
Phase C: healthy	Max deviation	amplitude	0	-4.08%	-0.21%
		angle		-1°	4°

The results in the highlighted column show that the maximum deviation both in the current amplitude and the phase angle is in the faulted phase A, belonging to the series coil current (amplitude) and the common coil current (phase angle): both are well within Criterion 2. In summary, the case “bottom 10%” shows an excellent prediction of the currents by the coil volume internal fault full model despite the shortcomings of the existing core circuit.

5.3.2 The Case “Bottom 30%”

The corresponding tables with results in Appendix M are Tables M.3 and M.4.

First, inspect the delta current predicted by the coil volume internal fault leakage model with the initial focus on the option where the fault current is matched. It is marked with red in Table 5.3 below, which is a snapshot from Table M.3.

Table 5.3. Delta current prediction:
the coil volume internal fault leakage model, case "bottom 30%".

I, A_{peak}	Proprietary model: core is included	Coil volume internal fault leakage model: Cylindrical approach, K_{cyl} neglected		
		X_{DNF} omitted	$X_{DNF} = 0.954 \text{ m}\Omega$ for matching I_F	$X_{DNF} = 1.321 \text{ m}\Omega$ for matching I_Δ
Delta current	102.6@-79°	393@-12°	137.6@-74° $I_\Delta > I_{\Delta_proprietary}$	102.9@-79°

Table 5.3 shows that the delta current, marked with red, is higher than that of the proprietary model, which agrees with Criterion 1. This result also suggests that the delta current amplitude, marked with red, should be reduced by 1.3 times to match that of the proprietary model, where the core is included. Attachment of the re-derived core circuit should reduce the delta current amplitude, followed up by fine-tuning of the X_{DNF} value. Otherwise, at the present state of the research, it is achieved by setting $X_{DNF} = 1.321 \text{ m}\Omega$, refer to Table M.3.

Second, based on Tables M.3 and M.4, the summary data for the case "bottom 30%" are given in Table 5.4 below. It shows the maximum per phase deviations of the current amplitudes (in percent) and phase angles (in degrees): the coil volume internal fault full model with respect with the proprietary model. The column of primary interest, where the delta current is matched, is highlighted in green, with the maximum deviations marked with red. The column where the fault current is matched is the intermediate step and its results are discussed further in Subsection 5.3.6.2.

For the same case produced by the coil volume internal fault leakage model, these deviations are insignificantly different, refer to Table M.4.

Table 5.4. Maximum per phase deviations of results: the coil volume internal fault full model with respect to the proprietary model, case "bottom 30%".

			Proprietary model: core is included	Coil volume internal fault full model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750 \text{ mH}$	
				$X_{DNF} = 0.954 \text{ m}\Omega$ for matching I_F	$X_{DNF} = 1.321 \text{ m}\Omega$ for matching I_Δ
Phase A: faulted	Max deviation	amplitude	0	-2.9%	-5.11% - series
		angle		3°	6° - common
Phase B: healthy	Max deviation	amplitude	0	-0.92%	-0.06%
		angle		2°	1°
Phase C: healthy	Max deviation	amplitude	0	-2.24%	-0.37%
		angle		-1°	-1°

The results in the highlighted column show that the maximum deviation both in the current amplitude and the phase angle is in the faulted phase A, belonging to the series coil current (amplitude) and the common coil current (phase angle): both are well within Criterion 2.

In summary, the case “bottom 30%” shows an excellent prediction of the currents by the coil volume internal fault full model despite the shortcomings of the existing core circuit.

5.3.3 The Case “Bottom 50%”

This is the single T2T fault in the middle of the phase A common coil and it is illustrated in Fig. 5.1. The corresponding tables with results in Appendix M are Tables M.5 and M.6.

First, inspect the delta current predicted by the coil volume internal fault leakage model with the initial focus on the option where the fault current is matched. It is marked with red in Table 5.5 below, which is a snapshot from Table M.5.

Table 5.5. Delta current prediction:
the coil volume internal fault leakage model, case “bottom 50%”.

I, A_{peak}	Proprietary model: core is included	Coil volume internal fault leakage model: Cylindrical approach, K_{cyl} neglected		
		X_{DNF} omitted	$X_{\text{DNF}} = 0.857 \text{ m}\Omega$ for matching I_F	$X_{\text{DNF}} = 0.75 \text{ m}\Omega$ for matching I_Δ
Delta current	117.3@-78°	275.3@-11°	105.1@-72° $I_\Delta < I_{\Delta_proprietary}$	117.2@-69°

Table 5.5 shows that the delta current, marked with red, is 10% lower than that of the proprietary model. This does not agree with Criterion 1: the delta current of the air-core Mirage should be higher than that of the iron-core Mirage. This result means that estimation by the Cylindrical approach with X_{DNF} set to match the fault current underestimates the delta current amplitude when the single T2T fault is located in the middle of the common coil.

Second, based on Tables M.5 and M.6, the summary data for the case “bottom 50%” are given in Table 5.6 below. It shows the maximum per phase deviations of the current amplitudes (in percent) and phase angles (in degrees): the coil volume internal fault full model with respect to the proprietary model. The column of primary interest, where the delta current is matched, is highlighted in green, with the maximum deviations marked with red. The column where the fault current is matched is the intermediate step and its results are discussed further in Subsection 5.3.6.2.

For the same case produced by the coil volume internal fault leakage model, these deviations are insignificantly different, refer to Table M.6.

The results in the highlighted column show that the maximum deviation both in the current amplitude and the phase angle is in the faulted phase A, belonging to the series coil current (amplitude) and the delta coil current (phase angle). The maximum phase angle deviation, belonging to the delta current, is propagated to the healthy phases. However, both the current amplitude and phase angle deviations are still well within Criterion 2. In summary,

the case “bottom 50%” shows an excellent prediction of the currents by the coil volume internal fault full model, despite the shortcomings of the existing core circuit.

Table 5.6. Maximum per phase deviations of results: the coil volume internal fault full model with respect to the proprietary model, case “bottom 50%”.

			Proprietary model: core is included	Coil volume internal fault full model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH	
				$X_{DNF} = 0.857$ m Ω for matching I_F	$X_{DNF} = 0.75$ m Ω for matching I_Δ
Phase A: faulted	Max deviation	amplitude	0	-3.79%	6.45% - series
		angle		5°	-9° - delta
Phase B: healthy	Max deviation	amplitude	0	0.77%	-0.63%
		angle		1°	-9° - delta
Phase C: healthy	Max deviation	amplitude	0	0.19%	0.51%
		angle		1°	-9° - delta

Since Criterion 1 is not satisfied and Criterion 2 is satisfied, the result for the case “bottom 50%” can be named as marginally acceptable. This indicates that the Cylindrical approach might not be a reliable current estimator with reference to the expectations stated in Subsection 3.4.2.2.

5.3.4 The Case “Top 30%”

The corresponding tables with results in Appendix M are Tables M.7 and M.8.

First, inspect the delta current predicted by the coil volume internal fault leakage model with the initial focus on the option where the fault current is matched. It is marked with red in Table 5.7 below, which is a snapshot from Table M.7.

Table 5.7. Delta current prediction:
the coil volume internal fault leakage model, case “top 30%”.

	Proprietary model: core is included	Coil volume internal fault leakage model: Cylindrical approach, K_{cyl} neglected		
		X_{DNF} omitted	$X_{DNF} = 0.968$ m Ω for matching I_F	$X_{DNF} = 0.45$ m Ω for matching I_Δ
Delta current	101.4@-79°	167.8@-11°	56.7@-74° $I_\Delta < I_{\Delta_proprietary}$	101.6@-58°

Table 5.7 shows that the delta current, marked with red, is 1.8 times lower than that of the proprietary model. This again does not agree with Criterion 1: the delta current of the air-core Mirage should be higher than that of the iron-core Mirage. This result means that estimation by the Cylindrical approach with X_{DNF} set to match the fault current underestimates the delta current amplitude when the single T2T fault is located at 30% of the common coil height counting from its top.

Second, based on Tables M.7 and M.8, the summary data for the case “top 30%” is given in Table 5.8 below. It shows the maximum per phase deviations of the current amplitudes (in percent) and phase angles (in degrees): the coil volume internal fault full model with respect to the proprietary model. The column of primary interest, where the delta current is matched, is highlighted in green, with the maximum deviations marked with red. The column where the fault current is matched is the intermediate step and its results are discussed further in Subsection 5.3.6.2.

For the same case produced by the coil volume internal fault leakage model, these deviations are insignificantly different, refer to Table M.8.

Table 5.8. Maximum per phase deviations of results: the coil volume internal fault full model with respect to the proprietary model, case “top 30%”.

			Proprietary model: core is included	Coil volume internal fault full model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH	
				$X_{DNF} = 0.968$ m Ω for matching I_F	$X_{DNF} = 0.45$ m Ω for matching I_Δ
Phase A: faulted	Max deviation	amplitude	0	3.81%	31.72% - series
		angle		4°	-21° - delta
Phase B: healthy	Max deviation	amplitude	0	1.78%	-1.52%
		angle		1°	-21° - delta
Phase C: healthy	Max deviation	amplitude	0	1.86%	0.59%
		angle		1°	-21° - delta

The results in the highlighted column show that the maximum deviation in the current amplitude and the phase angle is in the faulted phase A, belonging to the series coil current (amplitude) and the delta coil current (phase angle). The maximum phase angle deviation, belonging to the delta current, is propagated to the healthy phases. Both the current amplitude and phase angle deviations are outside Criterion 2. In summary, the case “top 30%” shows a poor prediction of the currents by the coil volume internal fault full model.

Since both criteria are not satisfied, the result for the case “top 30%” is not acceptable: prediction of the currents in the faulted phase is poor.

5.3.5 The Case “Top 10%”

The corresponding tables with results in Appendix M are Tables M.9 and M.10.

First, inspect the delta current predicted by the coil volume internal fault leakage model with the initial focus on the option where the fault current is matched. It is marked with red in Table 5.9 below, which is a snapshot from Table M.9.

Table 5.9 shows that the delta current, marked with red, is 3.8 times lower than that of the proprietary model. This again does not agree with Criterion 1: the delta current of the air-

core Mirage should be higher than that of the iron-core Mirage. This result means that estimation by the Cylindrical approach with X_{DNF} set to match the fault current highly underestimates the delta current amplitude when the single T2T fault is located at 10% of the common coil height counting from its top.

Table 5.9. Delta current prediction:
the coil volume internal fault leakage model, case “top 10%”.

I, A_{peak}	Proprietary model: core is included	Coil volume internal fault leakage model: Cylindrical approach, K_{cyl} neglected		
		X_{DNF} omitted	$X_{DNF} = 1.315 \text{ m}\Omega$ for matching I_F	$X_{DNF} = 0.17 \text{ m}\Omega$ for matching I_Δ
Delta current	60.8@-83°	69.7@-14°	16@-84° $I_\Delta < I_{\Delta_proprietary}$	60.6@-37°

Second, based on Tables M.9 and M.10, the summary data for the case “top 10%” is given in Table 5.10 below. It shows the maximum per phase deviations of the current amplitudes (in percent) and phase angles (in degrees): the coil volume internal fault full model with respect to the proprietary model. The column of primary interest, where the delta current is matched, is highlighted in green, with the maximum deviations marked with red. The column where the fault current is matched is the intermediate step and its results are discussed further in Subsection 5.3.6.2.

For the same case produced by the coil volume internal fault leakage model, these deviations are insignificantly different, refer to Table M.10.

Table 5.10. Maximum per phase deviations of results: the coil volume internal fault full model with respect to the proprietary model, case “top 10%”.

			Proprietary model: core is included	Coil volume internal fault full model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750 \text{ mH}$	
				$X_{DNF} = 1.315 \text{ m}\Omega$ for matching I_F	$X_{DNF} = 0.17 \text{ m}\Omega$ for matching I_Δ
Phase A: faulted	Max deviation	amplitude	0	3.38%	79.58% - series
		angle		5°	-62° - common
Phase B: healthy	Max deviation	amplitude	0	1.79%	2.12%
		angle		1°	-46°
Phase C: healthy	Max deviation	amplitude	0	1.95%	0.26%
		angle		1°	-46°

The results in the highlighted column show that the maximum deviation in the current amplitude and the phase angle is in the faulted phase A, belonging to the series coil current (amplitude) and the common coil current (phase angle). Both the current amplitude and phase angle deviations are outside Criterion 2. In summary, the case “top 10%” shows a poor prediction of the currents by the coil volume internal fault full model.

Since both criteria are not satisfied, the result for the case “top 10%” is not acceptable: prediction of the currents in the faulted phase is poor.

5.3.6 Lack of Symmetry in the Results due to the Cylindrical Approach

The results from the proprietary internal fault model show symmetry in currents for symmetrical T2T faults, i.e., the corresponding currents from the cases “bottom 10%” and “top 10%”, “bottom 30%” and “top 30%” are almost equal. Table M.13 contains the current phasors from the proprietary model for all the five T2T faults located at different heights of the phase A common coil. The data in this table has been provided by the transformer manufacturer and is also shown in Tables M.1, M.3, M.5, M.7, and M.9.

It is observed from Table M.13 that the maximum T2T fault current amplitude occurs when the shorted turn is located in the middle of the common coil, i.e., for the case “bottom 50%”. This is justified by the fact that the amplitudes of the overall leakage flux and the “do-nothing” leakage flux of the shorted turn have their minimums in the coil middle due to the low amplitude of the leakage flux radial component at this location. When this T2T fault moves towards either end of the coil, the leakage flux radial component becomes more prominent due to the fringing effect reaching its maximum at the coil ends. This leads to a higher self-reactance associated with the shorted turn and, consequently, to a smaller fault current amplitude.

To conveniently evaluate the symmetry of the results, the following tables are created for five T2T fault locations on the phase A common coil:

- X_{DNF} is omitted – Table M.14,
- X_{DNF} is such to match the fault current – Table M.15, and
- X_{DNF} is such to match the delta current – Table M.16.

These tables are also based on Tables M.1, M.3, M.5, M.7, and M.9.

It is observed from Tables M.14, M.15 and M.16 that the currents obtained from the coil volume internal fault full model do not demonstrate the expected symmetry of the proprietary model from Table M.13.

The reason for the lack of symmetry in the symmetrically located T2T faults can be explained by referring to Fig. 5.1 again. When this T2T fault changes its location, the binary reactances change. However, for each pair of symmetrical locations of a T2T fault, as represented by the simplified reality and illustrated in Fig. 5.2 below, the binary reactances should be symmetrically equal in the two sets.

For example, consider two symmetrically located T2T faults “bottom 10%” and “top 10%”. The binary reactance between the bottom portion of the faulted common coil and the healthy series coil for the case “bottom 10%” should be approximately equal to the binary reactance between the top portion of the faulted common coil and the healthy series coil

for the case “top 10%”. The same is true for the binary reactances where the delta coil is involved. And the same is true for the binary reactances between the portions of the faulted common coil. This property is responsible for similar amplitudes of the corresponding currents in a transformer for symmetrically located T2T faults. The proprietary model demonstrates these results.

Fig. 5.2. Symmetrically located T2T faults represented by simplified reality and the Segmented approach: two symmetrically equal sets of the binary reactances.

However, the bottom-to-top fault transition in the simplified reality from Fig. 5.2 is represented by the Cylindrical approach with the left-to-right fault transition as illustrated in Fig. 5.3, with reference to Subsection 3.4.2. Therefore, a set of symmetrically equal binary reactances cannot be obtained for a symmetrically located T2T fault if the coil volume internal fault model is based on the Cylindrical approach.

The results from Table M.11 in Appendix M confirm this: the binary reactances involving the sub-coils $C1$, $C2$, and $C3$ are distinct for each location, and this natural property of the Cylindrical approach leads to the lack of symmetry in the results. This is the main weakness of the Cylindrical approach which was expected to be improved by adjusting the values of X_{DNF} , refer to Subsection 3.4.2.2. Yet, the Segmented approach proposed for the future work and coinciding with the simplified reality for representation of T2T (and T2G) faults as illustrated in Figs. 3.19, 3.22, 5.1 a), and 5.2 should overcome this limitation, refer also to Subsection 3.4.3.

Fig. 5.3. Symmetrically located T2T faults represented by the Cylindrical approach: two distinct sets of the binary reactances.

A detailed analysis of the symmetry inspection in the results obtained from the coil volume internal fault full model is offered below.

5.3.6.1 X_{DNF} Is Omitted

Refer to Table M.14. This option of the coil volume internal fault full model produces unsymmetrical results. The leakage field of the faulted phase is underestimated, as expected in Subsection 3.4.2, leading to much higher current amplitudes compared to those of the proprietary model, except the common coil current. It has very low amplitudes due to the current polarity reversal introduced by the necessity to balance the existing ATs in the faulted phase A because of the poor leakage field representation.

For the left-to-right fault transition in Table M.14:

- the fault current amplitude gradually increases from case to case,
- the delta current amplitude considerably decreases from case to case,
- the series coil current amplitude (the H-terminal current) in the faulted phase gradually increases from case to case,
- the common coil current amplitudes in the faulted phase show poor symmetry,
- the X-terminal current amplitude in the faulted phase gradually decreases from case to case,
- consequently, the series coil current amplitudes, the common coil current amplitudes, and the X-terminal current amplitudes in the healthy phases do not show the expected symmetry.

Poor behavior of the first two currents causes a lack of symmetry in the results because their improper prediction affects the behavior of the other currents. These currents are forced to take their amplitudes and directions under the strict transformer rule: at all times, ATs on any pair of core legs in three- and four-legged transformers, as well as in transformers built from single-phase units, should always be equal [42].

5.3.6.2 X_{DNF} for Fault Current Matching

Refer to Table M.15. For the left-to-right fault transition, this option of the coil volume internal fault model produces the following results:

- the fault current amplitudes show the expected symmetry since they closely follow those of the proprietary model by adjusting the X_{DNF} value,
- the delta current amplitude is poorly estimated and decreases nonuniformly from case to case, showing reasonable values in the left vertical half and unrealistic values in the right vertical half,
- the series coil current amplitudes (the H-terminal current) in the faulted phase approximately show the expected symmetry,

- the common coil current amplitudes in the faulted phase show the expected symmetry,
- the X-terminal current amplitudes in the faulted phase approximately show the expected symmetry,
- due to the poor delta current prediction, the amplitudes of the series coil current, the common coil current, and the X-terminal current in the healthy phases do not show the expected symmetry.

Since the delta current is circulating inside the delta winding and nothing comes out of the Y-terminals, it might seem that it does not matter what is circulating inside the delta winding if the other currents look right. That is, the results from this option of the coil volume internal fault full model might seem attractive. However, it is necessary to remember that, in this case, the balancing ATs transferred from the faulted phase to the healthy phases do not correspond to those of the proprietary model. Therefore, the healthy phases do not show symmetry in the current amplitudes, but the faulted phase approximately shows the expected symmetry in the current amplitudes.

Table 5.11 below shows a summary of the analysis for the results from Table M.15. The idea of Table 5.11 is to show the current prediction performance by the left vertical half and the right vertical half of the faulted common coil with reference to Fig. 5.3, i.e., the current prediction performance for different locations of the single T2T fault. For this, deviations of the current amplitudes obtained from the coil volume internal fault full model (the fault current is matched) are calculated with respect to those of the proprietary model for each location. The results, given in Table 5.11, are based on the results from Tables 5.2, 5.4, 5.6, 5.8, and 5.10.

Table 5.11. Current prediction performance in different locations of the faulted common coil with reference to Fig. 5.3: deviations of current amplitudes with respect to those of the proprietary model, the coil volume internal fault full model where the fault current is matched.

I_{Δ} is excluded from comparison as indicated in Section 5.2 since it is poorly predicted					
I_f is matched – incorrect balancing ATs					
Faulted phase			Healthy phases		
Left vertical half: “bottom 10%” “bottom 30%”	Coil middle: “bottom 50%”	Right vertical half: “top 10%” “top 30%”	Left vertical half: “bottom 10%” “bottom 30%”	Coil middle: “bottom 50%”	Right vertical half: “top 10%” “top 30%”
Max deviation < 4%			Max deviation < 2%		
Some sort of symmetry in the current values			No symmetry in the current values		
Reasonable I_{Δ}	Unrealistic I_{Δ}		Reasonable I_{Δ}	Unrealistic I_{Δ}	

Analyzing Table 5.11, it seems that the current amplitudes are properly predicted, being well within Criterion 2. Although, there is no symmetry in the current amplitudes of the healthy phases and the delta current is poorly predicted for the right vertical half. However, as mentioned in Subsection 4.7.3.2, matching the fault current is only an intermediate step

towards obtaining the ultimate model since the poorly predicted delta current produces incorrect balancing ATs followed by incorrect fault impact on the healthy phases.

5.3.6.3 X_{DNF} for Delta Current Matching

Refer to Table M.16. For the left-to-right fault transition, this option of the coil volume internal fault model produces the following results:

- the fault current amplitudes are highly underestimated for the left vertical half, but still, are high enough to be considered a fault current, and highly overestimated for the right vertical half,
- the delta current amplitudes show the expected symmetry since they closely follow those of the proprietary model due to adjusting the X_{DNF} value,
- the series coil current amplitude (the H-terminal current) in the faulted phase gradually increases from case to case, and there is no symmetry in amplitudes since the series coil current is affected by poor prediction of the fault current,
- the common coil current in the faulted phase shows reasonable amplitudes and some of the expected behavior for the left vertical half, however, in the right vertical half the amplitudes are underestimated,
- the X-terminal current amplitudes show some of the expected symmetry in the faulted phase.

In this case, the balancing ATs transferred to the healthy phases correspond to those of the proprietary model for all T2T fault locations. This produces the expected symmetry of the current amplitudes in the healthy phases as well as an excellent agreement of these currents with those of the proprietary model. However, the fault current amplitudes do not show symmetry: underestimated in the left vertical half and overestimated in the right vertical half.

This option of the coil volume internal fault full model makes the currents in the faulted phase to show good agreement with those of the proprietary model in the left vertical half and in the coil middle. However, no agreement in the right vertical half is obtained. Yet, even these results make this option of the coil volume internal fault full model, where the delta current is matched, be more accurate than the other option, where the fault current is matched. More on this conclusion is given in the next Subsection 5.3.7.

Table 5.12 below shows a summary of the analysis for the results from Table M.16. The idea of Table 5.12 is the same as for Table 5.11. For this, deviations of the current amplitudes obtained from the coil volume internal fault full model (the delta current is matched) are calculated with respect to those of the proprietary model for each location. The results given in Table 5.12 are based on the results from Tables 5.2, 5.4, 5.6, 5.8, and 5.10.

Table 5.12. Current prediction performance in different locations of the faulted common coil with reference to Fig. 5.3: deviations of current amplitudes with respect to those of the proprietary model, the coil volume internal fault full model where the delta current is matched.

I_F is excluded from comparison as indicated in Section 5.2 since it is not so well predicted					
I_Δ is matched – correct balancing ATs					
Faulted phase			Healthy phases		
Left vertical half: “bottom 10%” “bottom 30%”	Coil middle: “bottom 50%”	Right vertical half: “top 10%” “top 30%”	Left vertical half: “bottom 10%” “bottom 30%”	Coil middle: “bottom 50%”	Right vertical half: “top 10%” “top 30%”
Max deviation < 7%		Max deviation < 80%	Max deviation < 0.7%		Max deviation < 2.1%
No symmetry in the current values			Some sort of symmetry in the current values		
I_F is still high enough to be considered a fault current					

Analyzing Tables 5.11 and 5.12 together, it is concluded that the current amplitudes in the healthy phases are affected by the delta current prediction, and the current amplitudes in the faulted phase are affected by the fault current prediction.

In Subsection 3.4.2.2, it is proposed that the main limitation of the Cylindrical approach can be improved with insertion of X_{DNF} and adjusting its value for each fault location. The results, shown in this and the following subsections, do not support this idea.

5.3.7 X_{DNF} Values

Table 5.13 below shows the values of X_{DNF} for the five locations of the single T2T fault. The highlighted cells in Table 5.13 are related to the cases, where prediction of the currents shows excellent results, refer to Tables 5.1, 5.3, and 5.5. Two of the rows in Table 5.13 show the lower and upper factors as per (4.10) for estimation of the X_{DNF} value, based on knowledge of the faulted turn resistance R_{C2} . In simple words, the lower and upper factors represent the ratio X_{DNF}/R_{C2} of the faulted turn depending on the fault location.

As can be observed from Table 5.13, when matching the fault current, the values of X_{DNF} show symmetry about the case “bottom 50%”. This occurs because, from one side, the proprietary model’s results show the symmetry about the middle horizontal line (refer to Table M.13). From another side, the amplitudes of the fault current from the coil volume internal fault full model without X_{DNF} are almost equal for all the cases – among all the five cases $I_{F_{min}} = 347,250$ (case “bottom 10%”) and $I_{F_{max}} = 351,530$ A_{peak} (case “top 10%”). The fault current amplitude shows a little gradual increase from case to case. Table 5.14 below compares the fault and delta current amplitudes for both models for all the fault locations.

Table 5.13. Values of X_{DNF} for single T2T fault on the common coil at different locations.

	The base case "bottom 10%"	Case "bottom 30%"	Case "bottom 50%"	Case "top 30%"	Case "top 10%"
Matching I_F					
X_{DNF} , m Ω	1.293	0.954	0.857	0.968	1.315
Lower factor for $R_{C2} = 0.37464$ m Ω as per (4.10): X_{DNF} / R_{C2}	3.5	2.5	2.3	2.6	3.5
Matching I_{Δ}					
X_{DNF} , m Ω	2.9	1.321	0.75	0.45	0.17
Upper factor for $R_{C2} = 0.37464$ m Ω as per (4.10): X_{DNF} / R_{C2}	7.7	3.5	2	1.2	0.5

Table 5.14. Comparison of the delta and fault currents: the coil volume internal fault full model versus the proprietary model.

I , A_{peak}	The base case "bottom 10%"	Case "bottom 30%"	Case "bottom 50%"	Case "top 30%"	Case "top 10%"
Coil volume internal fault full model: X_{DNF} is omitted					
I_F	347,250@167°	348,450@169°	349,548@170°	350,568@172°	351,530@173°
I_{Δ}	517.7@-13°	393@-12°	275.3@-11°	167.8@-11°	69.7@-14°
Coil volume internal fault full model: X_{DNF} to match I_F					
I_F	95,073@101°	124,469@106°	136,929@109°	124,463@107°	95,393@104°
I_{Δ}	138.8@-79°	137.6@-74°	105.1@-72°	56.7@-74°	16@-84°
Coil volume internal fault full model: X_{DNF} to match I_{Δ}					
I_F	44,686@93°	93,838@101°	152,199@112°	217,121@125°	311,889@150°
I_{Δ}	63.5@-87°	102.9@-79°	117.2@-69°	101.6@-58°	60.7@-37°
Proprietary model					
I_F	95,047@99°	124,491@103°	136,959@104°	124,427@103°	95,412@99°
I_{Δ}	63.5@-83°	102.6@-79°	117.3@-78°	101.4@-79°	60.8@-83°

The results of Table 5.14 summarize the statements about the fault and delta current matching: if the fault current is matched, the delta current is poorly predicted, and if the delta current is matched, the fault current is poorly predicted.

Also observed from Table 5.13, when matching the delta current, the X_{DNF} values do not show symmetry. This occurs because even if the delta current from the proprietary model shows the symmetry, the delta current from the coil volume internal fault model without X_{DNF} considerably decreases from case to case as shown in Table 5.14. Therefore, for this option, although insertion of X_{DNF} leads to symmetry of the delta current amplitudes, it does not lead to symmetry of the X_{DNF} values, and, consequently, of the fault current amplitudes in Table M.16.

That is why the upper factors in Table 5.13 and the corresponding values of X_{DNF} show this behavior. However, in the highlighted area, i.e., in the left vertical half, they both behave as desired:

- the higher values are towards the common coil bottom,
- the lower values are towards the middle of the common coil, and
- the same is true for the upper factor values X_{DNF}/R_{C2} in the last row.

Ideally, the last two rows in Table 5.13 should show symmetry of the values about the case “bottom 50%”, and this is expected for the Segmented approach.

Even if the delta current is matched, the influence of X_{DNF} cannot overcome the poor leakage flux representation when the single T2T fault is located in the right vertical half of the common coil as per Fig. 5.3. In other words, above the middle horizontal line as represented by the simplified reality in Fig. 5.2. This happens because, in the right vertical half, the delta coil becomes further away from the sub-coil $C2$, the width of the sub-coil $C1$ reduces and the width of the sub-coil $C3$ increases.

The delta coil is much smaller than the series coil. Therefore, when the single T2T fault moves from left to right as per Fig. 5.3, estimation of the binary reactances, where the delta coil is involved, becomes affected more than the binary reactances, where the series coil is involved, refer to Table M.11. As stated in Subsection 5.3.6, the vertical segmentation of the faulted common coil for simulation of single T2T faults produces two distinct sets of the binary reactances for symmetrically located T2T faults.

The results in Chapter 5 show that, as opposed to expectations, such a natural property of the Cylindrical approach cannot be compensated fully with insertion of the X_{DNF} element and adjustment of its values. Nevertheless, the prediction performance of the coil volume internal fault model is improved due to X_{DNF} if single T2T faults are simulated in the left vertical half of the faulted common coil. The successful prediction of the currents for the faulted Mirage is demonstrated in the highlighted columns of Table M.16 in Appendix M. The currents from the right vertical half should be neglected and replaced with the currents from the corresponding highlighted columns for symmetrical T2T faults.

5.4 Overview of the Results

Overall, the results show that the Cylindrical approach, as opposed to expectations, is not a reliable tool for prediction of the currents in a faulted transformer. However, it is a good start to explore transformer theory, find out whether the assumptions of the Cylindrical approach would be sufficient for representation of the leakage field pattern of a faulted transformer, and define the next directions for development of the coil volume internal fault full model.

The results of Chapter 5 demonstrate that out of the five locations of the single T2T fault along the common coil height, two show decent prediction of the currents (the cases

“bottom 10%” and “bottom 30%”), two show poor prediction of the currents (the cases “top 10%” and “top 30%”), and one shows marginally acceptable prediction of the currents (the case “bottom 50%”). These results belong to the option of the coil volume internal fault full model where the delta current is matched with that of the proprietary model. This option of the proposed model is better than the other option, where the fault current is matched. The performance is improved because the former produces correct balancing ATs carried out to the healthy phases and such leads to an accurate prediction of the currents in the healthy phases for all the fault locations. However, the fault current is poorly predicted but still keeps high amplitudes to be considered a fault current. The option of the proposed model, where the fault current is matched, on the other side, produces an accurate prediction of the currents in the faulted phase, except the delta current, for all the fault locations. Yet, the currents in the healthy phases are poorly predicted.

Analyzing these results, the important observations are as follows:

- accuracy of prediction of the current amplitudes in the healthy phases is affected by accuracy of the delta current prediction and
- accuracy of prediction of the current amplitudes in the faulted phase is affected by accuracy of the fault current prediction.

The results demonstrate that insertion of X_{DNF} and adjustment of its value does not fully compensate for the main weakness of the Cylindrical approach. Symmetrically located T2T faults do not produce similar sets of currents due to the vertical segmentation of the faulted common coil. However, the results:

- establish that X_{DNF} is an essential parameter of a shorted turn for the leakage field pattern representation of a faulted transformer and
- show that the prediction performance of the coil volume internal fault model is improved due to X_{DNF} if single T2T faults are simulated in the left vertical half of the faulted common coil.

The results of Chapter 5 also show that only the left vertical half, refer to Fig. 5.3, of the coil volume internal fault full model with the delta current matching produces good/acceptable current amplitudes and phase angles. The right vertical half produces highly inaccurate results. This occurs due to the nature of the vertical segmentation. In this case, the binary reactances are non-symmetrically affected by the changing widths of the sub-coils and their distances to the healthy coils when the single T2T fault moves from left to right, as illustrated in Fig. 5.3. A solution to this problem within the Cylindrical approach can be the following: since, in reality, two symmetrically located T2T faults produce two similar sets of currents, then the currents from the right vertical half should be replaced with the corresponding currents from the left vertical half. However, solutions for the series and delta coils might be different.

This demonstrates that the Cylindrical approach is not a reliable current estimator. However, the work performed for the coil volume internal fault model based on the

Cylindrical approach offers a solid background for development and implementation of a coil volume internal fault model based on the Segmented approach. In the case of symmetrically located T2T faults represented by the Segmented approach, the binary reactances should be symmetrically equal in the two sets. This property leads to similar amplitudes of the corresponding currents in a transformer for symmetrical T2T faults. The proprietary model demonstrates these results.

The research also proves the need to re-derive the core circuit in the faulted phase, because attachment of the adopted core circuit does not affect the delta current amplitude in the faulted Mirage, refer to Subsection 4.7.3.2.

6 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

6.1 Overview

In this dissertation, development and implementation of the coil volume internal fault model is performed for the common coil of a three-phase three-winding autotransformer. The transformer ratings are 144.9/72.5/13.8 kV, 60/60/12 MVA, YNa0d1. The coil volume internal fault model presented in this dissertation has two major goals.

The first goal is to create a transformer leakage model for simulation of internal faults based on a limited set of data: a typical test report and limited core window design information (widths of core legs, windings, and air gaps, average winding height, and the number of turns in each winding).

The second goal is to incorporate the magnetic core representation in the overall transformer model. The core circuit provides the interphase coupling which allows internal faults to be simulated during unbalanced operation and core saturation.

A brief overview of each goal is offered below.

6.1.1 Transformer Leakage Field Representation

The leakage representation of the Mirage autotransformer is estimated by the Cylindrical approach. This approach proposes the vertical segmentation of a faulted coil to form three sub-coils for simulation of T2T faults and two sub-coils for simulation of T2G faults. This configuration of a faulted coil provides an inexact estimation of the leakage field during internal faults. However, it is a quick and easy tool for the initial estimation of currents in a faulted transformer, as well as for understanding of the overall modeling process. A new parameter, such as a self-reactance of the shorted turn X_{DNF} , is introduced for improved representation of the leakage field during internal faults.

In this dissertation, development of the coil volume internal fault model based on the Cylindrical approach is presented for both T2T and T2G faults. However, the implementation and preliminary results in Chapter 4 are shown for the base case T2T fault on the phase A common coil. The full set of results in Chapter 5 consists of modeling a single T2T fault on the phase A common coil at five different coil locations. These locations include the single T2T fault in the middle of the coil and two pairs of symmetrical locations along the coil height.

The main attraction of the Cylindrical approach is that two (T2G faults) or three (T2T faults) sub-coils of the faulted coil form individual coils of the same height as the healthy coils. In this case, a faulted transformer remains an N -coil transformer, where all the coils

are cylindrical, yet with different N in the faulted phase. Therefore, calculation of the binary reactances is performed by applying the well-known energy method.

Next, formulation of the leakage admittance matrix of the faulted transformer is performed by applying Dommel's method. This method is suitable for transformers with any number of coils. Dommel's method has been proven to give reliable results. The two transformer components of ATPDraw, XFMR and BCTRAN, use this method for formulation of the leakage admittance matrix. The coil volume internal fault model, whose leakage admittance matrix is built using Dommel's method, is easily implemented in ATP.

Development of the Segmented approach, as an alternative to the Cylindrical approach, has also been attempted. The Segmented approach proposes the horizontal segmentation of a faulted coil to form three portions for simulation of T2T faults and two portions for simulation of T2G faults. This configuration of a faulted coil resembles the reality of an internal fault more closely than the Cylindrical approach. The Segmented approach also requires insertion of the self-reactance X_{DNF} in the shorted turn.

Development of the Segmented approach started at a late stage of the research and encountered a couple of obstacles preventing its quick development and implementation. These issues are documented in Subsection 3.4.3.3. Therefore, comprehensive work on development and implementation of the Segmented approach remains for future work. However, derivation of the preliminary formulas for calculation of the binary reactances using the Segmented approach is shown in Section 3.5.

6.1.2 Transformer Core Model

The leakage model of a healthy or faulted transformer, which does not contain interphase coupling, can only be used for simulations of balanced events without core saturation. However, when the core circuit is attached to the leakage representation, the interphase coupling is provided through the zero-sequence flux paths that are a part of the core circuit. The major advantage of the core circuit is easy access to the core parameters for modification of flux values.

Incorporation of the core circuit in the coil volume internal fault model is based on the same method developed for the Hybrid model of ATPDraw. According to this method, an artificial infinitely thin $(N+1)^{th}$ coil is located on each core leg. This coil is a mathematical formulation serving as an interface between the core circuit and the leakage representation of a transformer. Presence of this coil also allows accounting for the leakage flux between the inner most winding and the core leg in each phase.

To differentiate between the two options of the coil volume internal fault model, the option without the core circuit is named the coil volume internal fault leakage model and the option with the core circuit is named the coil volume internal fault full model.

The core circuit used in this dissertation is adopted from the Hybrid model (XFMR) of the Mirage autotransformer. This means that the coil volume internal fault full model utilizes the core circuit of the model for a healthy three-legged transformer. The core circuit is illustrated in Fig. 2.17.

It is expected that the core effect incorporated in the coil volume internal fault model should increase the transformer zero-sequence inductance and, consequently, reduce the delta current magnitude. This, in turn, should reduce the fault current magnitude.

6.2 Summary

The main observations from each chapter are given below.

6.2.1 Chapter 3 Summary

This chapter describes the coil volume internal fault model development. An introduction to its development is performed by introducing the leakage and full models of the healthy Mirage. An important aspect of the healthy Mirage's full model is that it is equivalent to the XFMR Mirage if the coil-to-core reactances are estimated as in the Hybrid model, as per (2.78) – (2.80). However, if they are estimated from design data, as per (3.5) – (3.7), then the XFMR Mirage and the healthy Mirage's full model are not equivalent.

An important observation of Chapter 3 is the shorted turn physics representation in Subsection 3.3.1. It is summarized in a few statements:

- The induced voltage in the shorted turn appears between two nodes which represent a mutually coupled inductor. This inductor is coupled with the healthy portions of the faulted coil and with the healthy coils of the faulted phase. The per-phase coupling is implemented in the leakage admittance matrix. The interphase coupling is implemented by attachment of the core circuit.
- The total voltage drop in the shorted turn is a sum of the voltage drops on:
 - the shorted turn resistance R_{C2} and
 - the shorted turn self-reactance X_{DNF} associated with the “do-nothing” leakage flux.
- The induced voltage and the voltage drop in the shorted turn are equal.
- The values of X_{DNF} are estimated by (3.14) assuming that the fault current from the proprietary model is used.
- The fault current circulating in the shorted turn is estimated by (4.9).

A summary of the internal fault representation by the Cylindrical and Segmented approaches is given in Subsection 3.4.4. The major part of Chapter 3 is dedicated to development of the coil volume internal fault leakage and full models.

6.2.2 Chapter 4 Summary

Implementation of the coil volume internal fault model is performed on the basis of the single T2T fault located at 10% of the common coil height counting from its bottom. This is equivalent to the height of 10% of the total common coil turns. This case is named the base case. Therefore, the results and observations of Chapter 4 are limited to the base case.

Chapter 4 introduces the following models of the Mirage autotransformer:

- the XFMR Mirage,
- the healthy Mirage's leakage model,
- the healthy Mirage's full model,
- the coil volume internal fault leakage model,
- the coil volume internal fault full model, and
- the proprietary model.

The first step towards implementation of the coil volume internal fault full model is implementation of the healthy Mirage's full model, where the coil-to-core reactances should be evaluated. Section 4.3 describes the details, and briefly, the observations can be summarized as follows:

- The coil-to-core reactances evaluated by (2.78) – (2.80) internally in the XFMR Mirage are compared with those of the healthy Mirage's full model estimated from design data by (3.5) – (3.7). This comparison shows that the empirical coefficient $K=0.5$ of the Hybrid model, described in Subsection 2.4.2, does not fit the design data of the Mirage.
- Table 4.2 shows this comparison and suggests that K cannot be uniform for all the coil-to-core reactances. The results show that $K=0.5$ should be increased by 28% for the series-to-core reactance, decreased by 8.6% for the common-to-core reactance, and decreased by 51% for the delta-to-core reactance in order to correspond to calculations based on the design data.

Implementation of the healthy Mirage's leakage and full models is described in Section 4.4. It shows that the Rogowski factor for Cylindrical windings K_{cyl} can be neglected for each binary reactance estimated by the Cylindrical approach since all its values are above 0.9, refer to Subsections 4.4.1.2 and 4.4.2.2.

From implementation of the shorted turn self-reactance X_{DNF} , described in Section 4.5, it is observed that

- X_{DNF} affects both the fault and delta currents. Therefore, two assumptions are developed for benchmarking of the coil volume internal fault model with the proprietary model, refer to Subsection 4.5.1:
 - Assumption 1, where the fault current of the two models is matched,

- Assumption 2, where the delta current of the two models is matched. This assumption produces more valuable results since, in this case, the fault impact on the healthy phases is the same in both models.
- Table 4.5 shows the values of X_{DNF} for both assumptions.

Implementation of the coil volume internal fault leakage and full models described in Section 4.6 shows that all the values of the Rogowski factor for cylindrical windings K_{cyl} are above 0.9 and therefore can be neglected. This occurs because all the coils and sub-coils have equal height and small radial sizes compared to the coil height. However, in the Segmented approach, the Rogowski factor for sandwich windings K_{snw} cannot be neglected for any binary reactance.

After implementation is complete, the coil volume internal fault model is subject to benchmarking. This is described in Section 4.7 and consists of Stage 1 and Stage 2 benchmarking.

Stage 1 benchmarking is performed against the models of the healthy Mirage. It confirms that the leakage representation and the core circuit attachment are properly performed. Therefore, the coil volume internal fault leakage and full models are qualified for application of the short and insertion of the X_{DNF} element.

Stage 2 benchmarks the base case T2T fault simulated by the coil volume internal fault leakage and full models against the simulation by the proprietary model. The models for Stage 2 are as follows:

- the proprietary model,
- the coil volume internal fault leakage model, the short is applied – three options,
- the coil volume internal fault full model, the short is applied – three options.

The three options include:

- X_{DNF} is not inserted,
- X_{DNF} is such to match the fault current as per Assumption 1, and
- X_{DNF} is such to match the delta current as per Assumption 2.

The main observation from Stage 2 benchmarking is that attachment of the core circuit adopted from the XFMR Mirage does not significantly affect the current amplitudes, as expected. This is concluded by comparing Tables I.14 and I.16, and the details are given in Subsections 4.7.3.

The Stage 2 benchmarking observations for the options of the coil volume internal fault leakage and full models from Tables I.14 and I.16 are the same and as follows:

- If X_{DNF} is not inserted, the fault current amplitude in the shorted turn is overestimated by 265% (refer to columns 2 and 3 versus column 1). Consequently,

the H-terminal currents are off, especially the one in the faulted phase A is overestimated by 74%. This poor result is expected as explained in Subsection 3.4.1.3.

- In this case, X_{DNF} is inserted to match the fault current of the two models (refer to columns 4 and 5 versus column 1), as per Assumption 1, in Subsection 4.5.1. The results from the coil volume internal fault leakage model show excellent agreement with the results from the proprietary model for the faulted phase A currents (less than $|-2\%|$ deviation in amplitudes) and the healthy phase B currents (less than $|\pm 3\%|$ deviation in amplitudes). However, the result for the healthy phase C is slightly less impressive:

- the series coil current amplitude deviates by 3.7% and
- the common coil current amplitude deviates by $|-4\%|$.

For this case, the maximum deviation among all the currents, except the delta current, from the results of the proprietary model is 4% (the common coil current in the healthy phase C). The delta current amplitude deviates by 119%.

- Matching the delta current, as per Assumption 2, seems more reasonable than matching the fault currents as explained in Subsection 4.5.1. If X_{DNF} is inserted to match the delta current of the two models (refer to columns 6 and 7 versus column 1), the results from the coil volume internal fault leakage model show perfect agreement with the results from the proprietary model for the healthy phases B and C. However, the result for the faulted phase A is slightly less impressive:

- the series coil current amplitude deviates by $|-7\%|$,
- the common coil current amplitude deviates by 2.4%, and
- the X-terminal current amplitude deviates by 0.7%.

For this case, the maximum deviation among all the currents, except the fault current in the shorted turn, from the results of the proprietary model is 7% (the series coil current in the faulted phase A). The fault current amplitude deviates by $|-47\%|$.

Overall, it seems that both options with the X_{DNF} element offer promising results:

- The pro for the option, where the fault current of both models is matched, is that the current amplitudes in the faulted phase A of the coil volume internal fault leakage model deviate from those of the proprietary model by less than 2% and slightly more for other phases (roughly, 3% for phase B and 4% for phase C).
- The pro for the option, where the delta current of both models is matched, is that the correct balancing ATs are transferred from the faulted phase to the healthy phases. This leads to the fact that the currents in the healthy phases B and C of the coil volume internal fault leakage model are identical to those of the proprietary model. The series coil current amplitude in the faulted phase A deviates by 7% while providing confidence that such an important parameter as the delta current is matched precisely between the two models.

It is important to explore the volts per turn values in the models for the healthy and faulted Mirage, refer to Section 4.8. The main observations are as follows:

- The models for healthy Mirage produce the same voltage of $133 V_{\text{peak}}$ for a single turn, refer to Tables 4.11 and 4.12.
- Table J.2 contains the results for the faulted Mirage: the coil volume internal fault leakage and full models for the base case, each for three options. The main observations are as follows:
 - The induced voltage in the studied turn reduces by a maximum of 3.8% among all the cases compared to Table 4.12. Averaging the six cases of Table J.2 yields a change from $133 V_{\text{peak}}$ (healthy) to $130 V_{\text{peak}}$ (faulted). Such a low percentage of induced voltage reduction, due to the base case T2T fault, leads to the assumption that the induced voltage in the shorted turn is equal to the output voltage of the healthy turn, also refer to Subsection 3.3.1.
 - As expected, the voltage drop in the faulted turn increases from an insignificant value, before the fault, up to the value almost equal the induced voltage during the fault and is 180 degrees out of phase with the induced voltage. When the “do-nothing” flux is considered in the faulted turn, the voltage drop occurs mainly over the X_{DNF} element. This is because the Ohms values of the X_{DNF} element are higher than the resistance value R_{C2} .
 - Again, the parameters of interest almost do not change when comparing the coil volume internal fault leakage and the full models. When a fault is applied, the core effect should be prominent and, as mentioned previously, the core representation should be upgraded.
 - The other observations from Table J.2 can be found in Subsection 4.8.3.

The leakage admittance matrix of a healthy transformer is singular and rank deficient which is observed in Table 4.13 for different matrices of the healthy Mirage, refer to Subsection 4.9.1. It is important to understand how sensitivity of the matrix elements changes after manipulations on the original matrix. Refer to Subsection 4.9.2 for the full study.

Part 1 of the sensitivity analysis is performed on the matrices of the healthy Mirage and the matrices of the coil volume internal fault leakage and full models. The two models of the faulted Mirage are structured for the base case but the short is not applied and X_{DNF} is not inserted. The sensitivity is studied on the matrices represented in p.u. values by perturbing the off-diagonal elements by $\pm 10\%$, one at a time. The singularity property of the modified matrices is preserved. All perturbations are performed on phase A only. The most sensitive elements are identified based on changes in the current amplitudes for the base load and short-circuit tests by at least $\pm 5\%$.

After the most sensitive elements have been identified in Part 1, they are perturbed further by $\pm 40\%$ and $\pm 80\%$ in Part 2 to observe the tendencies of these sensitive elements to affect the certain currents. Part 2 of the sensitivity analysis is performed on the matrices of the coil volume internal fault leakage and full models for the base case T2T fault with X_{DNF} omitted. The analysis of Part 2 helps to understand if the X_{DNF} element can be avoided and only perturbations of the matrix elements can bring the results from the coil volume internal fault model closer to the results from the proprietary model.

Part 1 of the sensitivity analysis contained in Subsections 4.9.2.2.1 and 4.9.2.2.2 shows:

- The $\pm 10\%$ perturbations on three off-diagonal elements in the 3×3 phase A submatrix of the healthy Mirage's leakage model identify the elements a_{12} and a_{23} to be the most sensitive, or in terms of the coil pairs, series – common and common – delta.
- The $\pm 10\%$ perturbations on six off-diagonal elements in the 4×4 phase A submatrix of the healthy Mirage's full model identify the elements a_{12} , a_{13} , and a_{23} to be the most sensitive. Attachment of the core circuit increased sensitivity of the element a_{13} related to the coil pair series – delta.
- The $\pm 10\%$ perturbations on ten off-diagonal elements in the 5×5 phase A submatrix of the coil volume internal fault leakage model (short is not applied) identify the elements a_{12} , a_{13} , a_{23} , and a_{45} to be the most sensitive, or in terms of the coil pairs, series – sub-coil $C1$, series – sub-coil $C2$, sub-coil $C1$ – sub-coil $C2$, and sub-coil $C3$ – delta. The sub-coil $C2$ corresponds to the single turn. The elements a_{12} , a_{23} , and a_{45} are located near the main diagonal.
- The $\pm 10\%$ perturbations on ten off-diagonal elements in the 6×6 phase A submatrix of the coil volume internal fault full model (short is not applied) identify the elements a_{12} , a_{13} , a_{23} , a_{35} and a_{45} to be the most sensitive. With the core circuit added, the element a_{35} , related to the binary reactance of the coil pair sub-coil $C2$ – delta, also becomes sensitive.

For detailed observations of Part 2, refer to Subsection 4.9.2.2.3. However, in short:

- Perturbations of single elements in the studied matrices of Part 2 do not produce the results which shift closer to those from the proprietary model. However, the results obtained show that perturbation of certain matrix elements leads to the desired changes in the faulted phase A current amplitudes: decrease of the series coil current amplitude, increase of the common coil current amplitude, and decrease of the fault current amplitude.
- The results from Part 2 of the sensitivity studies provide additional justification that the X_{DNF} element is needed. In general, it seems that simultaneous perturbation of several original matrix elements should be performed to bring the results from the coil volume internal fault model without X_{DNF} closer to the results from the proprietary model. For this, an optimization algorithm should be developed to identify which elements to perturb and by how much.
- It is hard to state firmly whether the best solution from this algorithm will be successful or not. However, the author of this dissertation is leaning towards “no”. This is because the leakage admittance matrix represents the coupled inductances and perturbations of the matrix elements do not seem to reduce the fault current effectively and, besides, such perturbations in one phase only increase the delta current. The X_{DNF} element, which is uncoupled from the matrix elements, effectively reduces the fault current and consequently the delta current. Perturbations of the elements in the healthy phases are not needed. Perturbations of the elements in one phase only will lead to improper delta current amplitude.

Therefore, only the faulted phase A should be manipulated to improve the current prediction, i.e., insertion of the self-reactance X_{DNF} in the shorted turn.

The study on the net mmf per leg and the mean net mmf per leg, shown in Section 4.10, is performed for the same set of the transformer models as for Stages 1 and 2 of benchmarking. For mmf calculations, the current amplitudes and their phase angles are used. With reference to Table 4.14, the observations for the models of the healthy Mirage are summarized as follows:

- Only one of the models – the healthy Mirage’s leakage model – shows the perfect mmf balance. This is because the unsymmetric core structure is not attached.
- When the core circuit is present, like in the XFMR Mirage and the healthy Mirage’s full model, the net mmf per leg in each phase increases by a different amount in each phase and the mean net mmf per leg increases by 40.1% compared to the healthy Mirage’s leakage model.
- When the phase A common coil is sectioned to form the coil volume internal fault leakage model, the delta current of $1.3 A_{peak}$ appears. It brings a slight mmf imbalance and increase of the mean net mmf per leg by 3.5% compared to the healthy Mirage’s leakage model. This occurs because the common coil in the healthy phases is represented as one piece but represented with three portions in the faulted phase. Since the delta current is small, segmentation of the common coil in the faulted phase only is acceptable.
- When the phase A common coil is sectioned and the core circuit is attached to form the coil volume internal fault full model, the mmf imbalance increases further. Compared to the healthy Mirage’s full model and the XFMR Mirage the rise is by 3.6% and compared to the coil volume internal fault leakage model by 40.3%.
- Increase of mmf due to the core presence is an expected result. A slight imbalance of the net mmf per leg between the phases and presence of the delta current due to a different representation of the common coils in the faulted and healthy phases is also an expected result.

It is important to observe how the net mmf per leg and the mean net mmf per leg of the faulted Mirage change compared to the healthy Mirage. With reference to Table 4.15:

- The mean net mmf per leg increases by 34.3% during the base case T2T fault.
- The net mmf in the faulted phase A increases by 33.7%. However, a more considerable rise of the net mmf occurs in the healthy phase B – by 82.5%, while in the healthy phase C there is a little reduction in the net mmf – by 1.3%.

The study for the models of the faulted Mirage is described in Subsection 4.10.2.3. For the coil volume internal fault leakage model with three options, it is summarized in Table 4.16, and for the coil volume internal fault full model in Table 4.17. Briefly, the observations are as follows:

- As observed from Table 4.16, the option, where $X_{DNF} = 2.9 \text{ m}\Omega$ for the delta current matching, produces the net mmf per leg and the mean net mmf per leg which are the closest to those of the proprietary model. This is because of the same ATs in the delta winding of the two models. The mean net mmf per leg is higher by 25.1% than that from the proprietary model.
- As observed from Table 4.17, the option, where $X_{DNF} = 2.9 \text{ m}\Omega$ for the delta current matching, produces the net mmf per leg and the mean net mmf per leg which are the closest to those of the proprietary model, likewise in Table 4.16. The mean net mmf per leg is higher by 23.8% than that from the proprietary model. Also, attachment of the core circuit brings more imbalance in the phasors of the net mmf per leg.
- As observed from Tables 4.16 and 4.17, when X_{DNF} is present in the shorted turn and the core circuit is attached, the mean net mmf per leg reduces insignificantly, refer to the dedicated comparison in Table 4.18. When the upgraded core circuit, suitable for simulation of internal faults, will be attached, then more mmf reduction is expected because the delta current should reduce considerably. Since the existing core model does not assist in reduction of the delta current during faults, this is achieved by adjusting the value of X_{DNF} .

Among all the options of the coil volume internal fault leakage and full models explored for mmf calculations in Subsection 4.10.2.3, the results closest to those of the proprietary model for the base case T2T fault are observed for the options, where X_{DNF} is inserted to match the delta current according to Assumption 2:

- In terms of the current phasors – the corresponding currents are almost equal in the coil volume internal fault leakage and full models, refer to Tables L.28 and L.38.
- In terms of the mean net mmf per leg – the coil volume internal fault full model shows a slightly better result over the coil volume internal fault leakage model, refer to Table 4.18. Therefore, even the adopted core circuit offers some effect of mmf reduction which, of course, should be much more prominent with the upgraded core circuit.

The difficulties encountered during implementation of the coil volume internal fault model are described in Section 4.11. Table 4.19 offers a summary on the coil volume internal fault leakage and full models for the base case T2T fault. The coil volume internal fault full model from Table 4.19, developed on the basis of the base case T2T fault, is adopted for generation of results in Chapter 5.

6.2.3 Chapter 5 Summary

Testing of the coil volume internal fault full model is carried out by simulating the single T2T fault at five different locations on the phase A common coil. An overview of these results and the related conclusions are described in Section 5.4.

6.3 Conclusions

Development and implementation of a transformer model for simulation of internal faults is a sophisticated task requiring detailed design data. This information is rarely available outside the transformer manufacturer. This dissertation is an attempt to derive a simplistic transformer model for simulation of internal faults. The model should be based on typical test report data and limited design data. This new model will be successful if it provides results close to those of a more complex higher-order proprietary model based on detailed design data. Ideally, such work should be performed for a number of transformers with different ratings.

The transformer leakage flux during internal faults is a complex phenomenon. Its accurate representation requires CAD information for a finite element model. In this dissertation, a faulted complex transformer coil is represented as a single-layered coil with uniformly distributed turns along the coil height. A general approach for calculation of the binary reactances during internal faults and consideration of the radial component of the leakage flux are suggested for representation of the faulted common coil. This simplified representation of a faulted coil during faults leads to errors in estimation of the leakage flux.

The results produced for this dissertation can be regarded as a solid intermediate step towards successful development of the coil volume internal fault model for simulation of transformer internal faults. In this dissertation, development and implementation of the coil volume internal fault model has been performed for the common coil of a three-phase three-winding autotransformer. The autotransformer ratings are 144.9/72.5/13.8 kV, 60/60/12 MVA, YNa0d1. Benchmarking of the proposed model has been performed with the proprietary model of a transformer manufacturer. The main findings of this dissertation are discussed below in Subsections 6.3.1 – 6.3.3.

6.3.1 Leakage Field Pattern Representation

The transformer leakage field pattern during internal faults is formed by the superposition of two representations:

- The leakage field of the mutual inductances between all the pairs formed by the healthy coils and two (T2G fault) or three (T2T fault) portions of the faulted coil as if they all are healthy. This defines the binary short-circuit reactances and, consequently, the leakage admittance matrix $[A]$ of the faulted transformer.
- The individual leakage field of the shorted turn (portion) links neither with the healthy portions of the faulted coil nor with the healthy coils. The individual leakage field is interpreted as the “do-nothing” flux and described as a stand-alone inductor X_{DNF} located in series with the circuit elements of the shorted turn (portion). Its value depends on the fault location.

The short is applied outside the leakage admittance matrix across all the circuit elements of the dedicated turn (portion): the turn resistance, X_{DNF} , and the mutual reactance (a part of the leakage admittance matrix).

6.3.2 Two Approaches for Leakage Field Pattern Representation

The leakage field pattern during internal faults can be realized in two approaches: Cylindrical and Segmented. Each approach performs:

- segmentation of the cross-sectional area of a faulted coil (the vertical segmentation by the Cylindrical approach and the horizontal segmentation by the Segmented approach),
- estimates the binary reactances of the faulted phase.

In the Cylindrical approach, the bottom-to-top fault transition on a real coil is represented by the left-to-right fault transition. This research showed that the Cylindrical approach is not a reliable current estimator due to the vertical segmentation of a faulted coil. However, the work performed for the coil volume internal fault model based on the Cylindrical approach offers a solid background for the development and implementation of a coil volume internal fault model based on the Segmented approach.

Representation of the leakage field during internal faults by the Cylindrical approach highly underestimates the radial component of the leakage flux. The X_{DNF} element does not fully compensate for the radial component as expected. The Cylindrical approach does not demonstrate symmetry in the results as shown by the proprietary model. Symmetrically located T2T faults on the common coil do not produce similar sets of currents. This occurs due to the nature of the vertical segmentation. In this case, the binary reactances are non-symmetrically affected by the changing widths of the sub-coils and their distances to the healthy coils when a single T2T fault moves from left to right.

Although the Cylindrical approach does not demonstrate symmetry in the results as shown by the proprietary model, the work:

- establishes that X_{DNF} is an essential parameter of a shorted turn for the leakage field pattern representation of a faulted transformer, and
- shows that the prediction performance of the coil volume internal fault model is improved due to X_{DNF} if single T2T faults are simulated in the left vertical half of the faulted common coil. In this case, the results are close to those of the proprietary model. The best results from the coil volume internal fault model are highlighted in Table M.16 in Appendix M. The results from the proprietary model are shown in Table M.13 in Appendix M.

Importance of matching the delta current and the fault current with those of the proprietary model has been studied. At this point, only one of them can be matched at a time. The option of the proposed model, where the delta current is matched, is better than the other

option, where the fault current is matched. The performance is improved because when the delta current is matched, the proposed model produces the same fault impact on the healthy phases as that of the proprietary model. In other words, the balancing ATs carried out to the healthy phases are the same in the two comparable models. In this case, the proposed model accurately predicts the currents in the healthy phases for all the fault locations. However, the fault current is poorly predicted. But, the fault current is still large as anticipated. The option of the proposed model, where the fault current is matched, produces an accurate prediction of the currents in the faulted phase, except the delta current, for all the fault locations. Yet, the currents in the healthy phases are poorly predicted.

Analyzing these results, the important observations are as follows:

- accuracy of prediction of the current amplitudes in the healthy phases is affected by accuracy of the delta current prediction, and
- accuracy of prediction of the current amplitudes in the faulted phase is affected by accuracy of the fault current prediction.

The Segmented approach offers a more realistic representation of the radial component of the leakage flux. In the case of symmetrically located T2T faults represented by the Segmented approach, the binary reactances should be symmetrically equal in the two sets. This property leads to similar amplitudes of the corresponding currents in a transformer for symmetrical T2T faults. The proprietary model demonstrates these results. Development of the Segmented approach encountered a few obstacles and, thus, it remains for future work.

The author does not recommend continuing development of the Cylindrical approach for internal faults on the series and delta coils. The recommendation is to proceed with development and implementation of the Segmented approach using the guidance of this dissertation.

The reason why the Cylindrical approach has been attempted as the basis for development and implementation of the coil volume internal fault model is the ease of its implementation in ATP. The Segmented approach will require additional effort to derive the formulas for calculation of the binary reactances and come up with a method for building the leakage admittance matrices if Dommel's method cannot be applied.

6.3.3 Core Model

The other important finding is the necessity to re-derive the core circuit. The core circuit adopted from the Hybrid model corresponds to a healthy transformer and, as per the results, is not suitable for simulation of internal faults by the coil volume internal fault model.

For the fault cases, attachment of the core circuit demonstrated a negligible effect on the current amplitudes for two reasons:

- The core circuit adopted from the XFMR Mirage does not properly interact with the leakage representation of the faulted Mirage. Therefore, a modification of the core circuit in the faulted phase is required.
- The zero-sequence inductance value of 750 mH set for the core model of the XFMR Mirage might not be the appropriate value. But the sensitivity analysis of this parameter shows that variation of the 750 mH value by $\pm 50\%$ does not affect the delta current amplitude. Therefore, the zero-sequence inductance might not be properly represented in the adopted core circuit.

In the faulted phase, the modified core circuit should provide interaction of the individual portions of the faulted coil with the corresponding core leg regions. At the same time, the healthy coils in the faulted phase should interact with these core leg regions as if they represent the one-piece core leg. The main expectation from the re-derived core circuit is that it should reduce the delta current amplitude during internal faults. A sufficient understanding of how to develop the modified core circuit occurred too late to be included in this dissertation. However, expectations for the re-derived core circuit and general guidance on its development are documented in Subsection 4.7.3.2.

The transformer manufacturer has not shared information about their core model. Perhaps, this information will be available in the future for reliable benchmarking of the coil volume internal fault model with the proprietary model.

6.4 Recommendations for Future Work

Recommendations for future work contain ideas for the next iteration on development and implementation of the coil volume internal fault model.

6.4.1 The Segmented Approach

Development of the Segmented approach can be continued from the ideas described in Subsection 3.4.3. In this case, it is also beneficial to start working with the leakage only representation $[A(N)]$ for ease of debugging, and then develop the $[A(N+I)]$ representation. The approximate outline of the work is as follows:

- Develop the formulas for calculation of the binary reactances, refer to the initial ideas in Subsection 3.5.1.1.1.
- Verify if Dommel's approach is valid for building the leakage admittance matrices. If not, find the way to build $[A(N)]$ and $[A(N+I)]$ or maybe even $[X]$.
- Carry out the same tests for the base case as documented in Chapter 4.
- For benchmarking with the proprietary model when the core circuit is not attached, adjust the X_{DNF} value for each T2T fault case to imitate the “do-nothing” flux of the shorted turn and:
 - to match the fault current as an intermediate step,
 - to match the delta current.

- Re-derive the core circuit as suggested in Subsection 4.7.3.2.
- For benchmarking with the proprietary model when the core circuit is attached, adjust the X_{DNF} value and the core circuit parameters for each T2T fault case.
- The success of the model will be confirmed if symmetrically located T2T faults produce similar sets of currents.

6.4.2 Other Recommendations

- For simulation of internal faults, the core circuit adopted from the Hybrid model should be re-derived, as per the directions in Subsection 4.7.3.2. For initial testing, before development of the Segmented approach, the re-derived core circuit can be a part of the coil volume internal fault full model based on the Cylindrical approach. Local saturations of the core due to internal faults can be explored with the re-derived core circuit.
- When the coil volume internal fault model is equipped with the re-derived core circuit, internal faults should be studied at different states of core saturation during:
 - energization,
 - other types of inrush currents,
 - geomagnetic disturbances, and
 - overvoltages.
- A method for estimation of the leakage reactance of a shorted turn or a portion X_{DNF} can be developed. The method should be independent of benchmarking with the proprietary model. Development of this method can be based on the reactance calculation of a solenoid located on a magnetic core. Perhaps, the X_{DNF} value depends on the transformer MVA capacity, coil voltage, coil resistance, and transformer short-circuit reactance.
- The temperature effect and the arc effect can also be included in the shorted turn reactance estimation.
- Capacitive effects in the coil volume internal fault model can be added. The resonant frequency of the R - L - C circuit of the shorted turn might also reduce the fault current. If the capacitance matrix $[C]$ is attached later, it does not need the $(N+1)^{th}$ coil because the core model is connected to ground.
- A wider range of the methods of calculating the transformer binary reactances should be explored.
- The method of geometric mean distances can be applied for estimation of the binary reactances of a faulted transformer. It is briefly described in Subsection 2.2.4.4.
- Continued cooperation with the transformer manufacturer is desired to benchmark the core model.
- Further research is needed to define better values for the empirical factor K used in the Hybrid model for estimation of the coil-to-core reactances.
- The transformer leakage admittance matrix is singular by definition. However, when it is connected with the grounded core circuit, the overall system becomes non-singular. A matrix representation of a transformer, including both the leakage and core models, can be derived to prove this.

- At the present state of the coil volume internal fault model, the insulation thickness is neglected. In the future, its approximate thickness can be included in the coil volume internal fault model.
- Suggestions for future work related to the sensitivity analysis on the matrix elements are located in Subsection 4.9.2.2.2.
- It will be beneficial to develop the $3I_0$ and $3I_2$ meters to measure imbalances in the ATP simulations.
- The coil volume internal fault model based on the Segmented approach can be extended further. It can accommodate the cases where the common coil consists of the main coil and the tap coil, as well as the cases where the series coil has built-in taps.
- As the final stage of development of the coil volume internal fault model based on the Segmented approach, it can be implemented in two options, i.e., be based on:
 - known design data and
 - typical design data.

7 REFERENCE LIST

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A Matlab Code for Calculation of Reactances for the Healthy Mirage's Full Model

```
% THE MIRAGE AUTOTRANSFORMER

% 60/60/12 MVA

% 144.9/72.5/13.8 kV

% wye-wye-delta YNa0d1


% COIL VOLUME METHOD for calculation of binary s.c. reactances between
% pairs of coils of a healthy transformer


% ROGOWSKI COEFFICIENTS APPLIED: yes or no - choose inside the code

% CORE INCLUDED: yes

% Healthy model uses CYLINDRICAL approach since all the coils are of equal
% heights. The SEGMENTED approach is used when the coils are of unequal
% heights.

% all dimensions are in mm

% air1, air2, air3 are the air ducts starting from the core on the left:

% core, air1, delta coil, air2, common coil, air3, series coil

clear all

clc

format long


% Simplified design data

height = 1496; % all the coils are of the same height

b_air1 = 19; % air duct width between core and delta

b_air2 = 32; % air duct width between delta and common

b_air3 = 44; % air duct width between common and series

b_d = 28; % delta coil width

b_c = 120; % common coil width
```

```

b_s = 95; % series coil width

% Number of turns

N_s = 444; % number of turns in series coil

N_c = 444; % number of turns in common coil

N_d = 140; % number of turns in delta coil

t1 = table(N_s,N_c,N_d)

% Mean turn circumference 2πR

% R is the radius from the center of the core to the mean turn

% Core leg diameter

Dia_core = 570;

L_mt_s = 2*pi*(Dia_core/2 + b_air1 + b_d + b_air2 + b_c + b_air3 + b_s/2);

L_mt_d = 2*pi*(Dia_core/2 + b_air1 + b_d/2);

L_mt_c = 2*pi*(Dia_core/2 + b_air1 + b_d + b_air2 + b_c/2);

L_mt_air1 = 2*pi*(Dia_core/2 + b_air1/2);

L_mt_air2 = 2*pi*(Dia_core/2 + b_air1 + b_d + b_air2/2);

L_mt_air3 = 2*pi*(Dia_core/2 + b_air1 + b_d + b_air2 + b_c + b_air3/2);

t2 = table(L_mt_s,L_mt_c,L_mt_d,L_mt_air1,L_mt_air2,L_mt_air3)

% The Rogowski factors K1

% K1 is used for cylindrical windings, K2 is used for sandwich windings.

% In the Cylindrical approach, all the windings are cylindrical.

% for Xsc

tau_sc = b_s + b_c + b_air3;

u_sc = height/tau_sc;

K1_Rog_sc = 1 - (1 - exp(-pi * u_sc))/(pi * u_sc);

```

```

% for Xsd

tau_sd = b_s + b_c + b_d + b_air2 + b_air3;

u_sd = height/tau_sd;

K1_Rog_sd = 1 - (1 - exp(-pi * u_sd))/(pi * u_sd);


% for Xcd

tau_cd = b_c + b_d + b_air2;

u_cd = height/tau_cd;

K1_Rog_cd = 1 - (1 - exp(-pi * u_cd))/(pi * u_cd);


% Coil-to-core


% for Xs-core

tau_score = b_s + b_c + b_d + b_air1 + b_air2 + b_air3;

u_score = height/tau_score;

K1_Rog_score = 1 - (1 - exp(-pi * u_score))/(pi * u_score);


% for Xc-core

tau_ccore = b_c + b_d + b_air1 + b_air2;

u_ccore = height/tau_ccore;

K1_Rog_ccore = 1 - (1 - exp(-pi * u_ccore))/(pi * u_ccore);


% for Xd-core

tau_dcore = b_d + b_air1;

u_dcore = height/tau_dcore;

K1_Rog_dcore = 1 - (1 - exp(-pi * u_dcore))/(pi * u_dcore);


t3 = table(K1_Rog_sc,K1_Rog_sd,K1_Rog_cd)

t4 = table(K1_Rog_score,K1_Rog_ccore,K1_Rog_dcore)

```



```

% THE ROGOWSKI FACTORS K1 YES OR NO -> CHOOSE MANUALLY

ROGOWSKI_K1 = 0 % 1 for YES, 0 for NO

if ROGOWSKI_K1 == 1

    height_sc = height/K1_Rog_sc;

    height_sd = height/K1_Rog_sd;

    height_cd = height/K1_Rog_cd;

    height_score = height/K1_Rog_score;

    height_ccore = height/K1_Rog_ccore;

    height_dcore = height/K1_Rog_dcore;

else

    height_sc = height;

    height_sd = height;

    height_cd = height;

    height_score = height;

    height_ccore = height;

    height_dcore = height;

end

% Binary inductances and reactances (Ohms)

% Area of the ampere-turns diagram for SC

ATD_sc = L_mt_s * b_s/3 + L_mt_c * b_c/3 + L_mt_air3 * b_air3;

L_sc = 4 * pi * 10^(-10) * (N_s)^2 / (height_sc) * ATD_sc;

X_sc = 2 * pi * 60 * L_sc;

% Area of the ampere-turns diagram for SD

ATD_sd = L_mt_s * b_s/3 + L_mt_d * b_d/3 + L_mt_air2 * b_air2 ...

        + L_mt_air3 * b_air3 + L_mt_c * b_c;

L_sd = 4 * pi * 10^(-10) * (N_s)^2 / (height_sd) * ATD_sd;

X_sd = 2 * pi * 60 * L_sd;

```

```

% Area of the ampere-turns diagram for CD

ATD_cd = L_mt_c * b_c/3 + L_mt_d * b_d/3 + L_mt_air2 * b_air2;

L_cd = 4 * pi * 10^(-10) * (N_c)^2 / (height_cd) * ATD_cd;

X_cd = 2 * pi * 60 * L_cd;

% Coil-to-core reactances

% Area of the ampere-turns diagram for S-Core

ATD_score = L_mt_s * b_s/3 + L_mt_air1 * b_air1 + L_mt_air2 * b_air2...
            + L_mt_air3 * b_air3 + L_mt_c * b_c + L_mt_d * b_d;

L_score = 4 * pi * 10^(-10) * (N_s)^2 / (height_score) * ATD_score;

X_score = 2 * pi * 60 * L_score;

% Area of the ampere-turns diagram for C-Core

ATD_ccore = L_mt_c * b_c/3 + L_mt_air1 * b_air1 + L_mt_air2 * b_air2 + L_mt_d * b_d;

L_ccore = 4 * pi * 10^(-10) * (N_c)^2 / (height_ccore) * ATD_ccore;

X_ccore = 2 * pi * 60 * L_ccore;

% Area of the ampere-turns diagram for D-Core

ATD_dcore = L_mt_d * b_d/3 + L_mt_air1 * b_air1;

L_dcore = 4 * pi * 10^(-10) * (N_d)^2 / (height_dcore) * ATD_dcore;

X_dcore = 2 * pi * 60 * L_dcore;

% The data to be taken as an input into the file where [A] is calculated

% Binary short-circuit reactances

t5 = table(X_sc,X_sd,X_cd)

t6 = table(X_score,X_ccore,X_dcore)

```

B Matlab Code for Calculation of Matrices for the Healthy Mirage's Full Model

```
% THE MIRAGE AUTOTRANSFORMER

% 60/60/12 MVA

% 144.9/72.5/13.8 kV

% wye-wye-delta YNa0d1


% COIL VOLUME METHOD for calculation of [A(N+1)] of the healthy transformer
% THE ROGOWSKI FACTOR APPLIED: depends on reactances which are entered here
% CORE INCLUDED: yes
% Impedances between the winding terminals are modified from the test report:

% Impedance hi to low 6.441% at 60 MVA
% Impedance hi to tertiary 4.837% at 12 MVA
% Impedance low to tertiary 2.786% at 12 MVA

% Corresponding file which calculates binary s.c. reactances between pairs
% of coils from simplified design data is "Reactances_CORE_healthy"
% IMPORTANT: To obtain the result A(N+1)] in the ATP format, the
% function "ABMatrixATPt.txt.m" should be in the same folder
% (It was created by Alejandro Avendano and Nicola Chiesa)

clear all

clc

freq = 60;

w=2*pi*freq;


% Part I - these few lines of the code are taken from the "SC_test_calc" code, refer the
Hybrid Model project

% Rated voltage levels, Volts (see the test report)

% Here, voltages corresponding to the simplified transformer are used

Vh_ll = 144.9*10^3;

Vl_ll = 72.5*10^3;
```

```

Vt_11 = 13.8*10^3;

% Rated voltages for coils, Volts

Vs = (Vh_11/sqrt(3)) - (Vl_11/sqrt(3));

Vc = Vl_11/sqrt(3);

Vd = Vt_11;

% Common base power is 100 MVA

Sbase = 100e6;

% Part II is written fully by Elizaveta Egorova

% Calculates [A(N+1)] for the healthy transformer based on data from
% Part I and the simplified design data.

% Binary short-circuit reactances between pairs of coils are
% taken from the corresponding % "Reactances..." code

% DATA USED FROM PART I:

%           - resistances of the coils (alternatively, can be
%           taken directly from the test report)

%           - voltages of the coils

%           - common base power 100 MVA

%           These above are calculated from the test report data

% DATA USED FROM THE CORRESPONDING "Reactances..." file:

%           - binary s.c. reactances between pairs of coils

%           These above are calculated from the simplified design data

% Resistances of healthy coils (taken from the test report here)

Rs = 0.292781; % series coil

Rc = 0.166339; % common coil

```

```

Rdelta = 0.034170; % delta coil

% Voltages of the healthy coils (from PART I)

Vs;

Vc;

Vd;

Vcore = Vd;

% phases A,B,C binary short-circuit reactances

% ***** This part is taken from the "Reactances..." file*****

% Here the Rogowski factors K1 are not applied

SC = 22.5336410276317;

SD = 41.3707186839129;

CD = 12.1846364383375;

SCore = 45.8938687962194;

CCore = 16.707786550644;

DCore = 0.333961675308722;

% *****end***

% phases A,B,C - base values per phase

Zbs = Vs^2/(Sbase/3);

Zbc = Vc^2/(Sbase/3);

Zbd = Vd^2/(Sbase/3);

% [A(N+1)] is built using 100 MVA base!!!

% phases A,B,C binary short-circuit reactances in p.u. (using base 100 MVA)

SCPU = SC/Zbs; % referred to series

SDPU = SD/Zbs; % referred to series

CDPU = CD/Zbc; % referred to common

SCorePU = SCore/Zbs; % referred to series

```

```

CCorePU = CCore/Zbc; % referred to common

DCorePU = DCore/Zbd; % referred to delta

t1 = table(SCPU,SDPU,CDPU)

t2 = table(SCorePU,CCorePU,DCorePU)


% FORMULATION OF [A(N+1)] BY DOMMEL'S APPROACH

% Reminder: the artificial (N+1)th coil is included here

% First, build the reduced reactance matrix [XRED] in p.u. for each phase

% Second, invert [XRED] to obtain [YRED] in p.u. for each phase

XREDA = zeros(3,3); % healthy phase A

XREDB = zeros(3,3); % healthy phase B

XREDC = zeros(3,3); % healthy phase C


% phase A reduced reactance matrix [XREDA] in p.u., Core is Nth coil

% Diagonal elements phase A

XREDA(1,1) = SCorePU;

XREDA(2,2) = CCorePU;

XREDA(3,3) = DCorePU;


% Off-diagonal elements phase A

XREDA(1,2) = 1/2*(SCorePU+CCorePU-SCPU); % ZSCred

XREDA(1,3) = 1/2*(SCorePU+DCorePU-SDPU); % ZSDred

XREDA(2,3) = 1/2*(CCorePU+DCorePU-CDPU); % ZCDred


% Lower half is symmetrical

XREDA(2,1) = XREDA(1,2);

XREDA(3,1) = XREDA(1,3);

XREDA(3,2) = XREDA(2,3);

XREDA;

```

```

YmtxpuA = inv(XREDA); % reduced [Y] in p.u. for phase A

% phase B reduced reactance matrix [XREDB] in p.u., Core is Nth coil

% Diagonal elements phase B

XREDB(1,1) = SCorePU;

XREDB(2,2) = CCorePU;

XREDB(3,3) = DCorePU;

% Off-diagonal elements phase B

XREDB(1,2) = 1/2*(SCorePU+CCorePU-SCPU); % ZSCred

XREDB(1,3) = 1/2*(SCorePU+DCorePU-SDPU); % ZSDred

XREDB(2,3) = 1/2*(CCorePU+DCorePU-CDPU); % ZCDred

% Lower half is symmetrical

XREDB(2,1)=XREDB(1,2);

XREDB(3,1)=XREDB(1,3);

XREDB(3,2)=XREDB(2,3);

XREDB;

YmtxpuB = inv(XREDB); % reduced [Y] in p.u. for phase B

% phase C reduced reactance matrix [XREDC] in p.u., Core is Nth coil

% Diagonal elements phase C

XREDC(1,1) = SCorePU;

XREDC(2,2) = CCorePU;

XREDC(3,3) = DCorePU;

% Off-diagonal elements phase C

XREDC(1,2) = 1/2*(SCorePU+CCorePU-SCPU); % ZSCred

XREDC(1,3) = 1/2*(SCorePU+DCorePU-SDPU); % ZSDred

XREDC(2,3) = 1/2*(CCorePU+DCorePU-CDPU); % ZCDred

```

```

    % Lower half is symmetrical

    XREDC(2,1) = XREDC(1,2);

    XREDC(3,1) = XREDC(1,3);

    XREDC(3,2) = XREDC(2,3);

    XREDC;

    YmtxpuC = inv(XREDC); % reduced [Y] in p.u. for phase C

% Third, convert [YRED] in p.u. to full [Y] in p.u. for each phase

% Full [Y] in p.u. for phase A

    YmtxpuA(1,4) = -sum(YmtxpuA(1,:));

    YmtxpuA(4,1) = YmtxpuA(1,4);

    YmtxpuA(2,4) = -sum(YmtxpuA(2,:));

    YmtxpuA(4,2) = YmtxpuA(2,4);

    YmtxpuA(3,4) = -sum(YmtxpuA(3,:));

    YmtxpuA(4,3) = YmtxpuA(3,4);

    YmtxpuA(4,4) = -sum(YmtxpuA(:,4));

% Full [Y] in p.u. for phase B

    YmtxpuB(1,4) = -sum(YmtxpuB(1,:));

    YmtxpuB(4,1) = YmtxpuB(1,4);

    YmtxpuB(2,4) = -sum(YmtxpuB(2,:));

    YmtxpuB(4,2) = YmtxpuB(2,4);

    YmtxpuB(3,4) = -sum(YmtxpuB(3,:));

    YmtxpuB(4,3) = YmtxpuB(3,4);

    YmtxpuB(4,4) = -sum(YmtxpuB(:,4));

% Full [Y] in p.u. for phase C

    YmtxpuC(1,4) = -sum(YmtxpuC(1,:));

    YmtxpuC(4,1) = YmtxpuC(1,4);

    YmtxpuC(2,4) = -sum(YmtxpuC(2,:));

```



```

YmtxpuC(4,2) = YmtxpuC(2,4);

YmtxpuC(3,4) = -sum(YmtxpuC(3,:));

YmtxpuC(4,3) = YmtxpuC(3,4);

YmtxpuC(4,4) = -sum(YmtxpuC(:,4));

% Fourth, create full [Y] in p.u., where all the three phases are included

Ymtxpu = zeros(12,12);

% Phase A sub-matrix

Ymtxpu(1,1:4) = YmtxpuA(1,:);

Ymtxpu(2,1:4) = YmtxpuA(2,:);

Ymtxpu(3,1:4) = YmtxpuA(3,:);

Ymtxpu(4,1:4) = YmtxpuA(4,:);

% Phase B sub-matrix

Ymtxpu(5,5:8) = YmtxpuB(1,:);

Ymtxpu(6,5:8) = YmtxpuB(2,:);

Ymtxpu(7,5:8) = YmtxpuB(3,:);

Ymtxpu(8,5:8) = YmtxpuB(4,:);

% Phase C sub-matrix

Ymtxpu(9,9:12) = YmtxpuC(1,:);

Ymtxpu(10,9:12) = YmtxpuC(2,:);

Ymtxpu(11,9:12) = YmtxpuC(3,:);

Ymtxpu(12,9:12) = YmtxpuC(4,:);

Ymtxpu;

% Fifth, convert full [Y] in p.u. to full [Y] in Siemens

% Base values of voltages and power are needed

```

```

Ymtx=Ymtxpu;

% Phase A sub-matrix
Ymtx(:,1) = Ymtx(:,1)/Vs;
Ymtx(1,:) = Ymtx(1,:)/Vs;
Ymtx(:,2) = Ymtx(:,2)/Vc;
Ymtx(2,:) = Ymtx(2,:)/Vc;
Ymtx(:,3) = Ymtx(:,3)/Vd;
Ymtx(3,:) = Ymtx(3,:)/Vd;
Ymtx(:,4) = Ymtx(:,4)/Vcore;
Ymtx(4,:) = Ymtx(4,:)/Vcore;

% Phase B sub-matrix
Ymtx(:,5) = Ymtx(:,5)/Vs;
Ymtx(5,:) = Ymtx(5,:)/Vs;
Ymtx(:,6) = Ymtx(:,6)/Vc;
Ymtx(6,:) = Ymtx(6,:)/Vc;
Ymtx(:,7) = Ymtx(:,7)/Vd;
Ymtx(7,:) = Ymtx(7,:)/Vd;
Ymtx(:,8) = Ymtx(:,8)/Vcore;
Ymtx(8,:) = Ymtx(8,:)/Vcore;

% Phase C sub-matrix
Ymtx(:,9) = Ymtx(:,9)/Vs;
Ymtx(9,:) = Ymtx(9,:)/Vs;
Ymtx(:,10) = Ymtx(:,10)/Vc;
Ymtx(10,:) = Ymtx(10,:)/Vc;
Ymtx(:,11) = Ymtx(:,11)/Vd;
Ymtx(11,:) = Ymtx(11,:)/Vd;
Ymtx(:,12) = Ymtx(:,12)/Vcore;

```

```

Ymtx(12,:) = Ymtx(12,+)/Vcore;

Ymtx = Ymtx*(Sbase/3);

% THE ALMOST FINAL TARGET OF THE CODE: [A(N+1)] in Ohms-1

% Not yet in the form to be inserted in the library component of an .acp file

A = w * Ymtx

% Resistance matrix of the windings [R(N+1)] in Ohms

% If it is uncommented, then [R(N+1)] is included in the AB matrix output.

% The result of the AB matrix output is intended to be used in .atp file.

% At this moment, it is more convenient not to include [R(N+1)] in the AB

% output, since winding resistances are included in the model as separate

% resistance elements in the .acp file. However, if the resistance elements

% are deleted from the .acp files, [R(N+1)] should be a part of the [AB] output.

% [R(N+1)] is zeroed out in the AB matrix output

Rmtx1(1,1) = 0;

Rmtx1(2,2) = 0;

Rmtx1(3,3) = 0;

Rmtx1(4,4) = 0;

Rmtx1(5,5) = 0;

Rmtx1(6,6) = 0;

Rmtx1(7,7) = 0;

Rmtx1(8,8) = 0;

Rmtx1(9,9) = 0;

Rmtx1(10,10) = 0;

Rmtx1(11,11) = 0;

Rmtx1(12,12) = 0;

```

```

% THE FINAL TARGET OF THE CODE: [A(N+1)] in Ohms-1

% Now, [A(N+1)] will be in the form to be inserted in the library component of
% an .acp file. AB matrix outputs are [A(N+1)] != 0 and [R(N+1)] = 0.

% It is convenient to think of [A(N+1)] as being represented with an upper
% triangle of a symmetric singular matrix.

% For insertion in the .atp file, some modification of node names might be needed.


% This line calls for the function located in the same folder

ABMatrixATPttx(A,Rmtx1,false,0)

```

C T2T Faults on the Series and Delta Coils

C.1 T2T Faults on the Series Coil: Leakage Representation

Fig C.1. Simulation of T2T faults on the phase A series coil of the Mirage: the leakage model $[A(N)]$.

C.1.1 Binary Reactances Estimated by the Cylindrical Approach

Fig. C.2. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the series coil: the leakage model $[A(N)]$, the Cylindrical approach.

$$X_{S1S2} = \omega \mu_0 \frac{N_{S1}^2}{h_R} \left(\frac{L_{mt,S1} \cdot b_{S1} + L_{mt,S2} \cdot b_{S2}}{3} \right) \quad (C.1)$$

$$X_{S1S3} = \omega \mu_0 \frac{N_{S1}^2}{h_R} \left(\frac{L_{mt,S1} \cdot b_{S1} + L_{mt,S3} \cdot b_{S3}}{3} + L_{mt,S2} \cdot b_{S2} \right) \quad (C.2)$$

$$X_{S1C} = \omega\mu_0 \frac{N_{S1}^2}{h_R} \left(\frac{L_{mt,S1} \cdot b_{S1} + L_{mt,C} \cdot b_C}{3} + L_{mt,S2} \cdot b_{S2} + L_{mt,S3} \cdot b_{S3} + L_{mt,a3} \cdot a_3 \right) \quad (C.3)$$

$$X_{S1D} = \omega\mu_0 \frac{N_{S1}^2}{h_R} \left(\frac{L_{mt,S1} \cdot b_{S1} + L_{mt,D} \cdot b_D}{3} + L_{mt,S2} \cdot b_{S2} + L_{mt,S3} \cdot b_{S3} + \right. \\ \left. L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (C.4)$$

$$X_{S2S3} = \omega\mu_0 \frac{N_{S2}^2}{h_R} \left(\frac{L_{mt,S2} \cdot b_{S2} + L_{mt,S3} \cdot b_{S3}}{3} \right) \quad (C.5)$$

$$X_{S2C} = \omega\mu_0 \frac{N_{S2}^2}{h_R} \left(\frac{L_{mt,S2} \cdot b_{S2} + L_{mt,C} \cdot b_C}{3} + L_{mt,S3} \cdot b_{S3} + L_{mt,a3} \cdot a_3 \right) \quad (C.6)$$

$$X_{S2D} = \omega\mu_0 \frac{N_{S2}^2}{h_R} \left(\frac{L_{mt,S2} \cdot b_{S2} + L_{mt,D} \cdot b_D}{3} + L_{mt,S3} \cdot b_{S3} + \right. \\ \left. L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (C.7)$$

$$X_{S3C} = \omega\mu_0 \frac{N_{S3}^2}{h_R} \left(\frac{L_{mt,S3} \cdot b_{S3} + L_{mt,C} \cdot b_C}{3} + L_{mt,a3} \cdot a_3 \right) \quad (C.8)$$

$$X_{S3D} = \omega\mu_0 \frac{N_{S3}^2}{h_R} \left(\frac{L_{mt,S3} \cdot b_{S3} + L_{mt,D} \cdot b_D}{3} + L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (C.9)$$

$$X_{CD} = \omega\mu_0 \frac{N_C^2}{h_R} \left(\frac{L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D}{3} + L_{mt,a2} \cdot a_2 \right) \quad (C.10)$$

C.1.2 [A(N)] Formulation

Fig. C.3. Admittance formulation of the faulted phase A for simulation of T2T faults on the series coil: the leakage model [A(N)].

$$[A_A] = \begin{bmatrix} a_{S1S1} & a_{S1S2} & a_{S1S3} & a_{S1C} & a_{S1D} \\ a_{S1S2} & a_{S2S2} & a_{S2S3} & a_{S2C} & a_{S2D} \\ a_{S1S3} & a_{S2S3} & a_{S3S3} & a_{S3C} & a_{S3D} \\ a_{S1C} & a_{S2C} & a_{S3C} & a_{CC} & a_{CD} \\ a_{S1D} & a_{S2D} & a_{S3D} & a_{CD} & a_{DD} \end{bmatrix} \quad (C.11)$$

C.2 T2T Faults on the Delta Coil: Leakage Representation

Fig. C.4. Simulation of T2T faults on the phase A delta coil of the Mirage: the leakage model $[A/N]$.

C.2.1 Binary Reactances Estimated by the Cylindrical Approach

Fig. C.5. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the delta coil: the leakage model $[A/N]$, the Cylindrical approach.

$$X_{SC} = \omega\mu_0 \frac{N_S^2}{h_R} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,C} \cdot b_C}{3} + L_{mt,a3} \cdot a_3 \right) \quad (C.12)$$

$$X_{SD1} = \omega\mu_0 \frac{N_S^2}{h_R} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,D1} \cdot b_{D1}}{3} + L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (C.13)$$

$$X_{SD2} = \omega\mu_0 \frac{N_S^2}{h_R} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,D2} \cdot b_{D2}}{3} + L_{mt,C} \cdot b_C + \right. \\ \left. L_{mt,D1} \cdot b_{D1} + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (C.14)$$

$$X_{SD3} = \omega\mu_0 \frac{N_{D3}^2}{h_R} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,D3} \cdot b_{D3}}{3} + L_{mt,C} \cdot b_C + L_{mt,D1} \cdot b_{D1} + \right. \\ \left. L_{mt,D2} \cdot b_{D2} + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (C.15)$$

$$X_{CD1} = \omega\mu_0 \frac{N_C^2}{h_R} \left(\frac{L_{mt,C} \cdot b_C + L_{mt,D1} \cdot b_{D1}}{3} + L_{mt,a2} \cdot a_2 \right) \quad (C.16)$$

$$X_{CD2} = \omega\mu_0 \frac{N_C^2}{h_R} \left(\frac{L_{mt,C} \cdot b_C + L_{mt,D2} \cdot b_{D2}}{3} + L_{mt,D1} \cdot b_{D1} + L_{mt,a2} \cdot a_2 \right) \quad (C.17)$$

$$X_{CD3} = \omega\mu_0 \frac{N_C^2}{h_R} \left(\frac{L_{mt,C} \cdot b_C + L_{mt,D3} \cdot b_{D3}}{3} + L_{mt,D1} \cdot b_{D1} + L_{mt,D2} \cdot b_{D2} + L_{mt,a2} \cdot a_2 \right) \quad (C.18)$$

$$X_{D1D2} = \omega\mu_0 \frac{N_{D1}^2}{h_R} \left(\frac{L_{mt,D1} \cdot b_{D1} + L_{mt,D2} \cdot b_{D2}}{3} \right) \quad (C.19)$$

$$X_{D1D3} = \omega\mu_0 \frac{N_{D1}^2}{h_R} \left(\frac{L_{mt,D1} \cdot b_{D1} + L_{mt,D3} \cdot b_{D3}}{3} + L_{mt,D2} \cdot b_{D2} \right) \quad (C.20)$$

$$X_{D2D3} = \omega\mu_0 \frac{N_{D2}^2}{h_R} \left(\frac{L_{mt,D2} \cdot b_{D2} + L_{mt,D3} \cdot b_{D3}}{3} \right) \quad (C.21)$$

C.2.2 [A(N)] Formulation

Fig.C.6. Admittance formulation of the faulted phase A for simulation of T2T faults on the delta coil: the leakage model $[A(N)]$.

$$[A_A] = \begin{bmatrix} a_{SS} & a_{SC} & a_{SD1} & a_{SD2} & a_{SD3} \\ a_{SC} & a_{CC} & a_{CD1} & a_{CD2} & a_{CD3} \\ a_{SD1} & a_{CD1} & a_{D1D1} & a_{D1D2} & a_{D1D3} \\ a_{SD2} & a_{CD2} & a_{D1D2} & a_{D2D2} & a_{D2D3} \\ a_{SD3} & a_{CD3} & a_{D1D3} & a_{D2D3} & a_{D3D3} \end{bmatrix} \quad (C.22)$$

C.3 T2T Faults on the Series Coil: the Full Model

Fig. C.7. Simulation of T2T faults on the phase A series coil of the Mirage: the full model $[A(N+1)]$.

C.3.1 Binary Reactances Estimated by the Cylindrical Approach

Fig. C.8. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the series coil: the full model $[A(N+1)]$, the Cylindrical approach.

$$X_{S1-Core} = \omega\mu_0 \frac{N_{S1}^2}{h_R} \left(\frac{L_{mt,S1} \cdot b_{S1}}{3} + L_{mt,S2} \cdot b_{S2} + L_{mt,S3} \cdot b_{S3} + L_{mt,C} \cdot b_C + \right. \\ \left. L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (C.23)$$

$$X_{S2-Core} = \omega\mu_0 \frac{N_{S2}^2}{h_R} \left(\frac{L_{mt,S2} \cdot b_{S2}}{3} + L_{mt,S3} \cdot b_{S3} + L_{mt,C} \cdot b_C + \right. \\ \left. L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (C.24)$$

$$X_{S3-Core} = \omega\mu_0 \frac{N_{S3}^2}{h_R} \left(\frac{L_{mt,S3} \cdot b_{S3}}{3} + L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D + \right. \\ \left. L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (C.25)$$

$$X_{C-Core} = \omega\mu_0 \frac{N_C^2}{h_R} \left(\frac{L_{mt,C} \cdot b_C}{3} + L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 \right) \quad (C.26)$$

$$X_{D-Core} = \omega\mu_0 \frac{N_D^2}{h_R} \left(\frac{L_{mt,D} \cdot b_D}{3} + L_{mt,a1} \cdot a_1 \right) \quad (C.27)$$

C.3.2 [A(N+1)] Formulation

Fig. C.9. Admittance formulation of the faulted phase A for simulation of T2T faults on the series coil: the full model [A(N+1)].

$$[A_A] = \begin{bmatrix} a_{S1S1} & a_{S1S2} & a_{S1S3} & a_{S1C} & a_{S1D} & a_{S1-Core} \\ a_{S1S2} & a_{S2S2} & a_{S2S3} & a_{S2C} & a_{S2D} & a_{S2-Core} \\ a_{S1S3} & a_{S2S3} & a_{S3S3} & a_{S3C} & a_{S3D} & a_{S3-Core} \\ a_{S1C} & a_{S2C} & a_{S3C} & a_{CC} & a_{CD} & a_{C-Core} \\ a_{S1D} & a_{S2D} & a_{S3D} & a_{CD} & a_{DD} & a_{D-Core} \\ a_{S1-Core} & a_{S2-Core} & a_{S3-Core} & a_{C-Core} & a_{D-Core} & a_{Core-Core} \end{bmatrix} \quad (C.28)$$

C.4 T2T Faults on the Delta Coil: the Full Model

Fig. C.10. Simulation of T2T faults on the phase A delta coil of the Mirage: the full model $[A(N+1)]$.

C.4.1 Binary Reactances Estimated by the Cylindrical Approach

Fig. C.11. Mmf distribution diagram of the faulted phase A for simulation of T2T faults on the delta coil: the full model $[A(N+1)]$, the Cylindrical approach.

$$X_{S-Core} = \omega \mu_0 \frac{N_S^2}{h_R} \left(\frac{L_{mt,S} \cdot b_S}{3} + L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D + \right. \\ \left. \frac{L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3}{3} \right) \quad (C.29)$$

$$X_{C-Core} = \omega\mu_0 \frac{N_C^2}{h_R} \left(\frac{L_{mt,C} \cdot b_C}{3} + L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 \right) \quad (C.30)$$

$$X_{D1-Core} = \omega\mu_0 \frac{N_{D1}^2}{h_R} \left(\frac{L_{mt,D1} \cdot b_{D1}}{3} + L_{mt,D2} \cdot b_{D2} + L_{mt,D3} \cdot b_{D3} + L_{mt,a1} \cdot a_1 \right) \quad (C.31)$$

$$X_{D2-Core} = \omega\mu_0 \frac{N_{D2}^2}{h_R} \left(\frac{L_{mt,D2} \cdot b_{D2}}{3} + L_{mt,D3} \cdot b_{D3} + L_{mt,a1} \cdot a_1 \right) \quad (C.32)$$

$$X_{D3-Core} = \omega\mu_0 \frac{N_{D3}^2}{h_R} \left(\frac{L_{mt,D3} \cdot b_{D3}}{3} + L_{mt,a1} \cdot a_1 \right) \quad (C.33)$$

C.4.2 [A(N+1)] Formulation

Fig. C.12. Admittance formulation of the faulted phase A for simulation of T2T faults on the delta coil: the full model [A(N+1)].

$$[A_A] = \begin{bmatrix} a_{SS} & a_{SC} & a_{SD1} & a_{SD2} & a_{SD3} & a_{S-Core} \\ a_{SC} & a_{CC} & a_{CD1} & a_{CD2} & a_{CD3} & a_{C-Core} \\ a_{SD1} & a_{CD1} & a_{D1D1} & a_{D1D2} & a_{D1D3} & a_{D1-Core} \\ a_{SD2} & a_{CD2} & a_{D1D2} & a_{D2D2} & a_{D2D3} & a_{D2-Core} \\ a_{SD3} & a_{CD3} & a_{D1D3} & a_{D2D3} & a_{D3D3} & a_{D3-Core} \\ a_{S-Core} & a_{C-Core} & a_{D1-Core} & a_{D2-Core} & a_{D3-Core} & a_{Core-Core} \end{bmatrix} \quad (C.34)$$

D T2G Faults on the Series and Delta Coils

D.1 T2G Faults on the Series Coil: Leakage Representation

Fig. D.1. Simulation of T2G faults on the phase A series coil of the Mirage: the leakage model $[A(N)]$.

D.1.1 Binary Reactances Estimated by the Cylindrical Approach

Fig. D.2. Mmf distribution diagram of the faulted phase A for simulation of T2G faults on the series coil: the leakage model $[A(N)]$, the Cylindrical approach.

$$X_{S1S2} = \omega \mu_0 \frac{N_{S1}^2}{h_{R_S1S2}} \left(\frac{L_{mt,S1} \cdot b_{S1} + L_{mt,S2} \cdot b_{S2}}{3} \right) \quad (D.1)$$

$$X_{S1C} = \omega \mu_0 \frac{N_{S1}^2}{h_{R_S1C}} \left(\frac{L_{mt,S1} \cdot b_{S1} + L_{mt,C} \cdot b_C}{3} + L_{mt,S2} \cdot b_{S2} + L_{mt,a3} \cdot a_3 \right) \quad (D.2)$$

$$X_{S1D} = \omega\mu_0 \frac{N_{S1}^2}{h_{R_S1D}} \left(\frac{L_{mt,S1} \cdot b_{S1} + L_{mt,D} \cdot b_D}{3} + L_{mt,S2} \cdot b_{S2} + \right. \\ \left. L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (D.3)$$

$$X_{S2C} = \omega\mu_0 \frac{N_{S2}^2}{h_{R_S2C}} \left(\frac{L_{mt,S2} \cdot b_{S2} + L_{mt,C} \cdot b_C}{3} + L_{mt,a3} \cdot a_3 \right) \quad (D.4)$$

$$X_{S2D} = \omega\mu_0 \frac{N_{S2}^2}{h_{R_S2D}} \left(\frac{L_{mt,S2} \cdot b_{S2} + L_{mt,D} \cdot b_D}{3} + L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (D.5)$$

$$X_{CD} = \omega\mu_0 \frac{N_C^2}{h_{R_CD}} \left(\frac{L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D}{3} + L_{mt,a2} \cdot a_2 \right) \quad (D.6)$$

D.1.2 [A(N)] Formulation

Fig. D.3. Admittance formulation for the faulted phase A for simulation of T2G faults on the series coil: the leakage model [A(N)].

$$[A_A] = \begin{bmatrix} a_{S1S1} & a_{S1S2} & a_{S1C} & a_{S1D} \\ a_{S1S2} & a_{S2S2} & a_{S2C} & a_{S2D} \\ a_{S1C} & a_{S2C} & a_{CC} & a_{CD} \\ a_{S1D} & a_{S2D} & a_{CD} & a_{DD} \end{bmatrix} \quad (D.7)$$

D.2 T2G Faults on the Delta Coil: Leakage Representation

Fig. D.4. Simulation of T2G faults on the phase A delta coil of the Mirage: the leakage model $[A(N)]$.

D.2.1 Binary Reactances Estimated by the Cylindrical Approach

Fig. D.5. Mmf distribution diagram of the faulted phase A for simulation of T2G faults on the delta coil: the leakage model $[A(N)]$, the Cylindrical approach.

$$X_{SC} = \omega \mu_0 \frac{N_S^2}{h_{R_SC}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,C} \cdot b_C}{3} + L_{mt,a3} \cdot a_3 \right) \quad (D.8)$$

$$X_{SD1} = \omega \mu_0 \frac{N_S^2}{h_{R_SD1}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,D1} \cdot b_{D1}}{3} + L_{mt,C} \cdot b_C + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3 \right) \quad (D.9)$$

$$X_{SD2} = \omega\mu_0 \frac{N_S^2}{h_{R_SD2}} \left(\frac{L_{mt,S} \cdot b_S + L_{mt,D2} \cdot b_{D2}}{3} + L_{mt,C} \cdot b_C + \right. \\ \left. \frac{L_{mt,D1} \cdot b_{D1} + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3}{3} \right) \quad (D.10)$$

$$X_{CD1} = \omega\mu_0 \frac{N_C^2}{h_{R_CD1}} \left(\frac{L_{mt,C} \cdot b_C + L_{mt,D1} \cdot b_{D1}}{3} + L_{mt,a2} \cdot a_2 \right) \quad (D.11)$$

$$X_{CD2} = \omega\mu_0 \frac{N_C^2}{h_{R_CD2}} \left(\frac{L_{mt,C} \cdot b_C + L_{mt,D2} \cdot b_{D2}}{3} + L_{mt,D1} \cdot b_{D1} + L_{mt,a2} \cdot a_2 \right) \quad (D.12)$$

$$X_{D1D2} = \omega\mu_0 \frac{N_{D1}^2}{h_{R_D1D2}} \left(\frac{L_{mt,D1} \cdot b_{D1} + L_{mt,D2} \cdot b_{D2}}{3} \right) \quad (D.13)$$

D.2.2 [A(N)] Formulation

Fig. D.6. Admittance formulation for the faulted phase A for simulation of T2G faults on the delta coil: the leakage model $[A(N)]$.

$$[A_A] = \begin{bmatrix} a_{SS} & a_{SC} & a_{SD1} & a_{SD2} \\ a_{SC} & a_{CC} & a_{CD1} & a_{CD2} \\ a_{SD1} & a_{CD1} & a_{D1D1} & a_{D1D2} \\ a_{SD2} & a_{CD2} & a_{D1D2} & a_{D2D2} \end{bmatrix} \quad (D.14)$$

D.3 T2G Faults on the Series Coil: the Full Model

Fig. D.7. Simulation of T2G faults on the phase A series coil of the Mirage: the full model $[A(N+1)]$.

D.3.1 Binary Reactances Estimated by the Cylindrical Approach

Fig. D.8. Mmf distribution diagram of the faulted phase A for simulation of T2G faults on the series coil: the full model $[A(N+1)]$, the Cylindrical approach.

$$X_{S1-Core} = \omega \mu_0 \frac{N_{S1}^2}{h_{R_S1-Core}} \left(\frac{L_{mt,S1} \cdot b_{S1}}{3} + L_{mt,S2} \cdot b_{S2} + L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D + \right. \\ \left. \frac{L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3}{L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3} \right) \quad (D.15)$$

D.4 T2G Faults on the Delta Coil: the Full Model

Fig. D.10. Simulation of T2G faults on the phase A delta coil of the Mirage: the full model $[A(N+1)]$.

D.4.1 Binary Reactances Estimated by the Cylindrical Approach

Fig. D.11. Mmf distribution diagram of the faulted phase A for simulation of T2G faults on the delta coil: the full model $[A(N+1)]$, the Cylindrical approach.

$$X_{S-Core} = \omega \mu_0 \frac{N_S^2}{h_{R_S-Core}} \left(\frac{L_{mt,S} \cdot b_S}{3} + L_{mt,C} \cdot b_C + L_{mt,D} \cdot b_D + \frac{L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 + L_{mt,a3} \cdot a_3}{3} \right) \quad (D.20)$$

$$X_{C-Core} = \omega\mu_0 \frac{N_C^2}{h_{R_C-Core}} \left(\frac{L_{mt,C} \cdot b_C}{3} + L_{mt,D} \cdot b_D + L_{mt,a1} \cdot a_1 + L_{mt,a2} \cdot a_2 \right) \quad (D.21)$$

$$X_{D1-Core} = \omega\mu_0 \frac{N_{D1}^2}{h_{R_D1-Core}} \left(\frac{L_{mt,D1} \cdot b_{D1}}{3} + L_{mt,D2} \cdot b_{D2} + L_{mt,a1} \cdot a_1 \right) \quad (D.22)$$

$$X_{D2-Core} = \omega\mu_0 \frac{N_{D2}^2}{h_{R_D2-Core}} \left(\frac{L_{mt,D2} \cdot b_{D2}}{3} + L_{mt,a1} \cdot a_1 \right) \quad (D.23)$$

D.4.2 [A(N+1)] Formulation

Fig. D.12. Admittance formulation for the faulted phase A for simulation of T2G faults on the delta coil: the full model [A(N+1)].

$$[A_A] = \begin{bmatrix} a_{SS} & a_{SC} & a_{SD1} & a_{SD2} & a_{S-Core} \\ a_{SC} & a_{CC} & a_{CD1} & a_{CD2} & a_{C-Core} \\ a_{SD1} & a_{CD1} & a_{D1D1} & a_{D1D2} & a_{D1-Core} \\ a_{SD2} & a_{CD2} & a_{D1D2} & a_{D2D2} & a_{D2-Core} \\ a_{S-Core} & a_{C-Core} & a_{D1-Core} & a_{D2-Core} & a_{Core-Core} \end{bmatrix} \quad (D.24)$$

E Stage 1 Benchmarking Tests

E.1 Short-Circuit Tests

E.1.1 Hi – Low Test

Values for voltage, current, and transformer losses are calculated as follows:

- Rated current: $I_{rated_line_peak} = \sqrt{2} \cdot \frac{S_{HX}/3}{V_{H,L-GND}} = \sqrt{2} \cdot \frac{60 \cdot 10^6 / 3}{(144900 / \sqrt{3})} = 338.1 \text{ A},$
- Applied voltage:

$$V_{applied_L-GND_peak} = \sqrt{2} \cdot \frac{V_{H,L-L}}{\sqrt{3}} \cdot Z_{HX_pu} = \sqrt{2} \cdot \frac{144900}{\sqrt{3}} \cdot 0.06441 = 7620 \text{ V},$$
- Transformer losses: $P_{HX} = 96610 \text{ W}.$

Figs. E.1 – E.3 show the ATP simulation results which agree with the calculations.

Fig. E.1. The XFMR Mirage: Short-circuit test Hi – Low. Applied voltage.

Fig. E.2. The XFMR Mirage: Short-circuit test Hi – Low. Rated current.

Fig. E.3. The XFMR Mirage: Short-circuit test Hi – Low. Standard 0° Y-Y phase shift between the rated input current (H) and the short-circuit current (X), phase A.

The measured power losses from ATP simulation: $P_{HX} = 96630 \text{ W}$.

E.1.2 Hi – Tertiary Test

Values for voltage, current, and transformer losses are calculated as follows:

- Rated current: $I_{rated_line_peak} = \sqrt{2} \cdot \frac{S_{HY}/3}{V_{H,L-GND}} = \sqrt{2} \cdot \frac{12 \cdot 10^6/3}{(144900/\sqrt{3})} = 67.6 \text{ A}$,
- Applied voltage:

$$V_{applied_L-GND_peak} = \sqrt{2} \cdot \frac{V_{H,L-L}}{\sqrt{3}} \cdot Z_{HY_pu} = \sqrt{2} \cdot \frac{144900}{\sqrt{3}} \cdot 0.04837 = 5722.7 \text{ V},$$
- Transformer losses: $P_{HY} = 18117 \text{ W}$.

Figs. E.4 – E.6 show the ATP simulation results which agree with the calculations.

Fig. E.4. The XFMR Mirage: Short-circuit test Hi – Tertiary. Applied voltage.

Fig. E.5. The XFMR Mirage: Short-circuit test Hi – Tertiary. Rated current.

Fig. E.6. The XFMR Mirage: Short-circuit test Hi – Tertiary. Standard 30° Y-Δ phase shift between the rated input current (H) and the short-circuit current (Y), phase A.

The measured power losses from ATP simulation: $P_{HY} = 18128 \text{ W}$.

E.1.3 Low – Tertiary Test

Values for voltage, current, and transformer losses are calculated as follows:

- Rated current: $I_{rated_line_peak} = \sqrt{2} \cdot \frac{S_{XY}/3}{V_{X,L-GND}} = \sqrt{2} \cdot \frac{12 \cdot 10^6 / 3}{(72500 / \sqrt{3})} = 135.1 \text{ A}$,
- Applied voltage:

$$V_{applied_L-GND_peak} = \sqrt{2} \cdot \frac{V_{X,L-L}}{\sqrt{3}} \cdot Z_{XY_pu} = \sqrt{2} \cdot \frac{72500}{\sqrt{3}} \cdot 0.02786 = 1649.2 \text{ V},$$
- Transformer losses: $P_{XY} = 18329 \text{ W}$.

Figs. E.7 – E.9 show the ATP simulation results which agree with the calculations.

Fig. E.7. The XFMR Mirage: Short-circuit test Low – Tertiary. Applied voltage.

Fig. E.8. The XFMR Mirage: Short-circuit test Low – Tertiary. Rated current.

Fig. E.9. The XFMR Mirage: Short-circuit test Low – Tertiary. Standard 30° Y-Δ phase shift between the rated input current (X) and the short-circuit current (Y), phase A.

The measured power losses from ATP simulation: $P_{XY} = 18357 \text{ W}$.

E.2 Open-Circuit Test

For this test, the voltage source is placed at the lowest voltage side (tertiary).

Values for voltages are calculated as follows:

- Applied voltage: $V_{\text{applied_L-GND_peak}}(Y) = \sqrt{2} \cdot \frac{V_{Y,L-L}}{\sqrt{3}} = \sqrt{2} \cdot \frac{13800}{\sqrt{3}} = 11267.7 \text{ V},$
- Induced voltage: $V_{\text{ind_L-GND_peak}}(H) = \sqrt{2} \cdot \frac{V_{H,L-L}}{\sqrt{3}} = \sqrt{2} \cdot \frac{144900}{\sqrt{3}} = 118310.4 \text{ V},$
- Induced voltage: $V_{\text{ind_L-GND_peak}}(X) = \sqrt{2} \cdot \frac{V_{X,L-L}}{\sqrt{3}} = \sqrt{2} \cdot \frac{72500}{\sqrt{3}} = 59196.0 \text{ V}.$

Figs. E.10 – E.12 show the ATP simulation results which agree with the calculations. Figs. E.13 - E.15 show the corresponding phase shifts between the voltages of different sides.

Fig. E.10. The XFMR Mirage: Open-circuit test. Applied voltage (Y).

Fig. E.11. The XFMR Mirage: Open-circuit test. Induced voltage (H).

Fig. E.12. The XFMR Mirage: Open-circuit test. Induced voltage (X).

Fig. E.13. The XFMR Mirage: Open-circuit test. Standard 0° Y-Y phase shift between the rated voltages, H and X, phase A.

Fig. E.14. The XFMR Mirage: Open-circuit test. Standard 30° Y- Δ phase shift between the rated voltages, H and Y, phase A.

Fig. E.15. The XFMR Mirage: Open-circuit test. Standard 30° Y- Δ phase shift between the rated voltages, X and Y, phase A.

F Transformer Circuits in ATPDraw

Fig. F.1. ATPDraw circuit of the healthy Mirage's leakage model.

Fig. F.2. ATPDraw circuit of the faulted Mirage:
the leakage model $[A(N)]$, the base case, X_{DINF} is not inserted.

single T2T fault on common coil phase A
located at 10% from bottom:
C3 - 9.9%, C2 - 0.225%, C1 - 89.865%
Cylindrical approach
XDNF: inserted
core: not included

Leakage admittance matrix [A]

POLARITY MARKING

60/60/12 MVA
Mirage autotransformer
Yg-Yg-D, Yna0d1
144.9/72.5/13.8 kV L-L

Base load 87.24 Ohms
referred to secondary side

Fig. F.3. ATPDraw circuit of the faulted Mirage:
the leakage model $[A/V]$, the base case, X_{DNF} is inserted.

G Transformer Matrices

When the Mirage is healthy, all three submatrices are identical, and only two of them are given in each table below to show that all the submatrices do not have interphase coupling.

When the Mirage has the base case T2T fault on phase A, its phase A submatrix is different from those of phases B and C, which remain healthy and identical. In this case, all the submatrices do not have interphase coupling also.

G.1 The Healthy Mirage's Leakage Model

Table G.1. $[A(N)]$ in Si built from the data of Table 3.1: the healthy Mirage's leakage model, K_{cy} neglected.

		Phase A			Phases B and C		
		<i>S</i>	<i>C</i>	<i>D</i>	<i>S</i>	<i>C</i>	<i>D</i>
Phase A	<i>S</i>	17.442064	-22.210076	14.535213	0		
	<i>C</i>	-22.210076	59.221358	-112.35483			
	<i>D</i>	14.535213	-112.35483	296.76539			
Phases B and C	<i>S</i>	0			17.442064	-22.210076	14.535213
	<i>C</i>				-22.210076	59.221358	-112.35483
	<i>D</i>				14.535213	-112.35483	296.76539

Table G.2. $[A(N)]$ in Si built from the data of Table 3.1: the healthy Mirage's leakage model, K_{cy} included.

		Phase A			Phases B and C		
		<i>S</i>	<i>C</i>	<i>D</i>	<i>S</i>	<i>C</i>	<i>D</i>
Phase A	<i>S</i>	18.278811	-22.615755	13.231209	0		
	<i>C</i>	-22.615755	60.153751	-113.95415			
	<i>D</i>	13.231209	-113.95415	305.56623			
Phases B and C	<i>S</i>	0			18.278811	-22.615755	13.231209
	<i>C</i>				-22.615755	60.153751	-113.95415
	<i>D</i>				13.231209	-113.95415	305.56623

Table G.3. $[R(N)]$ in Ohms built from the data of Table 3.1: healthy Mirage's leakage model.

		Phase A			Phases B and C		
		<i>S</i>	<i>C</i>	<i>D</i>	<i>S</i>	<i>C</i>	<i>D</i>
Phase A	<i>S</i>	0.292781	0	0	0		
	<i>C</i>	0	0.166339	0			
	<i>D</i>	0	0	0.03417			
Phases B and C	<i>S</i>	0			0.292781	0	0
	<i>C</i>				0	0.166339	0
	<i>D</i>				0	0	0.03417

G.2 The Healthy Mirage's Full Model

Table G.4. $[A(N+1)]$ in Si built from the data of Table 3.1 and design data of Fig. 3.1:
the healthy Mirage's full model, K_{cy} neglected.

		Phase A				Phases B and C			
		<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>	<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>
Phase A	<i>S</i>	17.451996	-22.288921	18.11737	-3.3730887	0			
	<i>C</i>	-22.288921	59.847282	-140.79234	26.777786				
	<i>D</i>	18.11737	-140.79234	1588.7625	-1216.5914				
	<i>Core</i>	-3.3730887	26.777786	-1216.5914	1145.5866				
Phases B and C	<i>S</i>	0				17.451996	-22.288921	18.11737	-3.3730887
	<i>C</i>					-22.288921	59.847282	-140.79234	26.777786
	<i>D</i>					18.11737	-140.79234	1588.7625	-1216.5914
	<i>Core</i>					-3.3730887	26.777786	-1216.5914	1145.5866

Table G.5. $[A(N+1)]$ in Si built from the data of Table 3.1 and design data of Fig. 3.1:
the healthy Mirage's full model, K_{cy} included.

		Phase A				Phases B and C			
		<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>	<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>
Phase A	<i>S</i>	18.300018	-22.728332	18.455577	-4.9471387	0			
	<i>C</i>	-22.728332	60.75135	-141.6869	26.261125				
	<i>D</i>	18.455577	-141.6869	1592.5588	-1218.6987				
	<i>Core</i>	-4.9471387	26.261125	-1218.6987	1154.0288				
Phases B and C	<i>S</i>	0				18.300018	-22.728332	18.455577	-4.9471387
	<i>C</i>					-22.728332	60.75135	-141.6869	26.261125
	<i>D</i>					18.455577	-141.6869	1592.5588	-1218.6987
	<i>Core</i>					-4.9471387	26.261125	-1218.6987	1154.0288

Table G.6. $[A(N+1)]$ in Si built as the Hybrid model from the data of Table 3.1: the XFMR Mirage.

		Phase A				Phases B and C			
		<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>	<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>
Phase A	<i>S</i>	19.067635	-24.281581	47.683838	-31.789225	0			
	<i>C</i>	-24.281581	61.86044	-154.56689	40.481823				
	<i>D</i>	47.683838	-154.56689	973.18529	-648.7902				
	<i>Core</i>	-31.789225	40.481823	-648.7902	622.29098				
Phases B and C	<i>S</i>	0				19.067635	-24.281581	47.683838	-31.789225
	<i>C</i>					-24.281581	61.86044	-154.56689	40.481823
	<i>D</i>					47.683838	-154.56689	973.18529	-648.7902
	<i>Core</i>					-31.789225	40.481823	-648.7902	622.29098

Table G.7. $[R(N+1)]$ in Ohms built from the data of Table 3.1: the healthy Mirage's full model.

		Phase A				Phases B and C			
		<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>	<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>
Phase A	<i>S</i>	0.292781	0	0	0	0			
	<i>C</i>	0	0.166339	0	0				
	<i>D</i>	0	0	0.03417	0				
	<i>Core</i>	0	0	0	0				
Phases B and C	<i>S</i>	0				0.292781	0	0	0
	<i>C</i>					0	0.166339	0	0
	<i>D</i>					0	0	0.03417	0
	<i>Core</i>					0	0	0	0

Table G.8. $[R(N+1)]$ in Ohms as the Hybrid model from the data of Table 3.1: the XFMR Mirage.

		Phase A				Phases B and C			
		<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>	<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>
Phase A	<i>S</i>	0.4149633	0	0	0	0			
	<i>C</i>	0	0.1488984	0	0				
	<i>D</i>	0	0	0.056536	0				
	<i>Core</i>	0	0	0	0				
Phases B and C	<i>S</i>	0				0.4149633	0	0	0
	<i>C</i>					0	0.1488984	0	0
	<i>D</i>					0	0	0.056536	0
	<i>Core</i>					0	0	0	0

G.3 The Coil Volume Internal Fault Leakage Model: the Base Case

Table G.9. $[A(N)]$ in Si built from the data of Table 3.1 and design data of Fig. 3.1:
the faulted Mirage's leakage model, K_{cy} neglected, the base case.

		Phase A					Phases B and C		
		<i>S</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>D</i>	<i>S</i>	<i>C</i>	<i>D</i>
Phase A	<i>S</i>	18.461973	-30.808057	4148.4096	-1.2762043	0.0980208	0		
	<i>C1</i>	-30.808057	128.22617	-37926.263	10.558068	-0.2775078			
	<i>C2</i>	4148.4096	-37926.263	142706418	-3079776.2	41651.324			
	<i>C3</i>	-1.2762043	10.558068	-3079776.2	79489.705	-2878.9092			
	<i>D</i>	0.0980208	-0.2775078	41651.324	-2878.9092	581.2773			
Phases B and C	<i>S</i>	0					17.442064	-22.210076	14.535213
	<i>C</i>						-22.210076	59.221358	-112.35483
	<i>D</i>						14.535213	-112.35483	296.76539

Table G.10. $[A(N)]$ in Si built from the data of Table 3.1 and design data of Fig. 3.1:
the faulted Mirage's leakage model, K_{cy} included, the base case.

		Phase A					Phases B and C		
		<i>S</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>D</i>	<i>S</i>	<i>C</i>	<i>D</i>
Phase A	<i>S</i>	19.364191	-31.643234	4487.42	-3.661392	-1.9572923	0		
	<i>C1</i>	-31.643234	130.36151	-38573.285	16.304684	-0.8754233			
	<i>C2</i>	4487.42	-38573.285	142922308	-3084175.7	42235.66			
	<i>C3</i>	-3.661392	16.304684	-3084175.7	79617.152	-2895.6022			
	<i>D</i>	-1.9572923	-0.8754233	42235.66	-2895.6022	590.15841			
Phases B and C	<i>S</i>	0					18.278811	-22.615755	13.231209
	<i>C</i>						-22.615755	60.153751	-113.95415
	<i>D</i>						13.231209	-113.95415	305.56623

Table G.11. $[A(N)]$ in Ohms built from the data of Table 3.1 and design data of Fig. 3.1:
the faulted Mirage's leakage model, the base case.

		Phase A					Phases B and C		
		<i>S</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>D</i>	<i>S</i>	<i>C</i>	<i>D</i>
Phase A	<i>S</i>	0.292781	0	0	0	0	0		
	<i>C1</i>	0	0.1494803	0	0	0			
	<i>C2</i>	0	0	0.0003746	0	0			
	<i>C3</i>	0	0	0	0.016484	0			
	<i>D</i>	0	0	0	0	0.03417			
Phases B and C	<i>S</i>	0					0.292781	0	0
	<i>C</i>						0	0.166339	0
	<i>D</i>						0	0	0.03417

G.4 The Coil Volume Internal Fault Full Model: the Base Case

Table G.12. $[A(N+1)]$ in Si built from the data of Table 3.1 and design data of Fig. 3.1:
the faulted Mirage's full model, K_{cyl} neglected, the base case.

		Phase A						Phases B and C			
		<i>S</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>D</i>	<i>Core</i>	<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>
Phase A	<i>S</i>	18.462367	-30.809842	4147.6682	-1.1674709	-0.0236821	0.1039147	0			
	<i>C1</i>	-30.809842	128.22128	-37921.588	10.392347	-0.1054636	-0.1457378				
	<i>C2</i>	4147.6682	-37921.588	142778311	-3085512.8	52968.177	-10088.07				
	<i>C3</i>	-1.1674709	10.392347	-3085512.8	79903.023	-3663.4168	699.25526				
	<i>D</i>	-0.0236821	-0.1054636	52968.177	-3663.4168	2043.1181	-1303.3591				
	<i>Core</i>	0.1039147	-0.1457378	-10088.07	699.25526	-1303.3591	1162.1566				
Phases B and C	<i>S</i>	0						17.451996	-22.288921	18.11737	-3.3730887
	<i>C</i>							-22.288921	59.847282	-140.79234	26.777786
	<i>D</i>							18.11737	-140.79234	1588.7625	-1216.5914
	<i>Core</i>							-3.3730887	26.777786	-1216.5914	1145.5866

Table G.13. $[A(N+1)]$ in Si built from the data of Table 3.1 and design data of Fig. 3.1:
the faulted Mirage's full model, K_{cyl} included, the base case.

		Phase A						Phases B and C			
		<i>S</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>D</i>	<i>Core</i>	<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>
Phase A	<i>S</i>	19.365997	-31.643228	4497.5987	-4.3682576	-0.5364241	-1.2771643	0			
	<i>C1</i>	-31.643228	130.35863	-38557.572	15.286495	0.9406069	-1.6201068				
	<i>C2</i>	4497.5987	-38557.572	142976689	-3089149	52345.849	-9054.1996				
	<i>C3</i>	-4.3682576	15.286495	-3089149	80011.942	-3662.0714	686.36349				
	<i>D</i>	-0.5364241	0.9406069	52345.849	-3662.0714	2049.7553	-1307.4474				
	<i>Core</i>	-1.2771643	-1.6201068	-9054.1996	686.36349	-1307.4474	1171.2593				
Phases B and C	<i>S</i>	0						18.300018	-22.728332	18.455577	-4.9471387
	<i>C</i>							-22.728332	60.75135	-141.6869	26.261125
	<i>D</i>							18.455577	-141.6869	1592.5588	-1218.6987
	<i>Core</i>							-4.9471387	26.261125	-1218.6987	1154.0288

Table G.14. $[A(N+1)]$ in Ohms built from the data of Table 3.1 and design data of Fig. 3.1:
the faulted Mirage's full model, the base case.

		Phase A						Phases B and C			
		<i>S</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>D</i>	<i>Core</i>	<i>S</i>	<i>C</i>	<i>D</i>	<i>Core</i>
Phase A	<i>S</i>	0.292781	0	0	0	0	0	0			
	<i>C1</i>	0	0.1494968	0	0	0	0				
	<i>C2</i>	0	0	0.0003747	0	0	0				
	<i>C3</i>	0	0	0	0.0164859	0	0				
	<i>D</i>	0	0	0	0	0.03417	0				
		0	0	0	0	0	0				
Phases B and C	<i>S</i>	0						0.292781	0	0	0
	<i>C</i>							0	0.166339	0	0
	<i>D</i>							0	0	0.03417	0
	<i>Core</i>							0	0	0	0

H .ATP Files

H.1 Complete .ATP File of the XFMR Mirage

```
BEGIN NEW DATA CASE
C -----
C Generated by ATPDRAW  October, Saturday 20, 2018
C A Bonneville Power Administration program
C by H. K. Høidalen at SEFAS/NTNU - NORWAY 1994-2016
C -----
POWER FREQUENCY 60.
C dT >< Tmax >< Xopt >< Copt ><Epsiln>
5.E-6 .1 60. 1.E-12
500 1 1 1 1 0 0 1 0
C 1 2 3 4 5 6 7 8
C 34567890123456789012345678901234567890123456789012345678901234567890
/BRANCH
C < n1 >< n2 ><ref1><ref2>< R >< L >< C >
C < n1 >< n2 ><ref1><ref2>< R >< A >< B ><Leng><><>0
C core data before dielectric test
C -----
C Nonlinear core representation
C -----
93X0001AT0001A 0.309146.784
0.0 0.0
0.077279102722 29.654487218
0.15455820544 39.230140221
0.30911641089 46.783757247
0.61823282177 51.768188179
1.2364656435 54.682449063
2.4729312871 56.269036251
4.9458625742 57.103233521
9.8917251484 57.541590306
19.783450297 57.787304883
39.566900593 57.851798931
9999.
X0001AT0001A 46742.
93X0001BT0001B 0.309146.784
0.0 0.0
0.077279102722 29.654487218
0.15455820544 39.230140221
0.30911641089 46.783757247
0.61823282177 51.768188179
1.2364656435 54.682449063
2.4729312871 56.269036251
4.9458625742 57.103233521
9.8917251484 57.541590306
19.783450297 57.787304883
39.566900593 57.851798931
9999.
X0001BT0001B 46742.
93X0001CT0001C 0.309146.784
0.0 0.0
0.077279102722 29.654487218
0.15455820544 39.230140221
0.30911641089 46.783757247
0.61823282177 51.768188179
1.2364656435 54.682449063
2.4729312871 56.269036251
4.9458625742 57.103233521
9.8917251484 57.541590306
19.783450297 57.787304883
39.566900593 57.851798931
9999.
X0001CT0001C 46742.
```

```

93X0001BXX0002      0.309145.049
      0.0      0.0
0.077279102722      27.016706851
0.15455820544      36.850280211
0.30911641089      45.048914993
0.61823282177      50.688003792
1.2364656435      54.073406031
2.4729312871      55.943894918
4.9458625742      56.933383895
9.8917251484      57.451270212
19.783450297      57.73373568
39.566900593      57.798229728
9999.
X0001BXX0002      38951.
93X0001CXX0002      0.309145.049
      0.0      0.0
0.077279102722      27.016706851
0.15455820544      36.850280211
0.30911641089      45.048914993
0.61823282177      50.688003792
1.2364656435      54.073406031
2.4729312871      55.943894918
4.9458625742      56.933383895
9.8917251484      57.451270212
19.783450297      57.73373568
39.566900593      57.798229728
9999.
X0001CXX0002      38951.
X0001AXX0002      1.E-6
      XX0002      1.E-6
T0001AX0001B      0.0936      500.
T0001BX0001C      0.0936      500.
T0001C      0.0936      500.
X0001A      1.E7
X0001B      1.E7
X0001C      1.E7
C -----
C [R] matrix
C -----
$VINTAGE, 1,
HA      HX      0.41496329175
HB      HY      0.41496329175
HC      HZ      0.41496329175
XA      XX      0.14889843653
XB      XY      0.14889843653
XC      XZ      0.14889843653
YA      YX      0.05653602122
YB      YY      0.05653602122
YC      YZ      0.05653602122
C -----
C [A] matrix
C -----
USE AR
01HX      XA      19.06763495      0.0
02XX      GNDA      -24.2815805      0.0
      61.860440418      0.0
03YX      YB      47.683837666      0.0
      -154.56688504      0.0
      973.18529354      0.0
04X0001AT0001A      -31.78922511      0.0
      40.481823287      0.0
      -648.79019569      0.0
      622.29097763      0.0
05HY      XB      0.0      0.0
      0.0      0.0
      0.0      0.0
      0.0      0.0
      19.06763495      0.0

```

06XY	GNDB	0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		-24.2815805	0.0
		61.860440418	0.0
07YY	YC	0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		47.683837666	0.0
		-154.56688504	0.0
		973.18529354	0.0
08X0001BT0001B		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		-31.78922511	0.0
		40.481823287	0.0
		-648.79019569	0.0
		622.29097763	0.0
09HZ	XC	0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		19.06763495	0.0
10XZ	GNDC	0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		-24.2815805	0.0
		61.860440418	0.0
11YZ	YA	0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		47.683837666	0.0
		-154.56688504	0.0
		973.18529354	0.0
12X0001CT0001C		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		-31.78922511	0.0
		40.481823287	0.0
		-648.79019569	0.0
		622.29097763	0.0
USE RL			
\$VINTAGE, 0,			
C base load to give rated currents			
	XA	87.24	1
	XB	87.24	1

XC	87.24					1
YA	1.E6					1
YB	1.E6					1
YC	1.E6					1
GNDA	1.E-6					0
GNDB	1.E-6					0
GNDC	1.E-6					0

/SWITCH

C	< n 1>	< n 2>	Tclose	><Top/Tde ><	Ie	><Vf/CLOP ><	type	>
SRCA	HA					MEASURING		1
SRCB	HB					MEASURING		1
SRCC	HC					MEASURING		1

/SOURCE

C	< n 1>	><	Ampl.	><	Freq.	><	Phase/T0><	A1	><	T1	><	TSTART	><	TSTOP	>
14SRCA			118310.355		60.							-1.		100.	
14SRCB			118310.355		60.		-120.					-1.		100.	
14SRCC			118310.355		60.		-240.					-1.		100.	

/OUTPUT

BLANK BRANCH

BLANK SWITCH

BLANK SOURCE

BLANK OUTPUT

BLANK PLOT

BEGIN NEW DATA CASE

BLANK

H.2 Matrices for the Healthy Mirage's Full Model

Here, $[A(N+I)]$ is based on design data of Fig. 3.1, K_{cyl} is neglected and $[R(N+I)]$ is based on resistance measurements taken from the test report in Table 3.1. Substituting these matrices in the .atp file of the XFMR Mirage located in Appendix H.1, the entire .atp file for the healthy Mirage's full model is obtained.

```

C -----
C [R] matrix
C -----
$VINTAGE, 1,
  HA    HX          0.292781
  HB    HY          0.292781
  HC    HZ          0.292781
  XA    XX          0.166339
  XB    XY          0.166339
  XC    XZ          0.166339
  YA    YX          0.03417
  YB    YY          0.03417
  YC    YZ          0.03417
C -----
C [A] matrix
C -----
  USE AR
01HX    XA          17.451996          0.0
02XX    GNDA        -22.288921          0.0
          59.847282          0.0
03YX    YB          18.11737          0.0
          -140.792336          0.0
          1588.762539          0.0
04X0001AT0001A      -3.373089          0.0
          26.777786          0.0
          -1216.591395          0.0
          1145.586604          0.0
05HY    XB          0.0          0.0
          0.0          0.0
          0.0          0.0
          0.0          0.0
          17.451996          0.0
06XY    GNDB          0.0          0.0
          0.0          0.0
          0.0          0.0
          0.0          0.0
          -22.288921          0.0
          59.847282          0.0
07YY    YC          0.0          0.0
          0.0          0.0
          0.0          0.0
          0.0          0.0
          18.11737          0.0
          -140.792336          0.0
          1588.762539          0.0
08X0001BT0001B      0.0          0.0
          0.0          0.0
          0.0          0.0
          0.0          0.0
          -3.373089          0.0
          26.777786          0.0
          -1216.591395          0.0
          1145.586604          0.0
09HZ    XC          0.0          0.0
          0.0          0.0
          0.0          0.0
          0.0          0.0
          0.0          0.0
          0.0          0.0

```

		0.0	0.0
		0.0	0.0
		17.451996	0.0
10XZ	GNDC	0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		-22.288921	0.0
		59.847282	0.0
11YZ	YA	0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		18.11737	0.0
		-140.792336	0.0
		1588.762539	0.0
12X0001CT0001C		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		0.0	0.0
		-3.373089	0.0
		26.777786	0.0
		-1216.591395	0.0
		1145.586604	0.0
USE RL			
\$VINTAGE, 0,			

H.3 Complete .ATP File for the Faulted Mirage's Full Model: the Base Case, X_{DNF} Is Not Inserted

```

BEGIN NEW DATA CASE
C -----BASE CASE: single T2T fault on the common coil
C -----position: 10% from bottom or 9.9-0.225-89.865%
C -----C3 - C2 - C1
C -----9.9 + 0.225 + 89.865 = 99.99 != 100 due to rounding
C -----without rounding their sum is 100
C -----Cylindrical approach
C -----The Rogowski factor K_cyl: neglected
C -----CORE: included, Zo=750 mH
C -----XDNF: not inserted
C -----Node naming: H1,H2,H3 represent the HV side
C -----X1,X2,X3 represent the LV side
C -----Y1,Y2,Y3 represent the TV side
C -----
C Generated by ATPDRAW October, Saturday 20, 2018
C A Bonneville Power Administration program
C by H. K. Høidalen at SEFAS/NTNU - NORWAY 1994-2016
C -----
POWER FREQUENCY 60.
C dT >< Tmax >< Xopt >< Copt ><Epsiln>
C 5.E-6 .1 60. 1.E-12
C 500 1 1 1 1 0 0 1 0
C 1 2 3 4 5 6 7 8
C 34567890123456789012345678901234567890123456789012345678901234567890
/BRANCH
C < n1 >< n2 ><ref1><ref2>< R >< L >< C >
C < n1 >< n2 ><ref1><ref2>< R >< A >< B ><Leng><><>0
C -----SMALL RESISTANCES FOR RENAMING HA,HB,HC TO H1,H2,H3-----
HA H1 1.E-6 0
HB H2 1.E-6 0
HC H3 1.E-6 0
C -----BASE LOAD-----
X1 87.24 1
X2 87.24 1
X3 87.24 1
C -----LARGE RESISTANCES TO GROUND FOR STABILITY-----
Y1 1.E6 1
Y2 1.E6 1
Y3 1.E6 1
F1 1.E6 0
F2 1.E6 0
C -----S.C. RESISTANCES-----
NOD02BF1 1.E-6 0
F2 NOD03B 1.E-6 0
C -----BACKBONE STRUCTURE-----
C -----nodes 6,10,14 are the core nodes ALPHA,BETA,GAMMA and not included
NOD01ANOD01B 1.E8 0
NOD02ANOD02B 1.E8 0
NOD03ANOD03B 1.E8 0
NOD04ANOD04B 1.E8 0
NOD05ANOD05B 1.E8 0
NOD07ANOD07B 1.E8 0
NOD08ANOD08B 1.E8 0
NOD09ANOD09B 1.E8 0
NOD11ANOD11B 1.E8 0
NOD12ANOD12B 1.E8 0
NOD13ANOD13B 1.E8 0
C -----WINDING CONNECTIONS-----
NOD01BX1 1.E-6 0
NOD04B 1.E-6 0
NOD05BY2 1.E-6 0
NOD07BX2 1.E-6 0
NOD08B 1.E-6 0
NOD09BY3 1.E-6 0

```

NOD11BX3	1.E-6	0
NOD12B	1.E-6	0
NOD13BY1	1.E-6	0
C -----		
C Nonlinear core representation		
C -----		
C -----CORE LEG PHASE A-----		
93ALPHA ALPHAP	0.309146.784	2
0.0	0.0	
0.077279102722	29.654487218	
0.15455820544	39.230140221	
0.30911641089	46.783757247	
0.61823282177	51.768188179	
1.2364656435	54.682449063	
2.4729312871	56.269036251	
4.9458625742	57.103233521	
9.8917251484	57.541590306	
19.783450297	57.787304883	
39.566900593	57.851798931	
9999.		
ALPHA ALPHAP	46742.	
C -----CORE LEG PHASE B-----		
93BETA BETAP	0.309146.784	2
0.0	0.0	
0.077279102722	29.654487218	
0.15455820544	39.230140221	
0.30911641089	46.783757247	
0.61823282177	51.768188179	
1.2364656435	54.682449063	
2.4729312871	56.269036251	
4.9458625742	57.103233521	
9.8917251484	57.541590306	
19.783450297	57.787304883	
39.566900593	57.851798931	
9999.		
BETA BETAP	46742.	
C -----CORE LEG PHASE C-----		
93GAMMA GAMMAP	0.309146.784	2
0.0	0.0	
0.077279102722	29.654487218	
0.15455820544	39.230140221	
0.30911641089	46.783757247	
0.61823282177	51.768188179	
1.2364656435	54.682449063	
2.4729312871	56.269036251	
4.9458625742	57.103233521	
9.8917251484	57.541590306	
19.783450297	57.787304883	
39.566900593	57.851798931	
9999.		
GAMMA GAMMAP	46742.	
C -----CORE YOKE BETWEEN PHASES A AND B		
93BETA GNDCOR	0.309145.049	2
0.0	0.0	
0.077279102722	27.016706851	
0.15455820544	36.850280211	
0.30911641089	45.048914993	
0.61823282177	50.688003792	
1.2364656435	54.073406031	
2.4729312871	55.943894918	
4.9458625742	56.933383895	
9.8917251484	57.451270212	
19.783450297	57.73373568	
39.566900593	57.798229728	
9999.		
BETA GNDCOR	38951.	
C -----CORE YOKE BETWEEN PHASES B AND C		
93GAMMA GNDCOR	0.309145.049	2

```

0.0      0.0
0.077279102722  27.016706851
0.15455820544  36.850280211
0.30911641089  45.048914993
0.61823282177  50.688003792
1.2364656435   54.073406031
2.4729312871   55.943894918
4.9458625742   56.933383895
9.8917251484   57.451270212
19.783450297   57.73373568
39.566900593   57.798229728
9999.
GAMMA GNDCOR      38951.
C -----SIDE RESISTANCES TO COMMON NODE
ALPHA GNDCOR      1.E-6
      GNDCOR      1.E-6
C -----Z0=750 mH per phase-----
ALPHAPBETA      0.0936  500.
BETAP GAMMA      0.0936  500.
GAMMAP      0.0936  500.
C -----(N+1)th COIL NODES ARE GROUNDED THRU LARGE RESISTANCES
ALPHA      1.E7
BETA      1.E7
GAMMA      1.E7
C -----
C [R] matrix
C -----
$VINTAGE, 1,
C -----SERIES COILS-----
H1      NOD01A      0.292781      1
H2      NOD07A      0.292781      1
H3      NOD11A      0.292781      1
C -----COMMON COILS-----
C ----portion C1----
NOD02AX1      0.149480317      1
C ----portion C2----
NOD03ANOD02B      0.000374637      1
C ----portion C3----
NOD04ANOD03B      0.016484045      1
C -----
NOD08AX2      0.166339      1
NOD12AX3      0.166339      1
C -----DELTA COILS-----
Y1      NOD05A      0.03417      1
Y2      NOD09A      0.03417      1
Y3      NOD13A      0.03417      1
C -----
C [A] MATRIX\n
C Created with Matlab function AMatrixATP(A) by N&A
C -----
USE AR
C <nod1><nod2><      ><      A=1/L      ><      B=R      >
01NOD01ANOD01B      000000018.461980000000000.000000
02NOD02ANOD02B      -00000030.808067000000000.000000
      000000128.226179000000000.000000
03NOD03ANOD03B      000004147.598875000000000.000000
      -00037925.150904000000000.000000
      142794134.557739000000000.000000
04NOD04ANOD04B      -00000001.220020000000000.000000
      000000010.480982000000000.000000
      -03085854.858528000000000.000000
      000079910.944697000000000.000000
05NOD05ANOD05B      -00000000.006647000000000.000000
      -00000000.133903000000000.000000
      000052975.365396000000000.000000
      -00003663.650225000000000.000000
      000002043.195218000000000.000000
06ALPHA ALPHAP      000000000.093324000000000.000000

```

13NOD13ANOD123

14GAMMA GAMMAP

```

000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000018.117370000000000.000000
-00000140.792336000000000.000000
000001588.762539000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
-00000003.373089000000000.000000
000000026.777786000000000.000000
-00001216.591395000000000.000000
000001145.586604000000000.000000

```

```

USE RL
$VINTAGE,0
C -----
/SWITCH
C < n 1>< n 2>< Tclose ><Top/Tde >< Ie ><Vf/CLOP >< type >
  SRCA  HA                      MEASURING                      1
  SRCB  HB                      MEASURING                      1
  SRCC  HC                      MEASURING                      1
C -----S.C. SWITCH FOR T2T FAULTS-----
  F1    F2          -1        10.                                1
/SOURCE
C < n 1><>< Ampl. >< Freq. ><Phase/T0>< A1 >< T1 >< TSTART >< TSTOP >
14SRCA  118310.355      60.          -1.          100.
14SRCB  118310.355      60.        -120.          -1.          100.
14SRCC  118310.355      60.        -240.          -1.          100.
/OUTPUT
  H1    H2    H3    X1    X2    X3
BLANK BRANCH
BLANK SWITCH
BLANK SOURCE
BLANK OUTPUT
BLANK PLOT
BEGIN NEW DATA CASE
BLANK

```


H.4 Complete .ATP File for the Faulted Mirage's Full Model: the Base Case, X_{DNF} Is Inserted for Fault Current Matching

```

BEGIN NEW DATA CASE
C -----BASE CASE: single T2T fault on the common coil
C -----position: 10% from bottom or 9.9-0.225-89.865%
C -----C3 - C2 - C1
C -----9.9 + 0.225 + 89.865 = 99.99 != 100 due to rounding
C -----without rounding their sum is 100
C -----Cylindrical approach
C -----The Rogowski factor K_cyl: neglected
C -----CORE: included, Zo=750 mH
C -----XDNF: inserted 1.293 mOhms to match the fault current
C -----Rdamp: inserted 10.285 Ohms in parallel to XDNF
C -----Node naming: H1,H2,H3 represent the HV side
C -----X1,X2,X3 represent the LV side
C -----Y1,Y2,Y3 represent the TV side
C -----
C Generated by ATPDRAW October, Saturday 20, 2018
C A Bonneville Power Administration program
C by H. K. Høidalen at SEFAS/NTNU - NORWAY 1994-2016
C -----
POWER FREQUENCY 60.
C dT >< Tmax >< Xopt >< Copt >< Epsiln>
5.E-6 .1 60. 1.E-12
500 1 1 1 1 0 0 1 0
C 1 2 3 4 5 6 7 8
C 34567890123456789012345678901234567890123456789012345678901234567890
/BRANCH
C < n1 >< n2 ><ref1><ref2>< R >< L >< C >
C < n1 >< n2 ><ref1><ref2>< R >< A >< B ><Leng><><>0
C -----SMALL RESISTANCES FOR RENAMING HA,HB,HC TO H1,H2,H3-----
HA H1 1.E-6 0
HB H2 1.E-6 0
HC H3 1.E-6 0
C -----BASE LOAD-----
X1 87.24 1
X2 87.24 1
X3 87.24 1
C -----LARGE RESISTANCES TO GROUND FOR STABILITY-----
Y1 1.E6 1
Y2 1.E6 1
Y3 1.E6 1
F1 1.E6 0
F2 1.E6 0
C -----S.C. STRUCTURE-----
NOD02BF1 1.E-6 0
F2 NOD03B 1.E-6 0
C -----BACKBONE STRUCTURE-----
C -----nodes 6,10,14 are the core nodes ALPHA,BETA,GAMMA and not included
NOD01ANOD01B 1.E8 0
NOD02ANOD02B 1.E8 0
NOD03ANOD03B 1.E8 0
NOD04ANOD04B 1.E8 0
NOD05ANOD05B 1.E8 0
NOD07ANOD07B 1.E8 0
NOD08ANOD08B 1.E8 0
NOD09ANOD09B 1.E8 0
NOD11ANOD11B 1.E8 0
NOD12ANOD12B 1.E8 0
NOD13ANOD13B 1.E8 0
C -----WINDING CONNECTIONS-----
NOD01BX1 1.E-6 0
NOD04B 1.E-6 0
NOD05BY2 1.E-6 0
NOD07BX2 1.E-6 0
NOD08B 1.E-6 0

```

```

NOD09BY3          1.E-6          0
NOD11BX3          1.E-6          0
      NOD12B       1.E-6          0
NOD13BY1          1.E-6          0
C -----
C Nonlinear core representation
C -----
C -----CORE LEG PHASE A
93ALPHA ALPHAP      0.309146.784      2
      0.0          0.0
0.077279102722    29.654487218
0.15455820544    39.230140221
0.30911641089    46.783757247
0.61823282177    51.768188179
1.2364656435     54.682449063
2.4729312871     56.269036251
4.9458625742     57.103233521
9.8917251484     57.541590306
19.783450297     57.787304883
39.566900593     57.851798931
      9999.
      ALPHA ALPHAP      46742.
C -----CORE LEG PHASE B
93BETA BETAP        0.309146.784      2
      0.0          0.0
0.077279102722    29.654487218
0.15455820544    39.230140221
0.30911641089    46.783757247
0.61823282177    51.768188179
1.2364656435     54.682449063
2.4729312871     56.269036251
4.9458625742     57.103233521
9.8917251484     57.541590306
19.783450297     57.787304883
39.566900593     57.851798931
      9999.
      BETA BETAP        46742.
C -----CORE LEG PHASE C
93GAMMA GAMMAP      0.309146.784      2
      0.0          0.0
0.077279102722    29.654487218
0.15455820544    39.230140221
0.30911641089    46.783757247
0.61823282177    51.768188179
1.2364656435     54.682449063
2.4729312871     56.269036251
4.9458625742     57.103233521
9.8917251484     57.541590306
19.783450297     57.787304883
39.566900593     57.851798931
      9999.
      GAMMA GAMMAP      46742.
C -----CORE YOKE BETWEEN PHASES A AND B
93BETA GNDCOR       0.309145.049      2
      0.0          0.0
0.077279102722    27.016706851
0.15455820544    36.850280211
0.30911641089    45.048914993
0.61823282177    50.688003792
1.2364656435     54.073406031
2.4729312871     55.943894918
4.9458625742     56.933383895
9.8917251484     57.451270212
19.783450297     57.73373568
39.566900593     57.798229728
      9999.
      BETA GNDCOR       38951.
C -----CORE YOKE BETWEEN PHASES B AND C

```

```

93GAMMA GNDCOR          0.309145.049          2
      0.0              0.0
0.077279102722      27.016706851
0.15455820544      36.850280211
0.30911641089      45.048914993
0.61823282177      50.688003792
1.2364656435      54.073406031
2.4729312871      55.943894918
4.9458625742      56.933383895
9.8917251484      57.451270212
19.783450297      57.73373568
39.566900593      57.798229728
      9999.
      GAMMA GNDCOR          38951.
C -----SIDE RESISTANCES TO COMMON NODE
      ALPHA GNDCOR          1.E-6
      GNDCOR          1.E-6
C -----Z0=750 mH per phase
      ALPHAPBETA          0.0936      500.
      BETAP GAMMA          0.0936      500.
      GAMMAP          0.0936      500.
C -----(N+1)th COIL NODES ARE GROUNDED THRU LARGE RESISTANCES
      ALPHA          1.E7
      BETA          1.E7
      GAMMA          1.E7
C -----
C [R] matrix
C -----
$VINTAGE, 1,
C -----SERIES COILS-----
      H1      NOD01A          0.292781          1
      H2      NOD07A          0.292781          1
      H3      NOD11A          0.292781          1
C -----COMMON COILS-----
C ----portion C1----
      NOD02AX1          0.149480317          1
C ----portion C2----
      XDNF      NOD02B          0.000374637          1
C ----portion C3----
      NOD04ANOD03B          0.016484045          1
C -----
      NOD08AX2          0.166339          1
      NOD12AX3          0.166339          1
C -----DELTA COILS-----
      Y1      NOD05A          0.03417          1
      Y2      NOD09A          0.03417          1
      Y3      NOD13A          0.03417          1
C -----XDNF and parallel resistance Rd-----
      NOD03AXDNF          .001293          0
      NOD03AXDNF          10.28541428          0
C -----
C [A] MATRIX\n
C Created with Matlab function AMatrixATP(A) by N&A
C -----
      USE AR
C <nod1><nod2><      ><      ><      A=1/L      ><      B=R      >
01NOD01ANOD01B          000000018.4619800000000000.000000
02NOD02ANOD02B          -00000030.8080670000000000.000000
          000000128.2261790000000000.000000
03NOD03ANOD03B          000004147.5988750000000000.000000
          -00037925.1509040000000000.000000
          142794134.5577390000000000.000000
04NOD04ANOD04B          -00000001.2200200000000000.000000
          000000010.4809820000000000.000000
          -03085854.8585280000000000.000000
          000079910.9446970000000000.000000
05NOD05ANOD05B          -00000000.0066470000000000.000000
          -00000000.1339030000000000.000000

```

13NOD13ANOD13B

```

000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000018.1173700000000000.000000
-00000140.7923360000000000.000000
000001588.7625390000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000003.3730890000000000.000000
000000026.7777860000000000.000000
-00001216.5913950000000000.000000
000001145.5866040000000000.000000

14GAMMA GAMMAP

USE RL
$VINTAGE,0
C -----
/SWITCH
C < n 1>< n 2>< Tclose ><Top/Tde >< Ie ><Vf/CLOP >< type >
  SRCA  HA                                MEASURING 1
  SRCB  HB                                MEASURING 1
  SRCC  HC                                MEASURING 1
C -----S.C. SWITCH FOR T2T FAULTS-----
C F1    F2          -1      10. 1
/SOURCE
C < n 1><>< Ampl. >< Freq. ><Phase/T0>< A1 >< T1 >< TSTART >< TSTOP >
14SRCA  118310.355    60.          -1.      100.
14SRCB  118310.355    60.      -120.    -1.      100.
14SRCC  118310.355    60.      -240.    -1.      100.
/OUTPUT
  H1    H2    H3    X1    X2    X3
BLANK BRANCH
BLANK SWITCH
BLANK SOURCE
BLANK OUTPUT
BLANK PLOT
BEGIN NEW DATA CASE
BLANK

```

H.5 Complete .ATP File for the Faulted Mirage's Full Model: the Base Case, X_{DNF} Is Inserted for Delta Current Matching

```

BEGIN NEW DATA CASE
C -----BASE CASE: single T2T fault on the common coil
C -----position: 10% from bottom or 9.9-0.225-89.865%
C -----C3 - C2 - C1
C -----9.9 + 0.225 + 89.865 = 99.99 != 100 due to rounding
C -----without rounding their sum is 100
C -----Cylindrical approach
C -----The Rogowski factor K_cyl: neglected
C -----CORE: included, Zo=750 mH
C -----XDNF: inserted 2.9 mOhms to match the delta current
C -----Rdamp: inserted 23.069 Ohms in parallel to XDNF
C -----Node naming: H1,H2,H3 represent the HV side
C -----X1,X2,X3 represent the LV side
C -----Y1,Y2,Y3 represent the TV side
C -----
C Generated by ATPDRAW October, Saturday 20, 2018
C A Bonneville Power Administration program
C by H. K. Høidalen at SEFAS/NTNU - NORWAY 1994-2016
C -----
POWER FREQUENCY 60.
C dT >< Tmax >< Xopt >< Copt >< Epsiln>
5.E-6 .1 60. 1.E-12
500 1 1 1 1 0 0 1 0
C 1 2 3 4 5 6 7 8
C 34567890123456789012345678901234567890123456789012345678901234567890
/BRANCH
C < n1 >< n2 ><ref1><ref2>< R >< L >< C >
C < n1 >< n2 ><ref1><ref2>< R >< A >< B ><Leng><>>0
C -----SMALL RESISTANCES FOR RENAMING HA,HB,HC TO H1,H2,H3-----
HA H1 1.E-6 0
HB H2 1.E-6 0
HC H3 1.E-6 0
C -----BASE LOAD-----
X1 87.24 1
X2 87.24 1
X3 87.24 1
C -----LARGE RESISTANCES TO GROUND FOR STABILITY-----
Y1 1.E6 1
Y2 1.E6 1
Y3 1.E6 1
F1 1.E6 0
F2 1.E6 0
C -----S.C. STRUCTURE-----
NOD02BF1 1.E-6 0
F2 NOD03B 1.E-6 0
C -----BACKBONE STRUCTURE-----
C -----nodes 6,10,14 are the core nodes ALPHA,BETA,GAMMA and not included
NOD01ANOD01B 1.E8 0
NOD02ANOD02B 1.E8 0
NOD03ANOD03B 1.E8 0
NOD04ANOD04B 1.E8 0
NOD05ANOD05B 1.E8 0
NOD07ANOD07B 1.E8 0
NOD08ANOD08B 1.E8 0
NOD09ANOD09B 1.E8 0
NOD11ANOD11B 1.E8 0
NOD12ANOD12B 1.E8 0
NOD13ANOD13B 1.E8 0
C -----WINDING CONNECTIONS-----
NOD01BX1 1.E-6 0
NOD04B 1.E-6 0
NOD05BY2 1.E-6 0
NOD07BX2 1.E-6 0
NOD08B 1.E-6 0

```

```

NOD09BY3          1.E-6          0
NOD11BX3          1.E-6          0
      NOD12B       1.E-6          0
NOD13BY1          1.E-6          0
C -----
C Nonlinear core representation
C -----
C -----CORE LEG PHASE A
93ALPHA ALPHAP      0.309146.784      2
      0.0          0.0
0.077279102722    29.654487218
0.15455820544    39.230140221
0.30911641089    46.783757247
0.61823282177    51.768188179
1.2364656435     54.682449063
2.4729312871     56.269036251
4.9458625742     57.103233521
9.8917251484     57.541590306
19.783450297     57.787304883
39.566900593     57.851798931
      9999.
      ALPHA ALPHAP      46742.
C -----CORE LEG PHASE B
93BETA BETAP        0.309146.784      2
      0.0          0.0
0.077279102722    29.654487218
0.15455820544    39.230140221
0.30911641089    46.783757247
0.61823282177    51.768188179
1.2364656435     54.682449063
2.4729312871     56.269036251
4.9458625742     57.103233521
9.8917251484     57.541590306
19.783450297     57.787304883
39.566900593     57.851798931
      9999.
      BETA BETAP        46742.
C -----CORE LEG PHASE C
93GAMMA GAMMAP      0.309146.784      2
      0.0          0.0
0.077279102722    29.654487218
0.15455820544    39.230140221
0.30911641089    46.783757247
0.61823282177    51.768188179
1.2364656435     54.682449063
2.4729312871     56.269036251
4.9458625742     57.103233521
9.8917251484     57.541590306
19.783450297     57.787304883
39.566900593     57.851798931
      9999.
      GAMMA GAMMAP      46742.
C -----CORE YOKE BETWEEN PHASES A AND B
93BETA GNDCOR       0.309145.049      2
      0.0          0.0
0.077279102722    27.016706851
0.15455820544    36.850280211
0.30911641089    45.048914993
0.61823282177    50.688003792
1.2364656435     54.073406031
2.4729312871     55.943894918
4.9458625742     56.933383895
9.8917251484     57.451270212
19.783450297     57.73373568
39.566900593     57.798229728
      9999.
      BETA GNDCOR       38951.
C -----CORE YOKE BETWEEN PHASES B AND C

```

```

93GAMMA GNDCOR          0.309145.049          2
      0.0              0.0
0.077279102722      27.016706851
0.15455820544      36.850280211
0.30911641089      45.048914993
0.61823282177      50.688003792
1.2364656435      54.073406031
2.4729312871      55.943894918
4.9458625742      56.933383895
9.8917251484      57.451270212
19.783450297      57.73373568
39.566900593      57.798229728
      9999.
      GAMMA GNDCOR          38951.
C -----SIDE RESISTANCES TO COMMON NODE
      ALPHA GNDCOR          1.E-6
      GNDCOR          1.E-6
C -----Z0=750 mH per phase
      ALPHAPBETA          0.0936      500.
      BETAP GAMMA          0.0936      500.
      GAMMAP          0.0936      500.
C -----(N+1)th COIL NODES ARE GROUNDED THRU LARGE RESISTANCES
      ALPHA          1.E7
      BETA          1.E7
      GAMMA          1.E7
C -----
C [R] matrix
C -----
$VINTAGE, 1,
C -----SERIES COILS-----
      H1      NOD01A          0.292781          1
      H2      NOD07A          0.292781          1
      H3      NOD11A          0.292781          1
C -----COMMON COILS-----
C ----portion C1----
      NOD02AX1          0.149480317          1
C ----portion C2----
      XDNF      NOD02B          0.000374637          1
C ----portion C3----
      NOD04ANOD03B          0.016484045          1
C -----
      NOD08AX2          0.166339          1
      NOD12AX3          0.166339          1
C -----DELTA COILS-----
      Y1      NOD05A          0.03417          1
      Y2      NOD09A          0.03417          1
      Y3      NOD13A          0.03417          1
C -----XDNF and parallel resistance Rd-----
      NOD03AXDNF          .0029          0
      NOD03AXDNF          23.068601247          0
C -----
C [A] MATRIX\n
C Created with Matlab function AMatrixATP(A) by N&A
C -----
      USE AR
C <nod1><nod2><      ><      ><      A=1/L      ><      B=R      >
01NOD01ANOD01B          000000018.4619800000000000.000000
02NOD02ANOD02B          -00000030.8080670000000000.000000
          000000128.2261790000000000.000000
03NOD03ANOD03B          000004147.5988750000000000.000000
          -00037925.1509040000000000.000000
          142794134.5577390000000000.000000
04NOD04ANOD04B          -00000001.2200200000000000.000000
          000000010.4809820000000000.000000
          -03085854.8585280000000000.000000
          000079910.9446970000000000.000000
05NOD05ANOD05B          -00000000.0066470000000000.000000
          -00000000.1339030000000000.000000

```


13NOD13ANOD13B

```

000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000018.1173700000000000.000000
-00000140.7923360000000000.000000
000001588.7625390000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000000.0000000000000000.000000
000000003.3730890000000000.000000
000000026.7777860000000000.000000
-00001216.5913950000000000.000000
000001145.5866040000000000.000000

14GAMMA GAMMAP

USE RL
$VINTAGE,0
C -----
/SWITCH
C < n 1>< n 2>< Tclose ><Top/Tde >< Ie ><Vf/CLOP >< type >
  SRCA  HA                                MEASURING 1
  SRCB  HB                                MEASURING 1
  SRCC  HC                                MEASURING 1
C -----S.C. SWITCH FOR T2T FAULTS-----
C F1    F2          -1      10. 1
/SOURCE
C < n 1><>< Ampl. >< Freq. ><Phase/T0>< A1 >< T1 >< TSTART >< TSTOP >
14SRCA  118310.355    60.          -1.      100.
14SRCB  118310.355    60.      -120.    -1.      100.
14SRCC  118310.355    60.      -240.    -1.      100.
/OUTPUT
  H1    H2    H3    X1    X2    X3
BLANK BRANCH
BLANK SWITCH
BLANK SOURCE
BLANK OUTPUT
BLANK PLOT
BEGIN NEW DATA CASE
BLANK

```

I Benchmarking results

I.1 Short-Circuit Tests: Benchmarking of the Coil Volume Internal Fault Leakage Model

Table I.1. Benchmarking of the coil volume internal fault leakage model: X_{DNF} is not inserted, the short is not applied.

	A_{peak}	XFMR Mirage (section 4.1)			Healthy Mirage's leakage model (subsection 4.4.1)						Coil Volume Internal Fault Leakage Model (subsection 4.6.1)					
		1			2			3			4			5		
					K_{cyl} neglected			K_{cyl} included			K_{cyl} neglected			K_{cyl} included		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Rated load	I_H	338.4			338.4			338.4			338.1	338.4	338.5	338.2	338.5	338.6
	I_X	676.0			676.2			676.4			676.2	676.2	676.3	676.4	676.4	676.4
	I_Y	11.2E-3			11.2E-3			11.2E-3			11.2E-3	11.2E-3	11.2E-3	11.2E-3	11.2E-3	11.2E-3
	I_Δ	6.5E-3			6.5E-3			6.5E-3			1.3	1.3	1.3	1.3	1.3	1.3
H – X s.c. test	I_H	338.1			338.1			357.8			336.7	338.1	338.1	355.7	357.8	357.8
	I_X	675.7			675.7			715.1			673.1	673.2	675.6	711.0	715.2	715.1
	I_Y	0.2E-3			0.2E-3			0.2E-3			0.2E-3	0.2E-3	0.2E-3	0.2E-3	0.2E-3	0.2E-3
	I_Δ	0.12E-3			0.12E-3			0.11E-3			0.6	0.6	0.6	0.12	0.12	0.12
H – Y s.c. test	I_H	67.6			67.6			72.2			68.0	67.6	67.6	72.6	72.2	72.2
	I_X	1.9E-3			1.9E-3			1.9E-3			1.9E-3	1.9E-3	1.9E-3	1.9E-3	1.9E-3	1.9E-3
	I_Y	710.0			710.2			757.8			712.0	712.0	710.2	759.8	759.9	757.8
	I_Δ	409.9			410.1			437.5			412.1	410.1	410.1	439.9	437.5	437.5
X – Y s.c. test	I_H	3.7E-3			2.1E-3			2.0E-3			2.1E-3	2.1E-3	2.1E-3	2.0E-3	2.0E-3	2.0E-3
	I_X	135.2			135.3			140.6			137.1	135.3	135.3	142.5	140.6	140.6
	I_Y	710.0			710.6			738.8			715.3	715.5	710.6	743.5	743.7	738.8
	I_Δ	409.9			410.3			426.6			415.8	410.3	410.3	432.1	426.6	426.6

I.2 Kron Reduction Analysis for the Coil Volume Internal Fault Leakage Model

The Kron reduction work was carried out by Hemanth Kumar Vemprala.

The initial matrix in Si is shown in Table G.9. The goal is to reduce each submatrix to 2×2 submatrix leaving series and delta coils in each phase. For this, each submatrix should be rearranged (see Tables I.3 and I.4) and then reduced (see Tables I.5 and I.6). Tables I.3 – I.6 show the matrix elements in Si, but Table I.7 shows results of the Kron reduction performed for matrices both in Si and p.u. values.

Table I.3. Rearranged phase A submatrix, Si: the leakage model.

	<i>S</i>	<i>D</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>
<i>S</i>	18.461973	0.0980208	-30.808057	4148.4096	-1.2762043
<i>D</i>	0.0980208	581.27730	-0.2775078	41651.324	-2878.9092
<i>C1</i>	-30.808057	-0.2775078	128.22617	-37926.263	10.558068
<i>C2</i>	4148.4096	41651.324	-37926.263	142706418	-3079776.2
<i>C3</i>	-1.2762043	-2878.9092	10.558068	-3079776.2	79489.7050

Table I.4. Rearranged phase B submatrix (phase C submatrix is identical), Si: the leakage model.

	<i>S</i>	<i>D</i>	<i>C</i>
<i>S</i>	17.442064	14.535213	-22.210076
<i>D</i>	14.535213	296.76539	-112.35483
<i>C</i>	-22.210076	-112.35483	59.221358

Table I.5. Reduced 2×2 phase A submatrix, Si: the leakage model.

	<i>S</i>	<i>D</i>
<i>S</i>	9.1125107	-27.601768
<i>D</i>	-27.601768	83.605674

Table I.6. Reduced 2×2 phase B submatrix (phase C submatrix is identical), Si: the leakage model.

	<i>S</i>	<i>D</i>
<i>S</i>	9.1125107	-27.601768
<i>D</i>	-27.601768	83.605675

The differences between the elements of Tables I.5 and I.6 are shown in Table I.7. Since the healthy phases B and C are identical, the differences between the elements of their reduced submatrices are zero.

Table I.7. Differences between the corresponding elements of 2×2 submatrices of phases A and B (C), Si and p.u.: the leakage model.

	<i>S</i>		<i>D</i>	
	Si	p.u.	Si	p.u.
<i>S</i>	-5.555E-10	2.141E-06	5.787E-08	-1.997E-06
<i>D</i>	5.787E-08	-1.997E-06	1.380E-06	6.535E-07

As can be concluded, the differences are almost zero indicating the faulted phase A common coil is properly sectioned.

I.3 Short-Circuit Tests: Benchmarking of the Coil Volume Internal Fault Full Model

Table I.8. Benchmarking of the coil volume internal fault full model: X_{DNF} is not inserted, the short is not applied.

	A _{peak}	XFMR Mirage Z ₀ = 750 mH (section 4.1)			Healthy Mirage's full model Z ₀ = 750 mH (subsection 4.4.2)						Coil Volume Internal Fault Full Model Z ₀ = 750 mH (subsection 4.6.2)						
		1			2			3			4			5			
					K _{cyl} neglected			K _{cyl} included			K _{cyl} neglected			K _{cyl} included			
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	
Rated load	I _H	338.4			338.5			338.5			338.2	338.5	338.6	338.3	338.6	338.7	
	I _X	676.0			676.2			676.4			676.2	676.2	676.3	676.4	676.3	676.4	
	I _Y	11.2E-3			11.2E-3			11.2E-3			11.2E-3	11.2E-3	11.2E-3	11.2E-3	11.2E-3	11.2E-3	
	I _Δ	6.5E-3			6.5E-3			6.5E-3			1.3	1.3	1.3	1.3	1.3	1.3	
	V _{peak across core}	3 legs	19459			19469			19473			19469	19467	19473	19472	19470	19476
		2 yokes	19242	19400		19252	19411		19255	19414		19252		19413		19255	
H – X s.c. test	I _H	338.1			338.1			357.8			336.7	338.1	338.1	355.7	357.8	357.8	
	I _X	675.7			675.8			715.1			673.1	675.6	675.6	711.0	715.2	715.1	
	I _Y	0.2E-3			0.2E-3			0.2E-3			0.2E-3	0.2E-3	0.2E-3	0.2E-3	0.2E-3	0.2E-3	
	I _Δ	0.12E-3			0.12E-3			0.11E-3			0.6	0.6	0.6	0.12	0.12	0.12	
	V _{peak across core}	3 legs	16.6			374.3			316.3			375.7	374.6	374.7	315.6	316.2	316.3
		2 yokes	16.6			371.7	374.5		314.1	316.5		373.1		374.9		313.6	
H – Y s.c. test	I _H	67.6			67.6			72.2			68.0	67.6	67.6	72.6	72.2	72.2	
	I _X	1.9E-3			1.9E-3			1.9E-3			1.9E-3	1.9E-3	1.9E-3	1.9E-3	1.9E-3	1.9E-3	
	I _Y	710.0			710.2			757.8			712.0	712.1	710.2	759.8	759.9	757.8	
	I _Δ	409.9			410.1			437.5			412.1	410.1	410.1	439.9	437.5	437.5	
	V _{peak across core}	3 legs	78.2			35.4			30.5			35.5	35.4	35.4	30.8	30.5	30.5
		2 yokes	78.2			35.1	35.4		30.3	30.6		35.2		35.4		30.5	
X – Y s.c. test	I _H	3.7E-3			3.7E-3			3.7E-3			3.8E-3	3.7E-3	3.7E-3	3.7E-3	3.7E-3	3.7E-3	
	I _X	135.2			135.3			140.6			137.1	135.3	135.3	142.5	140.6	140.6	
	I _Y	710.0			710.6			738.8			715.3	715.5	710.6	743.5	743.7	738.8	
	I _Δ	409.9			410.3			426.6			415.8	410.3	410.3	432.1	426.6	426.6	
	V _{peak across core}	3 legs	23.2			35.3			32.2			35.7	35.3	35.3	32.8	32.2	32.2
		2 yokes	23.2			35.0	35.3		32.0	32.3		35.3		35.3		32.4	

I.4 Kron Reduction Analysis for the Coil Volume Internal Fault Full Model

This Kron reduction work was carried out by Hemanth Kumar Vemprala.

The initial matrix in Si is shown in Table G.12. The goal is to reduce each submatrix to 3×3 submatrix leaving series, delta and $(N+1)^{th}$ coils in each phase. For this, each submatrix should be rearranged (see Tables I.9 and I.10) and then reduced (see Tables I.11 and I.12). Tables I.9 – I.12 show the matrix elements in Si, but Table I.13 shows results of the Kron reduction performed for matrices both in Si and p.u. values.

Table I.9. Rearranged phase A submatrix, Si: the full model.

	<i>S</i>	<i>D</i>	<i>Core</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>
<i>S</i>	18.46198022	-0.0066472	0.0933238	-30.808067	4147.5989	-1.2200196
<i>D</i>	-0.0066472	2043.1952	-1303.4706	-0.1339029	52975.365	-3663.6502
<i>Core</i>	0.0933238	-1303.4706	1162.1963	-0.1280406	-10096.706	699.68830
<i>C1</i>	-30.808067	-0.1339029	-0.1280406	128.22618	-37925.151	10.480982
<i>C2</i>	4147.5989	52975.365	-10096.706	-37925.151	142794135	-3085854.9
<i>C3</i>	-1.2200196	-3663.6502	699.68830	10.480982	-3085854.9	79910.945

Table I.10. Rearranged phase B submatrix (phase C submatrix is identical), Si: the full model.

	<i>S</i>	<i>D</i>	<i>Core</i>	<i>C</i>
<i>S</i>	17.451996	18.117370	-3.3730887	-22.288921
<i>D</i>	18.117370	1588.7625	-1216.5914	-140.79234
<i>Core</i>	-3.3730887	-1216.5914	1145.5866	26.777786
<i>C</i>	-22.288921	-140.79234	26.777786	59.847282

Table I.11. Reduced 3×3 phase A submatrix, Si: the full model.

	<i>S</i>	<i>D</i>	<i>Core</i>
<i>S</i>	9.1509318	-34.317922	6.5997623
<i>D</i>	-34.317922	1257.5448	-1153.5959
<i>Core</i>	6.5997623	-1153.5959	1133.6053

Table I.12. Reduced 3×3 phase B submatrix (phase C submatrix is identical), Si: the full model.

	<i>S</i>	<i>D</i>	<i>Core</i>
<i>S</i>	9.1509340	-34.317913	6.5997610
<i>D</i>	-34.317913	1257.5448	-1153.5959
<i>Core</i>	6.5997610	-1153.5959	1133.6053

The differences between the elements of Tables I.11 and I.12 are shown in Table I.13. Since the healthy phases B and C are identical, the differences between the elements of their reduced submatrices are zero.

Table I.13. Differences between the corresponding elements of 3×3 submatrices of phases A and B (C), Si and p.u.: the full model.

	S		D		Core	
	Si	p.u.	Si	p.u.	Si	p.u.
S	2.233E-06	-3.094E-06	8.417E-06	-7.470E-06	-1.2519E-06	1.0058E-06
D	8.417E-06	-7.470E-06	-1.083E-05	-8.616E-06	1.0964E-06	6.3243E-07
Core	-1.2519E-06	1.0058E-06	1.0964E-06	6.3243E-07	-4.7628E-07	8.7103E-07

As can be concluded, the differences are almost zero indicating the faulted phase A common coil is properly sectioned.

I.5 Stage 2 Benchmarking

I.5.1 The Coil Volume Internal Fault Leakage Model

Table I.14. Benchmarking against the base case T2T fault: the coil volume internal fault leakage model.

	Current, A_{peak}	Proprietary model: core is included	Coil Volume Internal Fault Leakage Model with the short applied: core is not included					
			X_{DNF} is not inserted		$X_{DNF} = 1.293 \text{ m}\Omega$ to match the fault current I_F		$X_{DNF} = 2.9 \text{ m}\Omega$ to match the delta current I_Δ	
			K_{cyl} neglected	K_{cyl} included	K_{cyl} neglected	K_{cyl} included	K_{cyl} neglected	K_{cyl} included
			1	2	3	4	5	6
Phase A	Series / H terminal	366.1	638.3	639.7	361.6	362.2	340.9	341.2
	Common	324.9	38.9 (polarity reversal)	38.0 (polarity reversal)	320.0	320.3	332.7	332.9
	X terminal	664.3	665.6	666.6	661.7	662.5	668.8	669.3
	Faulted turn	95,047.0	347,250.4	347,803.3	95,073.0	95,237.9	44,688.7	44,730.2
	Delta	63.5	517.7	515.9	138.8	138.1	63.5	63.1
	Y terminal	N/A	0.01	0.01	0.01	0.01	0.01	0.01
Phase B	Series / H terminal	330.1	371.1	372.1	321.2	321.5	329.5	329.7
	Common	345.3	310.6	310.6	353.3	353.3	345.9	345.9
	X terminal	675.3	662.6	663.9	673.7	674.1	675.3	675.5
	Delta	63.5	517.7	519.6	138.8	138.1	63.5	63.05
	Y terminal	N/A	0.01	0.01	0.01	0.01	0.01	0.01
Phase C	Series / H terminal	347.3	402.2	401.3	360.0	360.0	347.6	347.7
	Common	328.4	297.8	298.1	315.2	315.4	327.9	328.1
	X terminal	675.6	686.7	686.1	675.0	675.3	675.4	675.7
	Delta	63.5	517.7	519.6	138.8	138.1	63.5	63.05
	Y terminal	N/A	0.01	0.01	0.01	0.01	0.01	0.01
	ATPDraw circuit		Fig. F.2		Fig. F.3		Fig. F.3	

I.5.2 The Coil Volume Internal Fault Full Model

Table I.16. Benchmarking against the base case T2T fault: the coil volume internal fault full model.

	Current, A_{peak}	Proprietary model: core is included	Coil Volume Internal Fault Full Model with the short applied: core is included, $L_0 = 750$ mH					
			X_{DNF} is not inserted		$X_{DNF} = 1.293$ m Ω to match the fault current I_F		$X_{DNF} = 2.9$ m Ω to match the delta current I_Δ	
			K_{cyl} neglected	K_{cyl} included	K_{cyl} neglected	K_{cyl} included	K_{cyl} neglected	K_{cyl} included
			1	2	3	4	5	6
Phase A	Series / H terminal	366.1	638.4	639.8	361.7	362.3	341.0	341.3
	Common	324.9	39.0 (polarity reversal)	38.1 (polarity reversal)	319.9	320.2	332.6	332.8
	X terminal	664.3	665.6	666.6	661.7	662.5	668.8	669.3
	Faulted turn	95,047.0	347,226.6	347,781.5	95,067.1	95,232.5	44,685.9	44,727.6
	Delta	63.5	517.7	516.0	138.8	138.2	63.5	63.1
	Y terminal	N/A	0.01	0.01	0.01	0.01	0.01	0.01
Phase B	Series / H terminal	330.1	371.2	372.2	321.3	321.6	329.6	329.8
	Common	345.3	310.5	310.5	353.2	353.2	345.8	345.8
	X terminal	675.3	662.6	663.9	673.7	674.1	675.3	675.5
	Delta	63.5	517.7	516.0	138.8	138.2	63.5	63.1
	Y terminal	N/A	0.01	0.01	0.01	0.01	0.01	0.01
Phase C	Series / H terminal	347.3	402.3	401.4	360.1	360.1	347.8	347.8
	Common	328.4	297.6	298.0	315.0	315.2	327.7	327.9
	X terminal	675.6	686.7	686.1	675.0	675.3	675.4	675.7
	Delta	63.5	517.7	516.0	138.8	138.2	63.5	63.1
	Y terminal	N/A	0.01	0.01	0.01	0.01	0.01	0.01
	Complete .atp file		Appendix H.3		Appendix H.4		Appendix H.5	

I.5.3 Comparison of Tables I.14 and I.16

Table I.18. Comparison of the fault and delta currents from Tables I.14 and I.16.

	Current, A_{peak}	Proprietary model: core is included	Coil Volume Internal Fault Model with the short applied					
			X_{DNF} is not inserted		$X_{DNF} = 1.293 \text{ m}\Omega$ to match the fault current I_F		$X_{DNF} = 2.9 \text{ m}\Omega$ to match the delta current I_Δ	
			K_{cyl} neglected	K_{cyl} included	K_{cyl} neglected	K_{cyl} included	K_{cyl} neglected	K_{cyl} included
			1	2	3	4	5	6
Leakage model	Fault current I_F	95,047.0	347,250.4	347,803.3	95,073.0	95,237.9	44,688.7	44,730.2
	Delta current I_Δ	63.5	517.7	515.9	138.8	138.1	63.5	63.1
Full model	Fault current I_F	95,047.0	347,226.6	347,781.5	95,067.1	95,232.5	44,685.9	44,727.6
	Delta current I_Δ	63.5	517.7	516.0	138.8	138.2	63.5	63.1

I.5.4 Sensitivity Analysis for the Zero-Sequence Inductance

Table I.19. Sensitivity analysis for the zero-sequence inductance.

	Current, A_{peak}	Proprietary model: core is included	Coil Volume Internal Fault Model with the short applied X_{DNF} is not inserted				
			Leakage model: core is not included	Full model: core is included			
				In Advanced Core Settings of XFMR Mirage:			
				$L_0 = 750$ mH	$L_0 = 375$ mH	$L_0 = 1125$ mH	L_0 is omitted
			K_{cyl} neglected	K_{cyl} neglected	K_{cyl} neglected	K_{cyl} neglected	K_{cyl} neglected
		1	2	3	4	5	6
X_{DNF} is not inserted	Fault current I_F	95,047.0	347,250.4	347,226.6	347,226.7	347,226.5	347,118.6
	Delta current I_Δ	63.5	517.7	517.7	517.8	517.7	611.7
			Fig. F.2	Appendix H.3			
X_{DNF} 1.293 m Ω	Fault current I_F	95,047.0	95,073.0	95,067.1	95,067.1	95,067.0	95,067.9
	Delta current I_Δ	63.5	138.8	138.8	138.8	138.8	164.0
			Fig. F.3	Appendix H.4			
X_{DNF} 2.9 m Ω	Fault current I_F	95,047.0	44,688.7	44,685.9	44,685.9	44,685.9	44,686.4
	Delta current I_Δ	63.5	63.5	63.5	63.5	63.5	75.1
			Fig. F.3	Appendix H.5			

J Volts per Turn Analysis

Table J.1. Comparison for the measured lumped parameters and the calculated parameters of a single turn: the healthy Mirage.

Phase A voltages, V_{peak}		XFMR Mirage (section 4.1)		Healthy Mirage's leakage model (subsection 4.4.1)		Healthy Mirage's full model $L_0 = 750$ mH (subsection 4.4.2)	
		1		2		3	
				K_{cyl} neglected			
$V_{\text{COMMON}} = V_{X_LG}$: measured		58,976.1		58,995.3		58,993.3	
V_{out} : eq. (4.5)		132.83		132.87		132.87	
		Value at $e_{\text{ind}} = \text{max}$	amplitude	Value at $e_{\text{ind}} = \text{max}$	amplitude	Value at $e_{\text{ind}} = \text{max}$	amplitude
Lumped	e_{ind} : measured	59,026.4	59,026.4	59,051.5	59,051.5	59,049.5	59,049.5
	$V_{\text{drop}} = V_R$: measured	-50.28	50.28	-56.2	56.2	-56.18	56.18
Per turn	e_{ind} : eq. (4.6)	132.94		133.0		133.0	
	$V_{\text{drop}} = V_R$: eq. (4.7)	0.11		0.13		0.13	
V_{out} : eq. (4.8)		132.83		132.87		132.87	

Table J.2. Comparison of the faulted turn measured parameters: the faulted Mirage, the base case.
The percent values are calculated in comparison with Table 4.12.

Phase A measured parameters: voltages, V_{peak} current, I_{peak}	Coil Volume Internal Fault Leakage Model (subsection 4.6.1) X_{DNF} omitted		Coil Volume Internal Fault Leakage Model (subsection 4.6.1) X_{DNF} = 1.293 mΩ		Coil Volume Internal Fault Leakage Model (subsection 4.6.1) X_{DNF} = 2.9 mΩ		Coil Volume Internal Fault Full Model (subsection 4.6.2) X_{DNF} omitted		Coil Volume Internal Fault Full Model (subsection 4.6.2) X_{DNF} = 1.293 mΩ		Coil Volume Internal Fault Full Model (subsection 4.6.2) X_{DNF} = 2.9 mΩ	
	1		2		3		4		5		6	
	K_{cyl} neglected											
$V_{COMMON} = V_{X_{LG}}$	58,067.8 dropped by 1.6%		57,727.2 dropped by 2.1%		58,350.1 dropped by 1.1%		58,065.5 dropped by 1.6%		57,725.2 dropped by 2.1%		58,348.1 dropped by 1.1%	
	Value at $e_{ind} = \max$	magnitude	Value at $e_{ind} = \max$	amplitude	Value at $e_{ind} = \max$	amplitude	Value at $e_{ind} = \max$	amplitude	Value at $e_{ind} = \max$	amplitude	Value at $e_{ind} = \max$	amplitude
e_{ind}	130.79	130.79 ↓ by 1.7%	128.03	128.03 ↓ by 3.8%	130.69	130.69 ↓ by 1.8%	130.77	130.77 ↓ by 1.7%	128.03	128.03 ↓ by 3.8%	130.68	130.68 ↓ by 1.8%
$V_{drop1} = V_{R_{C2}}$	-130.09	130.09	-9.49	35.62	-2.20	16.74	-130.08	130.08	-10.05	35.61	-2.17	16.74
$V_{drop2} = V_{DNF}$	N/A		-118.49	122.93	-128.47	129.6	N/A		-117.8	122.92	-128.49	128.58
$V_{drop} = V_{drop1} + V_{drop2}$	-130.09	130.09	-127.98	127.99	-130.68	130.68	-130.08	130.08	-127.97	127.98	-130.66	130.66
V_{out_F}	0.69	0.69	0.06	0.19	0.01	0.09	0.69	0.69	0.05	0.19	0.01	0.09
I_F	347,250.4		95,073.0		44,688.7		347,226.6		95,067.1		44,685.9	
$I_{F_{est}}$ estimated by (4.9)	355,045.4		98,799.0		45,484.2		355,045.4		98,799.0		45,484.2	

K Sensitivity Analysis

K.1 The Healthy Mirage

Table K.1. +10% perturbation of the off-diagonal elements in $[A/V]$: the healthy Mirage's leakage model.

	A _{peak}	Healthy Mirage’s leakage model (subsection 4.4.1)											
		K _{cyl} neglected											
		Original matrix Table G.1			a ₁₂ + 10%			a ₁₃ + 10%			a ₂₃ + 10%		
		A	B	C	A	B	C	A	B	C	A	B	C
Base load	I _H	338.4			343.1	336.4	335.7	335.7	339.5	339.8	339.5	337.8	337.8
	I _X	676.2			676.5	676.9	675.6	676.2	675.9	676.6	676.2	676.4	676.1
	I _C	337.9			333.4	340.5	340	340.4	336.4	336.8	336.8	338.6	338.3
	I _Y	0.011			0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
	I _Δ	0.007			28.2	28.2	28.2	15.7	15.7	15.7	6.8	6.8	6.8
H – X s.c. test	I _H	338.1			383.0	338.1	338.1	325.7	337.8	337.9	339.1	338.4	338.3
	I _X	675.7			765.5	675.4	676.1	654.8	673.2	673.5	674.4	678.2	677.8
	I _C	337.6			382.5	337.4	338	329	335.4	335.6	335.2	339.8	339.4
	I _Y	0.0002			0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
	I _Δ	0.0001			0.9	0.9	0.9	11.4	11.4	11.4	10.4	10.4	10.4
H – Y s.c. test	I _H	67.6			71.4	67.6	67.6	64.3	67.6	67.6	73.5	67.6	67.6
	I _X	0.002			0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	I _C	67.6			71.4	67.6	67.6	64.3	67.6	67.6	73.5	67.6	67.6
	I _Y	710.2			729.8	730.3	710.2	693.2	692.8	710.2	740.8	741.5	710.2
	I _Δ	410.1			432.8	410.1	410.1	390.0	410.1	410.1	445.5	410.1	410.1
X – Y s.c. test	I _H	0.002			0.002	0.002	0.0002	0.002	0.002	0.002	0.002	0.002	0.002
	I _X	135.3			135.9	135.3	135.3	131.8	135.3	135.3	151.4	135.3	135.3
	I _C	135.3			135.9	135.3	135.3	131.8	135.3	135.3	151.4	135.3	135.3
	I _Y	710.6			712.3	712.3	710.6	701.6	701.2	710.6	752.4	754.5	710.6
	I _Δ	410.3			412.2	410.3	410.3	400.0	410.3	410.3	459.3	410.3	410.3

Table K.2. Percentage representation of the values from Table K.1 with respect to the original matrix given in Table G.1.

	A_{peak}	Healthy Mirage's leakage model (subsection 4.4.1)											
		K_{cyl} neglected											
		Original matrix Table G.1			$a_{12} + 10\%$			$a_{13} + 10\%$			$a_{23} + 10\%$		
		A	B	C	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100%			101.4	99.4	99.2	99.2	100.3	100.4	100.3	99.8	99.8
	I_X	100%			100	100.1	99.9	100	100	100.1	100	100	100
	I_C	100%			98.7	100.8	100.6	100.7	99.6	99.7	99.7	100.2	100.1
	I_Y	100%			100	100	100	100	100	100	100	100	100
	I_Δ	100%			4E5	4E5	4E5	2.2E5	2.2E5	2.2E5	9.7E4	9.7E4	9.7E4
H – X s.c. test	I_H	100%			113.3	100	100	96.3	99.9	99.9	100.3	100.1	100.1
	I_X	100%			113.3	100	100.1	96.9	99.6	99.7	99.8	100.4	100.3
	I_C	100%			113.3	99.9	100.1	97.5	99.3	99.4	99.3	100.7	100.5
	I_Y	100%			100	100	100	100	100	100	100	100	100
	I_Δ	100%			9E5	9E5	9E5	114E5	114E5	114E5	104E5	104E5	104E5
H – Y s.c. test	I_H	100%			105.6	100	100	95.1	100	100	108.7	100	100
	I_X	100%			100	100	100	100	100	100	100	100	100
	I_C	100%			105.6	100	100	95.1	100	100	108.7	100	100
	I_Y	100%			102.8	102.8	100	97.6	97.6	100	104.3	104.4	100
	I_Δ	100%			105.5	100	100	95.5	100	100	108.6	100	100
X – Y s.c. test	I_H	100%			100	100	100	100	100	100	100	100	100
	I_X	100%			100.4	100	100	97.4	100	100	111.9	100	100
	I_C	100%			100.4	100	100	97.4	100	100	111.9	100	100
	I_Y	100%			100.2	100.2	100	98.7	98.7	100	105.9	106.1	100
	I_Δ	100%			100.5	100	100	97.5	100	100	111.9	100	100

Table K.3. –10% perturbation of the off-diagonal elements in $[A(N)]$: the healthy Mirage's leakage model.

	A _{peak}	Healthy Mirage's leakage model (subsection 4.4.1)											
		K _{cyl} neglected											
		Original matrix Table G.1			a ₁₂ - 10%			a ₁₃ - 10%			a ₂₃ - 10%		
		A	B	C	A	B	C	A	B	C	A	B	C
Base load	I _H	338.4			332.3	340.8	341.8	340.8	337.3	337.0	337.0	339.0	339.1
	I _X	676.2			675.9	675.4	677.0	676.3	676.6	675.9	676.2	676.0	676.4
	I _C	337.9			343.5	334.7	335.3	335.5	339.3	338.9	339.2	337.1	337.4
	I _Y	0.011			0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
	I _Δ	0.007			35.3	35.3	35.3	14.6	14.6	14.6	8.1	8.1	8.1
H – X s.c. test	I _H	338.1			293.2	338.1	338.1	350.2	338.4	338.3	336.9	337.8	337.9
	I _X	675.7			586.0	676.1	675.4	696.3	678.3	678.0	677.4	672.9	673.4
	I _C	337.6			292.8	337.9	337.3	346.1	339.9	339.6	340.5	335.2	335.6
	I _Y	0.0002			0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
	I _Δ	0.0001			0.9	0.9	0.9	11.1	11.1	11.1	12.2	12.2	12.2
H – Y s.c. test	I _H	67.6			63.3	67.6	67.6	70.9	67.6	67.6	61.4	67.6	67.6
	I _X	0.002			0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	I _C	67.6			63.3	67.6	67.6	70.9	67.6	67.6	61.4	67.6	67.6
	I _Y	710.2			687.8	687.3	710.2	727.3	727.7	710.2	678.2	677.5	710.2
	I _Δ	410.1			383.7	410.1	410.1	429.9	410.1	410.1	372.4	410.1	410.1
X – Y s.c. test	I _H	0.002			0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	I _X	135.3			134.4	135.3	135.3	138.6	135.3	135.3	119.1	135.3	135.3
	I _C	135.3			134.4	135.3	135.3	138.6	135.3	135.3	119.1	135.3	135.3
	I _Y	710.6			708.4	708.3	710.6	719.1	719.5	710.6	669.5	667.6	710.6
	I _Δ	410.3			407.7	410.3	410.3	420.3	410.3	410.3	361.2	410.3	410.3

Table K.4. Percentage representation of the values from Table K.3 with respect to the original matrix given in Table G.1.

	A _{peak}	Healthy Mirage's leakage model (subsection 4.4.1)											
		K _{cyl} neglected											
		Original matrix Table G.1			a ₁₂ - 10%			a ₁₃ - 10%			a ₂₃ - 10%		
		A	B	C	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I _H	100%			98.2	100.7	101	100.7	99.7	99.6	99.6	100.2	100.2
	I _X	100%			100	99.9	100.1	100	100.1	100	100	100	100
	I _C	100%			101.7	99.1	99.2	99.3	100.4	100.3	100.4	99.8	99.9
	I _Y	100%			100	100	100	100	100	100	100	100	100
	I _Δ	100%			5E5	5E5	5E5	2E5	2E5	2E5	1.2E5	1.2E5	1.2E5
H – X s.c. test	I _H	100%			86.7	100	100	103.6	100.1	100.1	99.6	99.9	99.9
	I _X	100%			86.7	100	100	103	100.4	100.3	100.3	99.6	99.7
	I _C	100%			86.7	100.1	99.9	102.5	100.7	100.6	100.9	99.3	99.4
	I _Y	100%			100	100	100	100	100	100	100	100	100
	I _Δ	100%			9E5	9E5	9E5	111E5	111E5	111E5	122E5	122E5	122E5
H – Y s.c. test	I _H	100%			93.6	100	100	104.9	100	100	90.5	100	100
	I _X	100%			100	100	100	100	100	100	100	100	100
	I _C	100%			93.6	100	100	104.9	100	100	90.8	100	100
	I _Y	100%			96.8	96.8	100	102.4	102.5	100	95.5	95.4	100
	I _Δ	100%			93.6	100	100	104.8	100	100	90.8	100	100
X – Y s.c. test	I _H	100%			100	100	100	100	100	100	100	100	100
	I _X	100%			99.3	100	100	102.4	100	100	88	100	100
	I _C	100%			99.3	100	100	102.4	100	100	88	100	100
	I _Y	100%			99.7	99.7	100	101.2	101.3	100	94.2	93.9	100
	I _Δ	100%			99.4	100	100	102.4	100	100	88	100	100

Table K.5. +10% perturbation of the off-diagonal elements in $[A(N+1)]$: the healthy Mirage's full model.

	A_{peak}	Healthy Mirage's full model (subsection 4.4.2)																				
		K_{cyl} neglected																				
		Original matrix Table G.4			$a_{12} + 10\%$			$a_{13} + 10\%$			$a_{14} + 10\%$			$a_{23} + 10\%$			$a_{24} + 10\%$			$a_{34} + 10\%$		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.5			343.3	336.5	335.8	335.1	339.9	340.3	339.1	338.2	338.2	339.8	337.8	337.8	338.1	338.6	338.7	338.5	338.4	338.5
	I_X	676.2			676.5	676.9	675.6	676.1	675.7	676.7	676.2	676.3	676.1	676.2	676.4	676.0	676.2	676.2	676.3	676.2	676.2	676.2
	I_C	337.8			333.2	340.5	339.8	341	335.9	336.4	337.2	338.1	338	336.4	338.6	338.3	338.1	337.6	337.6	337.8	337.8	337.7
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	0.007			28.3	28.3	28.3	19.8	19.8	19.8	3.6	3.6	3.6	8.3	8.3	8.3	1.9	1.9	1.9	0.01	0.006	0.005
H – X s.c. test	I_H	338.1			383.2	338.1	338.1	322.7	337.7	337.8	340.9	338.2	338.2	339.4	338.5	338.4	337.8	338.0	338.1	338.1	338.1	338.1
	I_X	675.8			765.8	675.4	676.1	649.6	672.5	672.9	680.5	676.4	676.3	674.0	678.8	678.2	676.2	675.1	675.2	675.8	675.8	675.8
	I_C	337.6			382.6	337.4	338	326.9	334.8	335.1	339.6	338.2	338.1	334.7	340.3	339.9	338.3	337	337.1	337.6	337.6	337.6
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.0001			0.9	0.9	0.9	14.2	14.2	14.2	2.8	2.8	2.8	12.8	12.8	12.8	2.9	2.9	2.9	0.004	0.004	0.004
H – Y s.c. test	I_H	67.6			71.4	67.6	67.6	63.5	67.6	67.6	68.4	67.6	67.6	74.9	67.6	67.6	66.0	67.6	67.6	67.7	67.6	67.6
	I_X	0.002			0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	I_C	67.6			71.4	67.6	67.6	63.5	67.6	67.6	68.4	67.6	67.6	74.9	67.6	67.6	66	67.6	67.6	67.7	67.6	67.6
	I_Y	710.2			729.9	730.3	710.2	689.0	688.5	710.2	714.4	714.5	710.2	748.3	749.2	710.2	701.9	701.7	710.2	710.5	710.5	710.2
	I_Δ	410.1			432.9	410.1	410.1	385.1	410.1	410.1	414.9	410.1	410.1	454.1	410.1	410.1	400.3	410.1	410.1	410.4	410.1	410.1
X – Y s.c. test	I_H	0.004			0.004			0.004			0.004			0.004			0.004			0.004		
	I_X	135.3			135.9	135.3	135.3	130.9	135.3	135.3	136.1	135.3	135.3	155.5	135.3	135.3	130.9	135.3	135.3	135.4	135.3	135.3
	I_C	135.3			135.9	135.3	135.3	130.9	135.3	135.3	136.1	135.3	135.3	155.5	135.3	135.3	130.9	135.3	135.3	135.4	135.3	135.3
	I_Y	710.6			712.3	712.3	710.6	699.4	698.8	710.6	712.8	712.9	710.6	763.1	765.7	710.6	699.4	698.9	710.6	711.1	711.1	710.6
	I_Δ	410.3			412.2	410.3	410.3	397.0	410.3	410.3	412.9	410.3	410.3	471.7	410.3	410.3	397.0	410.3	410.3	410.8	410.3	410.3

Table K.6. Percentage representation of the values from Table K.5 with respect to the original matrix given in Table G.4.

	A_{peak}	Healthy Mirage's full model (subsection 4.4.2)																				
		K_{cyl} neglected																				
		Original matrix Table G.4			$a_{12} + 10\%$			$a_{13} + 10\%$			$a_{14} + 10\%$			$a_{23} + 10\%$			$a_{24} + 10\%$			$a_{34} + 10\%$		
		A	B	C	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100%			101.4	99.4	99.2	99	100.4	100.5	100.2	99.9	99.9	100.4	99.8	99.8	99.9	100	100.1	100	100	100
	I_X	100%			100	100.1	99.9	100	99.9	100.1	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100%			98.6	100.8	100.6	100.9	99.4	99.6	99.8	100.1	100.1	99.6	100.2	100.1	100.1	99.9	99.9	100	100	100
	I_Y	100%			100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100%			4E5	4E5	4E5	2.8E5	2.8E5	2.8E5	5.1E4	5.1E4	5.1E4	1.2E5	1.2E5	1.2E5	2.7E4	2.7E4	2.7E4	142.9	85.7	71.4
H – X s.c. test	I_H	100%			113.3	100	100	95.4	99.9	99.9	100.8	100	100	100.4	100.1	100.1	99.9	100	100	100	100	100
	I_X	100%			113.3	99.9	100	96.1	99.5	99.6	100.7	100.1	100.1	99.7	100.4	100.4	100.1	99.9	99.9	100	100	100
	I_C	100%			113.3	99.9	100.1	96.8	99.2	99.3	100.6	100.2	100.1	99.1	100.8	100.7	100.2	99.8	99.9	100	100	100
	I_Y	100%			100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100%			9E5	9E5	9E5	145E5	145E5	145E5	28E5	28E5	28E5	128E5	128E5	128E5	29E5	29E5	29E5	4E3	4E3	4E3
H – Y s.c. test	I_H	100%			105.6	100	100	93.9	100	100	101.2	100	100	110.8	100	100	97.6	100	100	100	100	100
	I_X	100%			100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100%			105.6	100	100	93.9	100	100	101.2	100	100	110.8	100	100	97.6	100	100	100	100	100
	I_Y	100%			102.8	102.8	100	97	96.9	100	100.6	100.6	100	105.4	105.5	100	98.8	98.8	100	100	100	100
	I_Δ	100%			105.6	100	100	93.9	100	100	101.2	100	100	110.7	100	100	97.6	100	100	100.1	100	100
X – Y s.c. test	I_H	100%			100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100%			100.4	100	100	96.7	100	100	100.6	100	100	114.9	100	100	96.7	100	100	100.1	100	100
	I_C	100%			100.4	100	100	96.7	100	100	100.6	100	100	114.9	100	100	96.7	100	100	100.1	100	100
	I_Y	100%			100.2	100.2	100	98.4	98.3	100	100.3	100.3	100	107.4	107.8	100	98.4	98.4	100	100.1	100.1	100
	I_Δ	100%			100.5	100	100	96.8	100	100	100.6	100	100	115	100	100	96.8	100	100	100.1	100	100

Table K.7. -10% perturbation of the off-diagonal elements in $[A(N+1)]$: the healthy Mirage's full model.

	A_{peak}	Healthy Mirage's full model (subsection 4.4.2)																				
		K_{cyl} neglected																				
		Original matrix Table G.4			a_{12} - 10%			a_{13} - 10%			a_{14} - 10%			a_{23} - 10%			a_{24} - 10%			a_{34} - 10%		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.5			332.4	340.8	342.0	341.5	337.2	336.8	337.9	338.7	338.8	336.7	339.2	339.4	338.8	338.3	338.3	338.5	338.4	338.5
	I_X	676.2			675.9	675.3	677.0	676.3	676.7	675.8	676.2	676.1	676.3	676.2	676.0	676.5	676.2	676.3	676.2	676.2	676.2	676.2
	I_C	337.8			343.4	334.6	335.1	334.8	339.5	339	338.3	337.4	337.5	339.5	336.8	337	337.5	338	337.8	337.8	337.8	337.7
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	0.007			35.5	35.5	35.5	18.1	18.1	18.1	3.6	3.6	3.6	10.4	10.4	10.4	1.8	1.8	1.8	0.003	0.009	0.01
H – X s.c. test	I_H	338.1			293.0	338.1	338.1	353.2	338.5	338.4	335.3	338.0	338.1	336.6	337.7	337.8	338.4	338.2	338.2	338.1	338.1	338.1
	I_X	675.8			585.7	676.1	675.4	701.3	678.9	678.5	671.0	675.1	675.2	677.8	672.1	672.8	675.3	676.4	676.2	675.8	675.8	675.8
	I_C	337.6			292.6	337.9	337.3	348.1	340.4	340.1	335.7	337.1	337.1	341.2	334.5	335	337	338.2	338.1	337.6	337.6	337.6
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.0001			0.9	0.9	0.9	13.8	13.8	13.8	2.8	2.8	2.8	15.6	15.6	15.6	2.8	2.8	2.8	0.005	0.005	0.005
H – Y s.c. test	I_H	67.6			63.3	67.6	67.6	71.7	67.6	67.6	66.8	67.6	67.6	59.8	67.6	67.6	69.2	67.6	67.6	67.6	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	67.6			63.3	67.6	67.6	71.7	67.6	67.6	66.8	67.6	67.6	59.8	67.6	67.6	69.2	67.6	67.6	67.6	67.6	67.6
	I_Y	710.2			687.7	687.2	710.2	731.5	732.1	710.2	706.1	706.0	710.2	669.9	669.0	710.2	718.4	718.6	710.2	709.9	709.9	710.2
	I_Δ	410.1			383.6	410.1	410.1	434.8	410.1	410.1	405.2	410.1	410.1	362.5	410.1	410.1	419.6	410.1	410.1	409.7	410.1	410.1
X – Y s.c. test	I_H	0.004			0.004			0.004			0.004			0.004			0.004			0.004		
	I_X	135.3			134.4	135.3	135.3	139.3	135.3	135.3	134.4	135.3	135.3	115.0	135.3	135.3	139.6	135.3	135.3	135.0	135.3	135.3
	I_C	135.3			134.4	135.3	135.3	139.3	135.3	135.3	134.4	135.3	135.3	115.0	135.3	135.3	139.6	135.3	135.3	135.0	135.3	135.3
	I_Y	710.6			708.4	708.3	710.6	721.1	721.6	710.6	708.4	708.2	710.6	659.2	656.9	710.6	721.6	722.2	710.6	710.0	710.0	710.6
	I_Δ	410.3			407.7	410.3	410.3	422.7	410.3	410.3	407.6	410.3	410.3	348.8	410.3	410.3	423.3	410.3	410.3	409.6	410.3	410.3

Table K.8. Percentage representation of the values from Table K.7 with respect to the original matrix given in Table G.4.

	A_{peak}	Healthy Mirage's full model (subsection 4.4.2)																				
		K_{cyl} neglected																				
		Original matrix Table G.4			$a_{12} - 10\%$			$a_{13} - 10\%$			$a_{14} - 10\%$			$a_{23} - 10\%$			$a_{24} - 10\%$			$a_{34} - 10\%$		
		A	B	C	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100%			98.2	100.7	101	100.9	99.6	99.5	99.8	100.1	100.1	99.5	100.2	100.3	100.1	99.9	99.9	100	100	100
	I_X	100%			100	99.9	100.1	100	100.1	99.9	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100%			101.7	99.1	99.2	99.1	100.5	100.4	100.1	99.9	99.9	100.5	99.7	99.8	99.9	100.1	100	100	100	100
	I_Y	100%			100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_A	100%			5.1E5	5.1E5	5.1E5	2.6E5	2.6E5	2.6E5	5.1E4	5.1E4	5.1E4	1.5E5	1.5E5	1.5E5	2.6E4	2.6E4	2.6E4	42.9	128.6	142.6
H – X s.c. test	I_H	100%			86.7	100	100	104.5	100.1	100.1	99.2	100	100	99.6	99.9	99.9	100.1	100	100	100	100	100
	I_X	100%			86.7	100	99.9	103.8	100.5	100.4	99.3	99.9	99.9	100.3	99.5	99.6	99.9	100.1	100.1	100	100	100
	I_C	100%			86.7	100.1	99.9	103.1	100.8	100.7	99.4	99.9	99.9	101.1	99.1	99.2	99.8	100.2	100.1	100	100	100
	I_Y	100%			100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_A	100%			9E5	9E5	9E5	138E5	138E5	138E5	28E5	28E5	28E5	156E5	156E5	156E5	28E5	28E5	28E5	5E3	5E3	5E3
H – Y s.c. test	I_H	100%			93.6	100	100	106.1	100	100	98.8	100	100	88.5	100	100	102.4	100	100	100	100	100
	I_X	100%			100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100%			93.6	100	100	106.1	100	100	98.8	100	100	88.5	100	100	102.4	100	100	100	100	100
	I_Y	100%			96.8	96.8	100	103	103.1	100	99.4	99.4	100	94.3	94.2	100	101.2	101.2	100	100	100	100
	I_A	100%			93.5	100	100	106	100	100	98.8	100	100	88.4	100	100	102.3	100	100	99.9	100	100
X – Y s.c. test	I_H	100%			100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100%			99.3	100	100	103	100	100	99.3	100	100	85	100	100	103.2	100	100	99.8	100	100
	I_C	100%			99.3	100	100	103	100	100	99.3	100	100	85	100	100	103.2	100	100	99.8	100	100
	I_Y	100%			99.7	99.7	100	101.5	101.5	100	99.7	99.7	100	92.8	92.4	100	101.5	101.6	100	99.9	99.9	100
	I_A	100%			99.4	100	100	103	100	100	99.3	100	100	85	100	100	103.2	100	100	99.8	100	100

K.2 Sectioned Common Coil but the Fault Is Not Applied

Table K.9. +10% perturbation of the off-diagonal elements in $[A(N)]$:
the coil volume internal fault leakage model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 1.

	A_{peak}	Coil Volume Internal Fault Leakage Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.1)																	
		K_{CYI} neglected																	
		Original matrix Table G.9			$a_{12} + 10\%$			$a_{13} + 10\%$			$a_{14} + 10\%$			$a_{15} + 10\%$			$a_{23} + 10\%$		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.1	338.4	338.5	343.6	336.3	335.4	334.2	340.1	340.1	338.2	338.4	338.4	338.1	338.5	338.5	339.5	337.9	337.8
	I_X	676.2	676.2	676.3	676.5	677	675.5	676	675.6	676.8	676.2	676.2	676.3	676.2	676.2	676.3	676.3	676.4	676.1
	I_C	338.1	337.8	337.8	332.9	340.8	340.2	341.9	335.6	336.1	338	337.8	337.8	338.1	337.8	337.8	336.8	338.5	338.3
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	30.9	30.9	30.9	24.9	24.9	24.9	1	1	1	1.4	1.4	1.4	6.6	6.6	6.6
H – X s.c. test	I_H	336.7	338.1	338.1	391	338	338.1	312.1	337.9	337.9	337	338.1	338.1	336.6	338.1	338.1	339.9	338.4	338.3
	I_X	673.1	675.6	675.6	782.4	674.6	675.5	626.7	673.9	673.9	673.7	675.7	675.7	673	675.6	675.6	676.3	677.9	677.5
	I_C	336.4	337.5	337.5	391.5	336.6	337.4	314.6	336	336	336.7	337.6	337.6	336.4	337.5	337.5	336.4	339.5	339.2
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	3.5	3.5	3.5	8.8	8.8	8.8	0.4	0.4	0.4	0.6	0.6	0.6	9.2	9.2	9.2
H – Y s.c. test	I_H	68	67.6	67.6	72.2	67.6	67.6	64.3	67.6	67.6	68	67.6	67.6	68	67.6	67.6	71.2	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	72.2	67.6	67.6	64.3	67.6	67.6	68	67.6	67.6	68	67.6	67.6	71.2	67.6	67.6
	I_Y	712.0	712.0	710.2	733.8	734.4	710.2	692.9	692.5	710.2	712.3	712.3	710.2	711.9	711.9	710.2	729	729.5	710.2
	I_Δ	412.1	410.1	410.1	437.4	410.1	410.1	389.7	410.1	410.1	412.4	410.1	410.1	412	410.1	410.1	431.9	410.1	410.1
X – Y s.c. test	I_H	0.002			0.002			0.002			0.002			0.002			0.002		
	I_X	137.1	135.3	135.3	137.6	135.3	135.3	135	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	143.5	135.3	135.3
	I_C	137.1	135.3	135.3	137.6	135.3	135.3	135	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	143.5	135.3	135.3
	I_Y	715.3	715.5	710.6	716.7	717	710.6	710	710	710.6	715.4	715.6	710.6	715.2	715.4	710.6	731.9	733	710.6
	I_Δ	415.8	410.3	410.3	417.5	410.3	410.3	409.6	410.3	410.3	415.9	410.3	410.3	415.7	410.3	410.3	435.4	410.3	410.3

Table K.9. +10% perturbation of the off-diagonal elements in $[A(N)]$:
the coil volume internal fault leakage model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 2.

	A_{peak}	Coil Volume Internal Fault Leakage Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.1)																	
		K_{cyl} neglected																	
		Original matrix Table G.9			$a_{24} + 10\%$			$a_{25} + 10\%$			$a_{34} + 10\%$			$a_{35} + 10\%$			$a_{45} + 10\%$		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.1	338.4	338.5	338.1	338.5	338.5	338.1	338.4	338.5	338.2	338.4	338.5	338.1	338.5	338.5	338.1	338.4	338.5
	I_X	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3
	I_C	338.1	337.8	337.8	338.1	337.7	337.8	338.1	337.8	337.8	338.1	337.8	337.8	338.1	337.8	337.8	338.1	337.8	337.8
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.4	1.4	1.4	1.2	1.2	1.2
H – X s.c. test	I_H	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1
	I_X	673.1	675.6	675.6	673.1	675.6	675.6	673.1	675.6	675.6	673	675.7	675.7	673.2	675.6	675.6	673.1	675.7	675.7
	I_C	336.4	337.5	337.5	336.4	337.5	337.5	336.4	337.5	337.5	336.3	337.6	337.6	336.5	337.5	337.5	336.4	337.6	337.6
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	0.7	0.7	0.7	0.5	0.5	0.5	0.3	0.3	0.3	0.7	0.7	0.7	0.4	0.4	0.4
H – Y s.c. test	I_H	68.0	67.6	67.6	67.9	67.6	67.6	68	67.6	67.6	68.4	67.6	67.6	66.5	67.6	67.6	71	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	67.9	67.6	67.6	68	67.6	67.6	68.4	67.6	67.6	66.5	67.6	67.6	71	67.6	67.6
	I_Y	712.0	712.0	710.2	711.7	711.8	710.2	712.1	712.1	710.2	714	714.1	710.2	704.5	704.4	710.2	727.5	727.9	710.2
	I_Δ	412.1	410.1	410.1	411.8	410.1	410.1	412.2	410.1	410.1	414.5	410.1	410.1	403.4	410.1	410.1	430.1	410.1	410.1
X – Y s.c. test	I_H	0.002			0.002			0.002			0.002			0.002			0.002		
	I_X	137.1	135.3	135.3	137	135.3	135.3	137.1	135.3	135.3	138.4	135.3	135.3	132.1	135.3	135.3	147.9	135.3	135.3
	I_C	137.1	135.3	135.3	137	135.3	135.3	137.1	135.3	135.3	138.4	135.3	135.3	132.1	135.3	135.3	147.9	135.3	135.3
	I_Y	715.3	715.5	710.6	715	715.2	710.6	715.4	715.6	710.6	718.6	719	710.6	702.6	702.2	710.6	743.1	744.7	710.6
	I_Δ	415.8	410.3	410.3	415.5	410.3	410.3	415.9	410.3	410.3	419.7	410.3	410.3	400.8	410.3	410.3	448.5	410.3	410.3

Table K.10. Percentage representation of the values from Table K.9
with respect to the original matrix given in Table G.9 – piece 1.

	A_{peak}	Coil Volume Internal Fault Leakage Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.1)																	
		K_{cyl} neglected																	
		Original matrix Table G.9			$a_{12} + 10\%$			$a_{13} + 10\%$			$a_{14} + 10\%$			$a_{15} + 10\%$			$a_{23} + 10\%$		
		A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100	100	100	101.6	99.4	99.1	98.8	100.5	100.5	100	100	100	100	100	100	100.4	99.9	99.8
	I_X	100	100	100	100	100.1	99.9	100	99.9	100.1	100	100	100	100	99.5	100	100	100	100
	I_C	100	100	100	98.5	100.9	100.7	101.1	99.3	99.5	100	100	100	100	100	100	99.6	100.2	100.1
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	2.4E3	2.4E3	2.4E3	1.9E3	1.9E3	1.9E3	76.9	76.9	76.9	107.7	107.7	107.7	507.7	507.7	507.7
H – X s.c. test	I_H	100	100	100	116.1	100	100	92.7	99.9	99.9	100.1	100	100	100	100	100	101	100.1	100.1
	I_X	100	100	100	116.2	100	100	93.1	100	99.7	100.1	100	100	100	100	100	100.5	100.3	100.3
	I_C	100	100	100	116.4	99.7	100	93.5	99.6	99.6	100.1	100	100	100	100	100	100	100.6	100.5
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	583.3	583.3	583.3	1.5E3	1.5E3	1.5E3	66.7	66.7	66.7	100	100	100	1.5E3	1.5E3	1.5E3
H – Y s.c. test	I_H	100	100	100	106.2	100	100	94.6	100	100	100	100	100	100	100	100	104.7	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	106.2	100	100	94.6	100	100	100	100	100	100	100	100	104.7	100	100
	I_Y	100	100	100	103.1	103.1	100	97.3	97.3	100	100	100	100	100	100	100	102.4	102.5	100
	I_Δ	100	100	100	106.1	100	100	94.6	100	100	100.1	100	100	100	100	100	104.8	100	100
X – Y s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100.4	100	100	96.1	100	100	100	100	100	100	100	100	104.7	100	100
	I_C	100	100	100	100.4	100	100	96.1	100	100	100	100	100	100	100	100	104.7	100	100
	I_Y	100	100	100	100.2	100.2	100	98.1	98	100	100	100	100	100	100	100	102.3	102.4	100
	I_Δ	100	100	100	100.4	100	100	96.2	100	100	100	100	100	100	100	100	104.7	100	100

Table K.10. Percentage representation of the values from Table K.9
with respect to the original matrix given in Table G.9 – piece 2.

	A_{peak}	Coil Volume Internal Fault Leakage Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.1)																	
		K_{cyl} neglected																	
		Original matrix Table G.9			$a_{24} + 10\%$			$a_{25} + 10\%$			$a_{34} + 10\%$			$a_{35} + 10\%$			$a_{45} + 10\%$		
		A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_A	100	100	100	107.7	107.7	107.7	100	100	100	92.3	92.3	92.3	107.7	107.7	107.7	92.3	92.3	92.3
H – X s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_A	100	100	100	116.7	116.7	116.7	83.3	83.3	83.3	50	50	50	116.7	116.7	116.7	66.7	66.7	66.7
H – Y s.c. test	I_H	100	100	100	99.9	100	100	100	100	100	100.6	100	100	97.8	100	100	104.4	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	99.9	100	100	100	100	100	100.6	100	100	97.8	100	100	104.4	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100.3	100.3	100	98.9	98.9	100	102.2	102.2	100
	I_A	100	100	100	99.9	100	100	100	100	100	100.6	100	100	97.9	100	100	104.4	100	100
X – Y s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	99.9	100	100	100	100	100	100.9	100	100	96.4	100	100	107.9	100	100
	I_C	100	100	100	99.9	100	100	100	100	100	100.9	100	100	96.4	100	100	107.9	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100.5	100.5	100	98.2	98.1	100	103.9	104.1	100
	I_A	100	100	100	99.9	100	100	100	100	100	100.9	100	100	96.4	100	100	107.9	100	100

Table K.11. -10% perturbation of the off-diagonal elements in $[A(N)]$:
the coil volume internal fault leakage model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 1.

	A_{peak}	Coil Volume Internal Fault Leakage Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.1)																	
		K_{cyl} neglected																	
		Original matrix Table G.9			$a_{12} - 10\%$			$a_{13} - 10\%$			$a_{14} - 10\%$			$a_{15} - 10\%$			$a_{23} - 10\%$		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.1	338.4	338.5	330.9	341.3	342.7	341.6	337	336.6	338.1	338.5	338.5	338.2	338.4	338.5	336.5	339.2	339.4
	I_X	676.2	676.2	676.3	675.8	675.1	677.2	676.4	676.7	675.8	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676	676.5
	I_C	338.1	337.8	337.8	344.9	333.9	334.7	334.8	339.7	339.3	338.1	337.7	337.8	338.1	337.8	337.8	339.7	336.8	337.1
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	43.9	43.9	43.9	19.3	19.3	19.3	1.6	1.6	1.6	1.2	1.2	1.2	11.1	11.1	11.1
H – X s.c. test	I_H	336.7	338.1	338.1	282.4	338.2	338.1	360.8	338.3	338.3	336.4	338.1	338.1	336.8	338.1	338.1	332.8	337.7	337.9
	I_X	673.1	675.6	675.6	563.7	676.7	675.7	718.7	677.4	677.4	672.5	675.6	675.6	673.3	675.6	675.6	669.2	672.9	673.4
	I_C	336.4	337.5	337.5	281.3	338.5	337.6	357.9	339.1	339.1	336.1	337.5	337.5	336.5	337.6	337.5	336.4	335.1	335.5
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	2.5	2.5	2.5	7.5	7.5	7.5	0.7	0.7	0.7	0.5	0.5	0.5	12.4	12.4	12.4
H – Y s.c. test	I_H	68.0	67.6	67.6	62.9	67.6	67.6	71.5	67.6	67.6	67.9	67.6	67.6	68	67.6	67.6	64.2	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	62.9	67.6	67.6	71.5	67.6	67.6	67.9	67.6	67.6	68	67.6	67.6	64.2	67.6	67.6
	I_Y	712.0	712.0	710.2	686	685.4	710.2	730.1	730.6	710.2	711.7	711.8	710.2	712.1	712.2	710.2	692.5	692.1	710.2
	I_Δ	412.1	410.1	410.1	381.5	410.1	410.1	433.2	410.1	410.1	411.8	410.1	410.1	412.3	410.1	410.1	389.3	410.1	410.1
X – Y s.c. test	I_H	0.002			0.002			0.002			0.002			0.002			0.002		
	I_X	137.1	135.3	135.3	136.3	135.3	135.3	138.9	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	129.6	135.3	135.3
	I_C	137.1	135.3	135.3	136.3	135.3	135.3	138.9	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	129.6	135.3	135.3
	I_Y	715.3	715.5	710.6	713.4	713.5	710.6	719.9	720.3	710.6	715.2	715.4	710.6	715.3	715.6	710.6	696.1	695.5	710.6
	I_Δ	415.8	410.3	410.3	413.5	410.3	410.3	421.2	410.3	410.3	415.7	410.3	410.3	415.9	410.3	410.3	393.1	410.3	410.3

Table K.11. -10% perturbation of the off-diagonal elements in $[A(N)]$:
the coil volume internal fault leakage model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 2.

	A_{peak}	Coil Volume Internal Fault Leakage Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.1)																	
		K_{cyl} neglected																	
		Original matrix Table G.9			$a_{24} - 10\%$			$a_{25} - 10\%$			$a_{34} - 10\%$			$a_{35} - 10\%$			$a_{45} - 10\%$		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.1	338.4	338.5	338.2	338.4	338.5	338.1	338.4	338.5	338.1	338.5	338.5	338.1	338.4	338.5	338.1	338.5	338.5
	I_X	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3
	I_C	338.1	337.8	337.8	338.1	337.8	337.8	338.1	337.8	337.8	338.1	337.7	337.8	338.1	337.8	337.8	338.1	337.8	337.8
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	1.2	1.2	1.2	1.3	1.3	1.3	1.5	1.5	1.5	1.2	1.2	1.2	1.4	1.4	1.4
H – X s.c. test	I_H	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1
	I_X	673.1	675.6	675.6	673.2	675.7	675.7	673.1	675.2	675.2	673.2	675.6	675.6	673.1	675.7	675.7	673.2	675.6	675.6
	I_C	336.4	337.5	337.5	336.4	337.6	337.6	336.4	337.5	337.5	336.5	337.5	337.5	336.4	337.6	337.6	336.5	337.5	337.5
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	0.4	0.4	0.4	0.6	0.6	0.6	0.8	0.8	0.8	0.4	0.4	0.4	0.7	0.7	0.7
H – Y s.c. test	I_H	68.0	67.6	67.6	68	67.6	67.6	68	67.6	67.6	67.5	67.6	67.6	69.3	67.6	67.6	64.3	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	68	67.6	67.6	68	67.6	67.6	67.5	67.6	67.6	69.3	67.6	67.6	64.3	67.6	67.6
	I_Y	712.0	712.0	710.2	712.3	712.3	710.2	711.9	712	710.2	709.5	709.5	710.2	718.9	719.2	710.2	693.1	692.7	710.2
	I_Δ	412.1	410.1	410.1	412.4	410.1	410.1	412	410.1	410.1	409.2	410.1	410.1	420.2	410.1	410.1	389.9	410.1	410.1
X – Y s.c. test	I_H	0.002			0.002			0.002			0.002			0.002			0.002		
	I_X	137.1	135.3	135.3	137.2	135.3	135.3	137	135.3	135.3	135.5	135.3	135.3	141.8	135.3	135.3	124.7	135.3	135.3
	I_C	137.1	135.3	135.3	137.2	135.3	135.3	137	135.3	135.3	135.5	135.3	135.3	141.8	135.3	135.3	124.7	135.3	135.3
	I_Y	715.3	715.5	710.6	715.6	715.8	710.6	715.2	715.4	710.6	711.3	711.3	710.6	727.4	728.2	710.6	683.7	682.5	710.6
	I_Δ	415.8	410.3	410.3	416.1	410.3	410.3	415.7	410.3	410.3	411.1	410.3	410.3	430.1	410.3	410.3	378.3	410.3	410.3

Table K.12. Percentage representation of the values from Table K.11
with respect to the original matrix given in Table G.9 – piece 1.

	A_{peak}	Coil Volume Internal Fault Leakage Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.1)																	
		K_{cyl} neglected																	
		Original matrix Table G.9			$a_{12} - 10\%$			$a_{13} - 10\%$			$a_{14} - 10\%$			$a_{15} - 10\%$			$a_{23} - 10\%$		
		A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100	100	100	97.9	100.9	101.2	101	99.6	99.4	100	100	100	100	100	100	99.5	100.2	100.3
	I_X	100	100	100	99.9	99.8	100.1	100	100.1	99.9	100	100	100	100	99.5	100	100	100	100
	I_C	100	100	100	102	98.8	99.1	99	100.6	100.4	100	100	100	100	100	100	100.5	99.7	99.8
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	3.4E3	3.4E3	3.4E3	1.5E3	1.5E3	1.5E3	123.1	123.1	123.1	92.3	92.3	92.3	853.8	853.8	853.8
H – X s.c. test	I_H	100	100	100	83.9	100	100	107.2	100.1	100.1	99.9	100	100	100	100	100	98.8	99.9	99.9
	I_X	100	100	100	83.7	100.5	100	106.8	100.6	100.3	99.9	100	100	100	100.4	100	99.4	99.6	99.7
	I_C	100	100	100	83.6	100.3	100	106.4	100.5	100.5	99.9	100	100	100	100	100	100	99.3	99.4
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	416.7	416.7	416.7	1250	1250	1250	116.7	116.7	116.7	83.3	83.3	83.3	2.1E3	2.1E3	2.1E3
H – Y s.c. test	I_H	100	100	100	92.5	100	100	105.1	100	100	99.9	100	100	100	100	100	94.4	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	92.5	100	100	105.1	100	100	99.9	100	100	100	100	100	94.4	100	100
	I_Y	100	100	100	96.3	96.3	100	102.5	102.6	100	100	100	100	100	100	100	97.3	97.2	100
	I_Δ	100	100	100	92.6	100	100	105.1	100	100	99.9	100	100	100	100	100	94.5	100	100
X – Y s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	99.4	100	100	101.3	100	100	100	100	100	100	100	100	94.5	100	100
	I_C	100	100	100	99.4	100	100	101.3	100	100	100	100	100	100	100	100	94.5	100	100
	I_Y	100	100	100	99.7	99.7	100	100.6	100.7	100	100	100	100	100	100	100	97.3	97.2	100
	I_Δ	100	100	100	99.4	100	100	101.3	100	100	100	100	100	100	100	100	94.5	100	100

Table K.12. Percentage representation of the values from Table K.11
with respect to the original matrix given in Table G.9 – piece 2.

	A_{peak}	Coil Volume Internal Fault Leakage Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.1)																	
		K_{cyl} neglected																	
		Original matrix Table G.9			$a_{24} - 10\%$			$a_{25} - 10\%$			$a_{34} - 10\%$			$a_{35} - 10\%$			$a_{45} - 10\%$		
		A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	92.3	92.3	92.3	100	100	100	115.4	115.4	115.4	92.3	92.3	92.3	107.7	107.7	107.7
H – X s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100.4	100	100	100	99.9	99.9	100.4	100	100	100.4	100	100	100.4	100
	I_C	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	66.7	66.7	66.7	100	100	100	133.3	133.3	133.3	66.7	66.7	66.7	116.7	116.7	116.7
H – Y s.c. test	I_H	100	100	100	100	100	100	100	100	100	99.3	100	100	101.9	100	100	94.6	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	100	100	100	100	100	99.3	100	100	101.9	100	100	94.6	100	100
	I_Y	100	100	100	100	100	100	100	100	100	99.6	99.6	100	101	101	100	97.3	97.3	100
	I_Δ	100	100	100	100.1	100	100	100	100	100	99.3	100	100	102	100	100	94.6	100	100
X – Y s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100.1	100	100	99.9	100	100	98.8	100	100	103.4	100	100	91	100	100
	I_C	100	100	100	100.1	100	100	99.9	100	100	98.8	100	100	103.4	100	100	91	100	100
	I_Y	100	100	100	100	100	100	100	100	100	99.4	99.4	100	101.7	101.8	100	95.6	95.4	100
	I_Δ	100	100	100	100.1	100	100	100	100	100	98.9	100	100	103.4	100	100	91	100	100

Table K.13. +10% perturbation of the off-diagonal elements in $[A/(N+1)]$:
the coil volume internal fault full model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 1.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{cyl} neglected																	
		Original matrix Table G.12			$a_{12} + 10\%$			$a_{13} + 10\%$			$a_{14} + 10\%$			$a_{15} + 10\%$			$a_{16} + 10\%$		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.2	338.5	338.6	343.7	336.4	335.6	334.3	340.2	340.9	338.3	338.5	338.6	338.2	338.5	338.6	338.2	338.5	338.6
	I_X	676.2	676.2	676.3	676.5	677	675.2	676	675.6	676.8	676.2	676.2	676.2	676.2	676.2	676.3	676.2	676.2	676.3
	I_C	338	337.7	337.6	332.8	340.7	340	341.8	335.5	336	337.9	337.7	337.7	338	337.7	337.6	338	337.6	337.6
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	30.9	30.9	30.9	24.9	24.9	24.9	1	1	1	1.3	1.3	1.3	1.4	1.4	1.4
H – X s.c. test	I_H	336.7	338.1	338.1	391	338	338.1	312.1	337.9	337.9	337	338.1	338.1	336.7	338.1	338.1	336.6	338.1	338.1
	I_X	673.1	675.6	675.6	782.5	674.6	675.5	626.7	673.9	673.9	673.7	675.7	675.7	673.1	675.6	675.6	673	675.6	675.6
	I_C	336.4	337.5	337.5	391.5	336.6	337.4	314.6	336	336	336.7	337.6	337.6	336.4	337.5	337.5	336.4	337.5	337.5
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	3.5	3.5	3.5	8.8	8.8	8.8	0.4	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6
H – Y s.c. test	I_H	68	67.6	67.6	72.2	67.6	67.6	64.3	67.6	67.6	68	67.6	67.6	68	67.6	67.6	68	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	72.2	67.6	67.6	64.3	67.6	67.6	68	67.6	67.6	68	67.6	67.6	68	67.6	67.6
	I_Y	712	712.1	710.2	733.8	734.4	710.2	692.9	692.5	710.2	712.3	712.3	710.2	712	712.1	710.2	711.9	711.9	710.2
	I_Δ	412.1	410.1	410.1	437.4	410.1	410.1	389.7	410.1	410.1	412.4	410.1	410.1	412.1	410.1	410.1	412	410.1	410.1
X – Y s.c. test	I_H	0.004			0.004			0.004			0.004			0.004			0.004		
	I_X	137.1	135.3	135.3	137.6	135.3	135.3	135	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3
	I_C	137.1	135.3	135.3	137.6	135.3	135.3	135	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3
	I_Y	715.3	715.5	710.6	716.7	717	710.6	710	710	710.6	715.4	715.6	710.6	715.3	715.5	710.6	715.2	715.4	710.6
	I_Δ	415.8	410.3	410.3	417.5	410.3	410.3	409.6	410.3	410.3	415.9	410.3	410.3	415.8	410.3	410.3	415.7	410.3	410.3

Table K.13. +10% perturbation of the off-diagonal elements in $[A(N+1)]$:
the coil volume internal fault full model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 2.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{cyl} neglected																	
		Original matrix Table G.12			$a_{23} + 10\%$			$a_{24} + 10\%$			$a_{25} + 10\%$			$a_{26} + 10\%$			$a_{34} + 10\%$		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.2	338.5	338.6	339.6	337.9	337.9	338.2	338.5	338.6	338.2	338.5	338.6	338.2	338.5	338.6	338.3	338.5	338.6
	I_X	676.2	676.2	676.3	676.2	676.4	676.1	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3
	I_C	338	337.7	337.6	336.7	338.4	338.2	338	337.6	337.6	338	337.7	337.6	338	337.7	337.6	337.9	337.7	337.6
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	6.6	6.6	6.6	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2
H – X s.c. test	I_H	336.7	338.1	338.1	339.9	338.4	338.3	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1
	I_X	673.1	675.6	675.6	676.3	677.9	677.5	673.1	675.6	675.6	673.1	675.6	675.6	673.1	675.6	675.6	673	675.7	675.7
	I_C	336.4	337.5	337.5	336.4	339.6	339.2	336.4	337.5	337.5	336.4	337.5	337.5	336.4	337.5	337.5	336.3	337.6	337.6
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	9.2	9.2	9.2	0.7	0.7	0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.3	0.3	0.3
H – Y s.c. test	I_H	68	67.6	67.6	71.2	67.6	67.6	67.9	67.6	67.6	68	67.6	67.6	68	67.6	67.6	68.4	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	71.2	67.6	67.6	67.9	67.6	67.6	68	67.6	67.6	68	67.6	67.6	68.4	67.6	67.6
	I_Y	712	712.1	710.2	729	729.5	710.2	711.7	711.8	710.2	712	712.1	710.2	712	712.1	710.2	714	714.1	710.2
	I_Δ	412.1	410.1	410.1	431.9	410.1	410.1	411.8	410.1	410.1	412.2	410.1	410.1	412.2	410.1	410.1	414.5	410.1	410.1
X – Y s.c. test	I_H	0.004			0.004			0.004			0.004			0.004			0.004		
	I_X	137.1	135.3	135.3	143.5	135.3	135.3	137	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	138.4	135.3	135.3
	I_C	137.1	135.3	135.3	143.5	135.3	135.3	137	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	138.4	135.3	135.3
	I_Y	715.3	715.5	710.6	731.9	733	710.6	715	715.2	710.6	715.3	715.6	710.6	715.3	715.6	710.6	718.6	719	710.6
	I_Δ	415.8	410.3	410.3	435.4	410.3	410.3	415.5	410.3	410.3	415.9	410.3	410.3	415.9	410.3	410.3	419.7	410.3	410.3

Table K.13. +10% perturbation of the off-diagonal elements in $[A(N+1)]$:
the coil volume internal fault full model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 3.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{cyl} neglected																	
		Original matrix Table G.12			$a_{35} + 10\%$			$a_{36} + 10\%$			$a_{45} + 10\%$			$a_{46} + 10\%$			$a_{56} + 10\%$		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.2	338.5	338.6	338.2	338.5	338.6	338.2	338.5	338.6	338.3	338.5	338.6	338.2	338.5	338.6	338.2	338.5	338.6
	I_X	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3
	I_C	338	337.7	337.6	338	337.6	337.6	338	337.7	337.6	338	337.7	337.6	338	337.7	337.6	338	337.7	337.6
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.3	1.3	1.3	1.3	1.3	1.3
H – X s.c. test	I_H	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1
	I_X	673.1	675.6	675.6	673.2	675.6	675.6	673.1	675.6	675.6	673	675.7	675.7	673.1	675.6	675.6	673.1	675.6	675.6
	I_C	336.4	337.5	337.5	336.5	337.5	337.5	336.4	337.5	337.5	336.4	337.6	337.6	336.4	337.5	337.5	336.4	337.5	337.5
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	0.8	0.8	0.8	0.5	0.5	0.5	0.4	0.4	0.4	0.6	0.6	0.6	0.6	0.6	0.6
H – Y s.c. test	I_H	68	67.6	67.6	66.1	67.6	67.6	68.4	67.6	67.6	71.7	67.6	67.6	66.9	67.6	67.6	68	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	66.1	67.6	67.6	68.4	67.6	67.6	71.7	67.6	67.6	66.9	67.6	67.6	68	67.6	67.6
	I_Y	712	712.1	710.2	702.4	702.2	710.2	714.1	714.2	710.2	731.2	731.7	710.2	706.6	706.5	710.2	712.3	712.4	710.2
	I_Δ	412.1	410.1	410.1	400.9	410.1	410.1	414.5	410.1	410.1	434.5	410.1	410.1	405.8	410.1	410.1	412.5	410.1	410.1
X – Y s.c. test	I_H	0.004			0.004			0.004			0.004			0.004			0.004		
	I_X	137.1	135.3	135.3	130.7	135.3	135.3	138.5	135.3	135.3	150.6	135.3	135.3	133.5	135.3	135.3	137.3	135.3	135.3
	I_C	137.1	135.3	135.3	130.7	135.3	135.3	138.5	135.3	135.3	150.6	135.3	135.3	133.5	135.3	135.3	137.3	135.3	135.3
	I_Y	715.3	715.5	710.6	699	698.5	710.6	718.9	719.3	710.6	750.2	752.1	710.6	706	705.8	710.6	715.8	716	710.6
	I_Δ	415.8	410.3	410.3	396.6	410.3	410.3	420	410.3	410.3	456.7	410.3	410.3	404.8	410.3	410.3	416.4	410.3	410.3

Table K.14. Percentage representation of the values from Table K.13 with respect to the original matrix given in Table G.12 – piece 1.

[illegible]

Table K.14. Percentage representation of the values from Table K.13 with respect to the original matrix given in Table G.12 – piece 2.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{cyl} neglected																	
		Original matrix Table G.12			$a_{23} + 10\%$			$a_{24} + 10\%$			$a_{25} + 10\%$			$a_{26} + 10\%$			$a_{34} + 10\%$		
		A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100	100	100	100.4	99.8	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	99.6	100.2	100.2	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	507.7	507.7	507.7	107.7	107.7	107.7	100	100	100	100	100	100	92.3	92.3	92.3
H – X s.c. test	I_H	100	100	100	101	100.1	100.1	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100.5	100.3	100.3	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	100.6	100.5	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	1.5E3	1.5E3	1.5E3	116.7	116.7	116.7	83.3	83.3	83.3	83.3	83.3	83.3	50	50	50
H – Y s.c. test	I_H	100	100	100	104.7	100	100	99.9	100	100	100	100	100	100	100	100	100.6	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	104.7	100	100	99.9	100	100	100	100	100	100	100	100	100.6	100	100
	I_Y	100	100	100	102.4	102.4	100	100	100	100	100	100	100	100	100	100	100.3	100.3	100
	I_Δ	100	100	100	104.8	100	100	99.9	100	100	100	100	100	100	100	100	100.6	100	100
X – Y s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	104.7	100	100	99.9	100	100	100	100	100	100	100	100	100.9	100	100
	I_C	100	100	100	104.7	100	100	99.9	100	100	100	100	100	100	100	100	100.9	100	100
	I_Y	100	100	100	102.3	102.4	100	100	100	100	100	100	100	100	100	100	100.5	100.5	100
	I_Δ	100	100	100	104.7	100	100	99.9	100	100	100	100	100	100	100	100	100.9	100	100

Table K.14. Percentage representation of the values from Table K.13 with respect to the original matrix given in Table G.12 – piece 3.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{CYI} neglected																	
		Original matrix Table G.12			$a_{35} + 10\%$			$a_{36} + 10\%$			$a_{45} + 10\%$			$a_{46} + 10\%$			$a_{56} + 10\%$		
		A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	107.7	107.7	107.7	100	100	100	92.3	92.3	92.3	100	100	100	100	100	100
H – X s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	133.3	133.3	133.3	83.3	83.3	83.3	66.7	66.7	66.7	100	100	100	100	100	100
H – Y s.c. test	I_H	100	100	100	97.2	100	100	100.6	100	100	105.4	100	100	98.4	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	97.2	100	100	100.6	100	100	105.4	100	100	98.4	100	100	100	100	100
	I_Y	100	100	100	98.7	98.6	100	100.3	100.3	100	102.7	102.8	100	99.2	99.2	100	100	100	100
	I_Δ	100	100	100	97.3	100	100	100.6	100	100	105.4	100	100	98.5	100	100	100.1	100	100
X – Y s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	95.3	100	100	101	100	100	109.8	100	100	97.4	100	100	100.1	100	100
	I_C	100	100	100	95.3	100	100	101	100	100	109.8	100	100	97.4	100	100	100.1	100	100
	I_Y	100	100	100	97.7	97.6	100	100.5	100.5	100	104.9	105.1	100	98.7	98.6	100	100.1	100.1	100
	I_Δ	100	100	100	95.4	100	100	101	100	100	109.8	100	100	97.4	100	100	100.1	100	100

Table K.15. -10% perturbation of the off-diagonal elements in $[A(N+1)]$:
the coil volume internal fault full model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 1.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{cyl} neglected																	
		Original matrix Table G.12			a_{12} - 10%			a_{13} - 10%			a_{14} - 10%			a_{15} - 10%			a_{16} - 10%		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.2	338.5	338.6	331	341.4	342.8	341.7	337.1	336.7	338.2	338.6	338.7	338.2	338.5	338.6	338.3	338.5	338.6
	I_X	676.2	676.2	676.3	675.7	675.1	677.2	676.4	676.7	675.8	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3
	I_C	338	337.7	337.6	344.8	333.8	334.5	334.6	339.6	339.1	338	337.6	337.6	338	337.7	337.6	338	337.7	337.6
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	43.9	43.9	43.9	19.3	19.3	19.3	1.6	1.6	1.6	1.3	1.3	1.3	1.2	1.2	1.2
H – X s.c. test	I_H	336.7	338.1	338.1	282.4	338.2	338.1	360.8	338.3	338.3	336.4	338.1	338.1	336.7	338.1	338.1	336.8	338.1	338.1
	I_X	673.1	675.6	675.6	563.7	676.7	675.7	718.7	677.4	677.4	672.5	675.6	675.6	673.1	675.6	675.6	673.3	675.6	675.7
	I_C	336.4	337.5	337.5	281.3	338.5	337.6	357.9	339.1	339.1	336.1	337.5	337.5	336.4	337.5	337.5	336.5	337.6	337.6
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	2.5	2.5	2.5	7.5	7.5	7.5	0.7	0.7	0.7	0.6	0.6	0.6	0.5	0.5	0.5
H – Y s.c. test	I_H	68	67.6	67.6	62.9	67.6	67.6	71.5	67.6	67.6	67.9	67.6	67.6	68	67.6	67.6	68	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	62.9	67.6	67.6	71.5	67.6	67.6	67.9	67.6	67.6	68	67.6	67.6	68	67.6	67.6
	I_Y	712	712.1	710.2	686	685.4	710.2	730.1	730.6	710.2	711.7	711.8	710.2	712	712	710.2	712.1	712.2	710.2
	I_Δ	412.1	410.1	410.1	381.5	410.1	410.1	433.2	410.1	410.1	411.8	410.1	410.1	412.1	410.1	410.1	412.3	410.1	410.1
X – Y s.c. test	I_H	0.004			0.004			0.004			0.004			0.004			0.004		
	I_X	137.1	135.3	135.3	136.3	135.3	135.3	138.9	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3
	I_C	137.1	135.3	135.3	136.3	135.3	135.3	138.9	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3
	I_Y	715.3	715.5	710.6	713.4	713.5	710.6	719.9	720.3	710.6	715.2	715.4	710.6	715.3	715.5	710.6	715.3	715.6	710.6
	I_Δ	415.8	410.3	410.3	413.5	410.3	410.3	421.2	410.3	410.3	415.7	410.3	410.3	415.8	410.3	410.3	415.9	410.3	410.3

Table K.15. -10% perturbation of the off-diagonal elements in $[A(N+1)]$:
the coil volume internal fault full model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 2.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{cyl} neglected																	
		Original matrix Table G.12			a_{23} - 10%			a_{24} - 10%			a_{25} - 10%			a_{26} - 10%			a_{34} - 10%		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.2	338.5	338.6	336.6	339.3	339.5	338.3	338.5	338.6	338.2	338.5	338.6	338.2	338.5	338.6	338.2	338.5	338.6
	I_X	676.2	676.2	676.3	676.2	676	676.5	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3
	I_C	338	337.7	337.6	339.6	336.7	337	337.9	337.7	337.6	338	337.7	337.6	338	337.7	337.6	338	337.6	337.6
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	11.1	11.1	11.1	1.2	1.2	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.5	1.5	1.5
H – X s.c. test	I_H	336.7	338.1	338.1	332.8	337.7	337.9	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1
	I_X	673.1	675.6	675.6	669.2	672.9	673.4	673.2	675.7	675.7	673.1	675.6	675.6	673.1	675.6	675.6	673.2	675.6	675.6
	I_C	336.4	337.5	337.5	336.4	335.1	335.5	336.4	337.6	337.6	336.4	337.5	337.5	336.4	337.5	337.5	336.5	337.5	337.5
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	12.4	12.4	12.4	0.4	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6	0.8	0.8	0.8
H – Y s.c. test	I_H	68	67.6	67.6	64.2	67.6	67.6	68	67.6	67.6	68	67.6	67.6	68	67.6	67.6	67.5	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	64.2	67.6	67.6	68	67.6	67.6	68	67.6	67.6	68	67.6	67.6	67.5	67.6	67.6
	I_Y	712	712.1	710.2	692.5	692.1	710.2	712.3	712.3	710.2	712	712	710.2	712	712	710.2	709.5	709.5	710.2
	I_Δ	412.1	410.1	410.1	389.3	410.1	410.1	412.4	410.1	410.1	412.1	410.1	410.1	412.1	410.1	410.1	409.2	410.1	410.1
X – Y s.c. test	I_H	0.004			0.004			0.004			0.004			0.004			0.004		
	I_X	137.1	135.3	135.3	129.6	135.3	135.3	137.2	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	135.5	135.3	135.3
	I_C	137.1	135.3	135.3	129.6	135.3	135.3	137.2	135.3	135.3	137.1	135.3	135.3	137.1	135.3	135.3	135.5	135.3	135.3
	I_Y	715.3	715.5	710.6	696.1	695.5	710.6	715.6	715.8	710.6	715.2	715.5	710.6	715.2	715.4	710.6	711.3	711.3	710.6
	I_Δ	415.8	410.3	410.3	393.1	410.3	410.3	416.1	410.3	410.3	415.7	410.3	410.3	415.7	410.3	410.3	411.1	410.3	410.3

Table K.15. -10% perturbation of the off-diagonal elements in $[A(N+1)]$:
the coil volume internal fault full model, the base case, the fault is not applied, X_{DNF} is not inserted – piece 3.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{cyl} neglected																	
		Original matrix Table G.12			$a_{35} - 10\%$			$a_{36} - 10\%$			$a_{45} - 10\%$			$a_{46} - 10\%$			$a_{56} - 10\%$		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	338.2	338.5	338.6	338.3	338.5	338.6	338.2	338.5	338.6	338.2	338.5	338.6	338.2	338.5	338.6	338.2	338.5	338.6
	I_X	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3	676.2	676.2	676.3
	I_C	338	337.7	337.6	338	337.7	337.6	338	337.7	337.6	338	337.6	337.6	338	337.7	337.6	338	337.7	337.6
	I_Y	0.011			0.011			0.011			0.011			0.011			0.011		
	I_Δ	1.3	1.3	1.3	1.2	1.2	1.2	1.3	1.3	1.3	1.5	1.5	1.5	1.3	1.3	1.3	1.3	1.3	1.3
H – X s.c. test	I_H	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1	336.7	338.1	338.1
	I_X	673.1	675.6	675.6	673	675.7	675.7	673.1	675.6	675.6	673.2	675.6	675.6	673.1	675.6	675.6	673.1	675.6	675.6
	I_C	336.4	337.5	337.5	336.4	337.6	337.6	336.4	337.5	337.5	336.5	337.5	337.5	336.4	337.5	337.5	336.4	337.5	337.5
	I_Y	0.0002			0.0002			0.0002			0.0002			0.0002			0.0002		
	I_Δ	0.6	0.6	0.6	0.4	0.4	0.4	0.6	0.6	0.6	0.8	0.8	0.8	0.5	0.5	0.5	0.6	0.6	0.6
H – Y s.c. test	I_H	68	67.6	67.6	69.7	67.6	67.6	67.6	67.6	67.6	63.2	67.6	67.6	68.9	67.6	67.6	67.9	67.6	67.6
	I_X	0.002			0.002			0.002			0.002			0.002			0.002		
	I_C	68	67.6	67.6	69.7	67.6	67.6	67.6	67.6	67.6	63.2	67.6	67.6	68.9	67.6	67.6	67.9	67.6	67.6
	I_Y	712	712.1	710.2	720.7	721	710.2	709.9	709.9	710.2	687.2	686.6	710.2	716.9	717.1	710.2	711.6	711.7	710.2
	I_Δ	412.1	410.1	410.1	422.3	410.1	410.1	409.6	410.1	410.1	382.9	410.1	410.1	417.8	410.1	410.1	411.7	410.1	410.1
X – Y s.c. test	I_H	0.004			0.004			0.004			0.004			0.004			0.004		
	I_X	137.1	135.3	135.3	143	135.3	135.3	135.6	135.3	135.3	121	135.3	135.3	140.4	135.3	135.3	136.8	135.3	135.3
	I_C	137.1	135.3	135.3	143	135.3	135.3	135.6	135.3	135.3	121	135.3	135.3	140.4	135.3	135.3	136.8	135.3	135.3
	I_Y	715.3	715.5	710.6	730.6	731.6	710.6	711.6	711.6	710.6	674.4	672.8	710.6	723.9	724.5	710.6	714.6	714.8	710.6
	I_Δ	415.8	410.3	410.3	433.9	410.3	410.3	411.4	410.3	410.3	367.1	410.3	410.3	425.9	410.3	410.3	415	410.3	410.3

[illegible]

Table K.16. Percentage representation of the values from Table K.15 with respect to the original matrix given in Table G.12 – piece 2.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{cyl} neglected																	
		Original matrix Table G.12			$a_{23} - 10\%$			$a_{24} - 10\%$			$a_{25} - 10\%$			$a_{26} - 10\%$			$a_{34} - 10\%$		
		A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100	100	100	99.5	100.2	100.3	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100.5	99.7	99.8	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	853.8	853.8	853.8	92.3	92.3	92.3	100	100	100	100	100	100	115.4	115.4	115.4
H – X s.c. test	I_H	100	100	100	98.8	99.9	99.9	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	99.4	99.6	99.7	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	99.3	99.4	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Δ	100	100	100	2.1E3	2.1E3	2.1E3	66.7	66.7	66.7	83.3	83.3	83.3	100	100	100	133.3	133.3	133.3
H – Y s.c. test	I_H	100	100	100	94.4	100	100	100	100	100	100	100	100	100	100	100	99.3	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	94.4	100	100	100	100	100	100	100	100	100	100	100	99.3	100	100
	I_Y	100	100	100	97.3	97.2	100	100	100	100	100	100	100	100	100	100	99.6	99.6	100
	I_Δ	100	100	100	94.5	100	100	100.1	100	100	100	100	100	100	100	100	99.3	100	100
X – Y s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	94.5	100	100	100.1	100	100	100.1	100	100	100	100	100	98.8	100	100
	I_C	100	100	100	94.5	100	100	100.1	100	100	100.1	100	100	100	100	100	98.8	100	100
	I_Y	100	100	100	97.3	97.2	100	100	100	100	100	100	100	100	100	100	99.4	99.4	100
	I_Δ	100	100	100	94.5	100	100	100.1	100	100	100	100	100	100	100	100	98.9	100	100

Table K.16. Percentage representation of the values from Table K.15 with respect to the original matrix given in Table G.12 – piece 3.

	A_{peak}	Coil Volume Internal Fault Full Model, the base case, fault is not applied, X_{DNF} is not inserted (subsection 4.6.2)																	
		K_{cyl} neglected																	
		Original matrix Table G.12			$a_{35} - 10\%$			$a_{36} - 10\%$			$a_{45} - 10\%$			$a_{46} - 10\%$			$a_{56} - 10\%$		
		A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %	A, %	B, %	C, %
Base load	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_A	100	100	100	92.3	92.3	92.3	100	100	100	115.4	115.4	115.4	100	100	100	100	100	100
H – X s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_Y	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_A	100	100	100	66.7	66.7	66.7	100	100	100	133.3	133.3	133.3	83.3	83.3	83.3	100	100	100
H – Y s.c. test	I_H	100	100	100	102.5	100	100	99.4	100	100	92.9	100	100	101.3	100	100	100	100	100
	I_X	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_C	100	100	100	102.5	100	100	99.4	100	100	92.9	96.4	100	101.3	100	100	100	100	100
	I_Y	100	100	100	101.2	101.2	100	99.7	99.7	100	96.5	100	100	100.7	100.7	100	100	100	100
	I_A	100	100	100	102.5	100	100	99.4	100	100	92.9	100	100	101.4	100	100	100	100	100
X – Y s.c. test	I_H	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	I_X	100	100	100	104.3	100	100	98.9	100	100	88.3	100	100	102.4	100	100	99.8	100	100
	I_C	100	100	100	104.3	100	100	98.9	100	100	88.3	100	100	102.4	100	100	99.8	100	100
	I_Y	100	100	100	102.1	102.3	100	99.5	99.5	100	94.3	94	100	101.2	101.3	100	99.9	99.9	100
	I_A	100	100	100	104.4	100	100	98.9	100	100	88.3	100	100	102.4	100	100	99.9	100	100

K.3 Sectioned Common Coil and the Fault Is Applied

Table K.17. +40% perturbation of the off-diagonal elements in $[A(N)]$:
the coil volume internal fault leakage model, the base case, the fault is applied, X_{DNF} is not inserted.

	A_{peak}	Proprietary model, fault is applied			Coil Volume Internal Fault Leakage Model, fault is applied, X_{DNF} is not inserted (subsection 4.6.1)											
					K_{CYI} neglected											
					$a_{12} + 40\%$			$a_{13} + 40\%$			$a_{23} + 40\%$			$a_{45} + 40\%$		
					A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	366.1	330.1	347.3	676.6	358.8	375.4	580.6	388.5	444.7	659.6	363.8	386.9	634.9	373.1	404.8
	I_X	664.3	675.3	675.6	671	667.9	683	656.3	654.8	691.5	667.6	665.6	684.6	665.7	662.1	687.2
	I_C	324.9	345.3	328.4	27.8	316.2	313	83.2	313.6	276.3	21.3	313.3	306	41.3	309.6	296.9
	I_F	95,047	-	-	350050	-	-	342067	-	-	348708	-	-	347332	-	-
	I_Y	N/A			0.011	0.011	0.011	0.012	0.010	0.011	0.012	0.011	0.011	0.012	0.010	0.011
	I_Δ	63.5	63.5	63.5	321.4	321.4	321.4	800.6	800.6	800.6	406.5	406.5	406.5	539.2	539.2	539.2

Table K.18. -40% perturbation of the off-diagonal elements in $[A(N)]$:
the coil volume internal fault leakage model, the base case, the fault is applied, X_{DNF} is not inserted.

	A_{peak}	Proprietary model, fault is applied			Coil Volume Internal Fault Leakage Model, fault is applied, X_{DNF} is not inserted (subsection 4.6.1)											
					K_{CYI} neglected											
					$a_{12} - 40\%$			$a_{13} - 40\%$			$a_{23} - 40\%$			$a_{45} - 40\%$		
					A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	366.1	330.1	347.3	499.3	407.5	504.4	671	360.4	379.2	584.5	388.2	442.4	649.7	364.6	393.2
	I_X	664.3	675.3	675.6	635.6	644.6	696.2	669.8	667.1	683.5	660.1	665.2	691.4	665.1	664.5	685.1
	I_C	324.9	345.3	328.4	139.1	339	247.4	23.3	315.1	310.6	87.8	312.6	277.8	32.7	314	301.4
	I_F	95,047	-	-	331645	-	-	349595	-	-	342668	-	-	346950	-	-
	I_Y	N/A			0.012	0.009	0.011	0.012	0.011	0.011	0.012	0.010	0.011	0.012	0.011	0.011
	I_Δ	63.5	63.5	63.5	1160	1160	1160	349.7	349.7	349.7	787.9	787.9	787.9	444.6	444.6	444.6

Table K.19. +80% perturbation of the off-diagonal elements in $[A(N)]$:
the coil volume internal fault leakage model, the base case, the fault is applied, X_{DNF} is not inserted.

	A_{peak}	Proprietary model, fault is applied			Coil Volume Internal Fault Leakage Model, fault is applied, X_{DNF} is not inserted (subsection 4.6.1)											
					K_{cyI} neglected											
					$a_{12} + 80\%$			$a_{13} + 80\%$			$a_{23} + 80\%$			$a_{45} + 80\%$		
					A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	366.1	330.1	347.3	694.4	352.7	363.7	455.5	420.9	538.3	671.1	359.8	379	633.3	374	406.1
	I_X	664.3	675.3	675.6	673	670.4	681.1	629	638.9	698.7	668.6	667.2	683.4	665.8	661.8	687.4
	I_C	324.9	345.3	328.4	34.4	321.1	320.2	178.8	357.9	236	15.3	315.7	310.5	42.5	309.1	296.4
	I_F	95,047	-	-	351088	-	-	326952	-	-	349398	-	-	347370	-	-
	I_Y	N/A			0.011	0.011	0.011	0.012	0.009	0.011	0.012	0.011	0.011	0.012	0.010	0.011
	I_Δ	63.5	63.5	63.5	227.2	227.2	227.2	1363	1363	1363	346	346	346	549.5	549.5	549.5

Table K.20. -80% perturbation of the off-diagonal elements in $[A(N)]$:
the coil volume internal fault leakage model, the base case, the fault is applied, X_{DNF} is not inserted.

	A_{peak}	Proprietary model, fault is applied			Coil Volume Internal Fault Leakage Model, fault is applied, X_{DNF} is not inserted (subsection 4.6.1)											
					K_{cyI} neglected											
					$a_{12} - 80\%$			$a_{13} - 80\%$			$a_{23} - 80\%$			$a_{45} - 80\%$		
					A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	366.1	330.1	347.3	986.3	203.3	707.5	691.9	353.3	365.2	250.6	464.3	695.6	0.1E20 values in the .lis file		
	I_X	664.3	675.3	675.6	424	697.8	594.3	672	670.1	681.3	614.8	614.9	702.4			
	I_C	324.9	345.3	328.4	562.6	867	475.7	28.3	320.6	319.1	364.5	498.4	195.3			
	I_F	95,047	-	-	221924	-	-	350847	-	-	289356	-	-			
	I_Y	N/A			0.004	0.011	0.011	0.011	0.011	0.011	0.012	0.007	0.011			
	I_Δ	63.5	3179	3179	3179	3179	3179	238.9	238.9	238.9	2248	2248	2248			

Table K.21. +40% perturbation of the off-diagonal elements in $[A(N+1)]$:
the coil volume internal fault full model, the base case, the fault is applied, X_{DNF} is not inserted.

	A_{peak}	Proprietary model, fault is applied			Coil Volume Internal Fault Full Model, fault is applied, X_{DNF} is not inserted (subsection 4.6.2)														
					K_{cyl} neglected														
					$a_{12} + 40\%$			$a_{13} + 40\%$			$a_{23} + 40\%$			$a_{35} + 40\%$			$a_{45} + 40\%$		
					A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	366.1	330.1	347.3	676.7	358.9	375.5	580.6	388.6	444.8	659.7	363.9	387.1	642.2	369	399.3	634.5	373.5	405.4
	I_X	664.3	675.3	675.6	671	667.9	682.9	656.3	654.8	691.5	667.6	665.6	684.6	665.4	663.2	686.2	665.7	662	687.3
	I_C	324.9	345.3	328.4	28	316.1	312.8	83.1	313.5	276.1	21.4	313.2	305.9	39.6	311.6	298.7	41.7	309.3	296.5
	I_F	95,047	-	-	350032	-	-	342036	-	-	348689	-	-	347130	-	-	347319	-	-
	I_Y	N/A			0.012	0.011	0.011	0.012	0.010	0.011	0.012	0.011	0.011	0.012	0.011	0.011	0.012	0.010	0.011
	I_D	63.5	63.5	63.5	321.4	321.4	321.4	800.7	800.7	800.7	406.5	406.5	406.5	493.4	493.4	493.4	542.6	542.7	542.7

Table K.22. -40% perturbation of the off-diagonal elements in $[A(N+1)]$:
the coil volume internal fault full model, the base case, the fault is applied, X_{DNF} is not inserted.

	A_{peak}	Proprietary model, fault is applied			Coil Volume Internal Fault Full Model, fault is applied, X_{DNF} is not inserted (subsection 4.6.2)														
					K_{cyl} neglected														
					$a_{12} - 40\%$			$a_{13} - 40\%$			$a_{23} - 40\%$			$a_{35} - 40\%$			$a_{45} - 40\%$		
					A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	366.1	330.1	347.3	499.4	407.6	504.6	671.1	360.5	379.3	584.5	388.3	442.5	636.2	372.5	404	660	359.4	385.4
	I_X	664.3	675.3	675.6	635.5	644.6	696.2	670	667.1	683.5	660	655.2	691.4	665.7	662.2	687	664.6	666.2	683.7
	I_C	324.9	345.3	328.4	139	339	247.2	23.4	315.1	310.5	87.8	312.5	277.6	40.5	309.8	297	30.6	317.1	305.1
	I_F	95,047	-	-	331603	-	-	349576	-	-	342634	-	-	347280	-	-	346634	-	-
	I_Y	N/A			0.012	0.009	0.011	0.012	0.011	0.011	0.012	0.010	0.011	0.012	0.010	0.011	0.012	0.011	0.011
	I_D	63.5	63.5	63.5	1160	1160	1160	349.7	349.7	349.7	788	788	788	532	532	532	379.9	379.9	379.9

Table K.23. +80% perturbation of the off-diagonal elements in $[A(N+1)]$:
the coil volume internal fault full model, the base case, the fault is applied, X_{DNF} is not inserted.

	A_{peak}	Proprietary model, fault is applied			Coil Volume Internal Fault Full Model, fault is applied, X_{DNF} is not inserted (subsection 4.6.2)														
					K_{cyl} neglected														
					$a_{12} + 80\%$			$a_{13} + 80\%$			$a_{23} + 80\%$			$a_{35} + 80\%$			$a_{45} + 80\%$		
					A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	366.1	330.1	347.3	694.5	352.8	363.8	455.6	421	538.3	671.2	359.9	379.1	650.5	364.4	392.9	632.8	374.5	406.6
	I_X	664.3	675.3	675.6	673	670.4	681.1	629	638.9	698.7	668.6	667.2	683.4	665.1	664.6	685	665.8	661.7	687.5
	I_C	324.9	345.3	328.4	34.6	321	320	178.7	357.8	235.8	15.5	315.6	310.4	32.6	314.1	301.5	42.9	308.9	296.1
	I_F	95,047	-	-	351062	-	-	326913	-	-	349381	-	-	346907	-	-	347357	-	-
	I_Y	N/A			0.011	0.011	0.011	0.012	0.009	0.011	0.012	0.011	0.011	0.012	0.011	0.011	0.012	0.010	0.011
	I_A	63.5	63.5	63.5	227.2	227.2	227.2	1363	1363	1363	346	346	346	440.7	440.7	440.7	553.1	553.1	553.1

Table K.24. -80% perturbation of the off-diagonal elements in $[A(N+1)]$:
the coil volume internal fault full model, the base case, the fault is applied, X_{DNF} is not inserted.

	A_{peak}	Proprietary model, fault is applied			Coil Volume Internal Fault Full Model, fault is applied, X_{DNF} is not inserted (subsection 4.6.2)														
					K_{cyl} neglected														
					$a_{12} - 80\%$			$a_{13} - 80\%$			$a_{23} - 80\%$			$a_{35} - 80\%$			$a_{45} - 80\%$		
					A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I_H	366.1	330.1	347.3	986.5	203.2	707.6	692	353.4	365.4	250.6	464.3	695.6	634.7	373.3	405.1	error message KILL=212 in the .lis file related to the iteration limit		
	I_X	664.3	675.3	675.6	424	697.8	594.3	672	670.1	681.3	614.7	614.9	702.4	665.7	662	687.2			
	I_C	324.9	345.3	328.4	562.7	867	475.9	28.5	320.5	318.9	364.4	498.4	195.1	41.5	309.4	296.6			
	I_F	95,047	-	-	221930	-	-	350831	-	-	289279	-	-	347314	-	-			
	I_Y	N/A			0.004	0.011	0.011	0.011	0.011	0.011	0.012	0.007	0.011	0.012	0.010	0.011			
	I_A	63.5	3179	3179	3180	3180	3180	238.9	238.9	238.9	2248	2248	2248	540.9	540.9	540.9			

Table K.25. Additional sensitivity check for element a_{45} , based on Tables K.20 and K.24:
the coil volume internal fault leakage and full models, the base case, the fault is applied, X_{DNF} is not inserted.

	A _{peak}	Coil Volume Internal Fault Model, fault is applied, X _{DNF} is not inserted																	
		K _{cyl} neglected																	
		a ₄₅ - 80%			a ₄₅ - 70%						a ₄₅ - 60%								
		Leakage model			Full model			Leakage model			Full model			Leakage model			Full model		
		A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
Base load	I _H	0.1E20 values in the .lis file			error message KILL=212 in the .lis file related to the iteration limit			822.4	351.4	248.2	error message KILL=212 in the .lis file related to the iteration limit			680.8	349.9	368.6	394.7	583.7	576.7
	I _X							651.7	696.1	665.1				663.5	669.7	681	666.2	629.9	725.7
	I _C							185.5	383.9	433.7				37.4	324.2	315.1	279.7	280.8	389.4
	I _F							337130	-	-				345933	-	-	344658	-	-
	I _Y							0.011	0.013	0.011				0.011	0.011	0.011	0.014	0.010	0.011
	I _Δ							724.4	724.3	724.3				244.2	244.2	244.2	2014	2014	2014

L Net MMF per Leg and Mean Net MMF per Leg

L.1 The Healthy Mirage

Table L.1. Current phasors and mmf of each coil in three phases: the XFMR Mirage.

Coil	XFMR Mirage						
	I, A_{peak}			N	mmf: $AT = I \times N$, Ampere-turns _{peak}		
	A	B	C		A	B	C
Series	338.4@-3.7°	338.3@-123.7°	338.4@116.3°	444	150,249.6@-3.7°	150,205.2@-123.7°	150,249.6@116.3°
Common	337.7@176.3°	337.7@56.3°	337.6@-63.7°	444	149,938.8@176.3°	149,938.8@56.3°	149,894.4@-63.7°
Delta	0.006@175.2°	0.006@55.2°	0.006@-64.8°	140	0.84@175.2°	0.84@55.2°	0.84@-64.8°

Table L.2. The net mmf per leg and the mean net mmf per leg: the XFMR Mirage.

	XFMR Mirage	
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C_A} + AT_{\Delta_A}$	310@-3.7°	310
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	265.6@-123.7°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	354.4@116.3°	

Table L.3. Difference in the net mmf per leg between the leg pairs: the XFMR Mirage.

	XFMR Mirage	
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	44.4@120°	14.3%@0
$ AT_B - AT_C @ \text{angle difference}$	88.8@120°	28.6%@0
$ AT_C - AT_A @ \text{angle difference}$	44.4@120°	14.3%@0

Table L.4. Current phasors and mmf of each coil in three phases: the healthy Mirage's leakage model.

Coil	Healthy Mirage's leakage model, K_{cyl} neglected						
	I, A_{peak}			N	$AT = I \times N, \text{Ampere-turns}_{peak}$		
	A	B	C		A	B	C
Series	338.4@-3.7°	338.4@-123.7°	338.4@116.3°	444	150,249.6@-3.7°	150,249.6@-123.7°	150,249.6@116.3°
Common	337.9@176.3°	337.9@56.3°	337.9@-63.7°	444	150,027.6@176.3°	150,027.6@56.3°	150,027.6@-63.7°
Delta	0.006@175.2°	0.006@55.2°	0.006@-64.7°	140	0.84@175.2°	0.84@55.2°	0.84@-64.7°

Table L.5. The net mmf per leg and the mean net mmf per leg: the healthy Mirage's leakage model.

	Healthy Mirage's leakage model, K_{cyl} neglected	
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C_A} + AT_{\Delta_A}$	221.2@-3.7°	221.2
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	221.2@-123.7°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	221.2@116.3°	

Table L.6. Difference in the net mmf per leg between the leg pairs: the healthy Mirage's leakage model.

	Healthy Mirage's leakage model, K_{cyl} neglected	
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	0@120°	0
$ AT_B - AT_C @ \text{angle difference}$	0@120°	0
$ AT_C - AT_A @ \text{angle difference}$	0@120°	0

Table L.7. Current phasors and mmf of each coil in three phases: the healthy Mirage's full model.

Coil	Healthy Mirage's full model, K_{cyl} neglected						
	I, A_{peak}			N	$AT = I \times N, \text{Ampere-turns}_{peak}$		
	A	B	C		A	B	C
Series	338.4@-3.7°	338.3@-123.7°	338.4@116.3°	444	150,249.6@-3.7°	150,205.2@-123.7°	150,249.6@116.3°
Common	337.7@176.3°	337.7@56.3°	337.6@-63.7°	444	149,938.8@176.3°	149,938.8@56.3°	149,894.4@-63.7°
Delta	0.006@175.2°	0.006@55.2°	0.006@-64.8°	140	0.84@175.2°	0.84@55.2°	0.84@-64.8°

Table L.8. The net mmf per leg and the mean net mmf per leg: the healthy Mirage's full model.

	Healthy Mirage's full model, K_{cyl} neglected	
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C_A} + AT_{\Delta_A}$	310@-3.7°	310
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	266.4@-123.7°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	355.2@116.3°	

Table L.9. Difference in the net mmf per leg between the leg pairs: the healthy Mirage's full model.

	Healthy Mirage's full model, K_{cyl} neglected	
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	44.4@120°	14.3%@0
$ AT_B - AT_C @ \text{angle difference}$	88.8@120°	28.6%@0
$ AT_C - AT_A @ \text{angle difference}$	44.4@120°	14.3%@0

L.2 Sectioned Common Coil but the Fault Is Not Applied

Table L.10. Current phasors and mmf of each coil in three phases:
the coil volume internal fault leakage model, the base case, the fault is not applied, X_{DNF} is not inserted.

Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is not applied							
Coil	I, A_{peak}			N	$AT = I \times N, \text{Ampere-turns}_{peak}$		
	A	B	C		A	B	C
Series	338.1@-3.7°	338.4@-123.7°	338.5@116.3°	444	150,116.4@-3.7°	150,249.6@-123.7°	150,294@116.3°
Common	C1	338.1@176.3°	337.8@56.3°	399	134,901.9@176.3°	149,983.2@56.3°	149,983.2@-63.7°
	C2	338.1@176.3°		1	338.1@176.3°		
	C3	338.1@176.3°		44	14,876.4@176.3°		
Delta	1.3@-3.7°	1.3@-3.5°	1.3@-4°	140	182@-3.7°	182@-3.5°	182@-4°

Table L.11. The net mmf per leg and the mean net mmf per leg:
the coil volume internal fault leakage model, the base case, the fault is not applied, X_{DNF} is not inserted.

Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is not applied		
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C1_A} + AT_{C2_A} + AT_{C3_A} + AT_{\Delta_A}$	182@-3.7°	228.9
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	235.2@-81.7°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	269.5@80.6°	

Table L.12. Difference in the net mmf per leg between the leg pairs:
the coil volume internal fault leakage model, the base case, the fault is not applied, X_{DNF} is not inserted.

Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is not applied		
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	53.2@78°	23.2%@65%
$ AT_B - AT_C @ \text{angle difference}$	34.3@162.3°	15@135.3%
$ AT_C - AT_A @ \text{angle difference}$	87.5@84.3°	38.2%@70.3%

Table L.13. Current phasors and mmf of each coil in three phases:
the coil volume internal fault full model, the base case, the fault is not applied, X_{DNF} is not inserted.

	Coil Volume Internal Fault Full Model, K_{cy} , neglected, the base case: fault is not applied								
	I, A_{peak}				N		$AT = I \times N, \text{Ampere-turns}_{peak}$		
Coil	A		B	C			A		B
Series	338.2@-3.7°		338.5@-123.7°	338.6@116.3°	444		150,160.8@-3.7°	150,294@-123.7°	150,338.4@116.3°
Common	C1	338@176.3°	337.7@56.3°	337.6@-63.7°	399	444	134,862@176.3°	149,938.8@56.3°	149,894.4@-63.7°
	C2	338@176.3°			1		338@176.3°		
	C3	338@176.3°			44		14,872@176.3°		
Delta	1.3@-3.7°		1.3@-3.4°	1.3@-3.9°	140		182@-3.7°	182@-3.4°	182@-3.9°

Table L.14. The net mmf per leg and mean net mmf per leg:
the coil volume internal fault full model, the base case, the fault is not applied, X_{DNF} is not inserted.

Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is not applied		
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C1_A} + AT_{C2_A} + AT_{C3_A} + AT_{\Delta_A}$	270.8@-3.7°	321.2
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	306.7@-92.9°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	386@92.2°	

Table L.15. Difference in the net mmf per leg between the leg pairs:
the coil volume internal fault full model, the base case, the fault is not applied, X_{DNF} is not inserted.

Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is not applied		
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	35.9@89.2°	11.2%@74.3%
$ AT_B - AT_C @ \text{angle difference}$	79.3@185.1°	24.7%@154.3%
$ AT_C - AT_A @ \text{angle difference}$	115.2@95.9°	35.9%@79.9%

L.3 Sectioned Common Coil and the Fault is Applied

L.3.1 The Proprietary Internal Fault Model

Table L.16. Current phasors and mmf of each coil in three phases: the proprietary model, the base case, the fault is applied.

	Proprietary model (core is included), the base case: fault is applied								
	I, A_{peak}			N		$AT = I \times N, \text{Ampere-turns}_{\text{peak}}$			
Coil	A		B			C	A	B	C
Series	366.1@-19		330.1@-124	347.3@116	444		162,548.4@-19	146,564@-124	154,201.2@116
Common	Healthy portion	324.9@-167	345.3@58	328.4@-63	443	444	143,930.7@-167	153,313.2@58	145,809.6@-63
	Shorted turn	95,047@99			1		95,047@99		
Delta	63.5@-83		63.5@-83	63.5@-83	140		8,890@-83	8,890@-83	8,890@-83

Table L.17. The net mmf per leg and the mean net mmf per leg: the proprietary model, the base case, the fault is applied.

	Proprietary model, the base case: fault is applied	
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{Healthy_A} + AT_{Shorted_A} + AT_{\Delta_A}$	414.5@-143.8	416.3
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	484.8@-40.3	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	349.7@-155.3	

Table L.18. Difference in the net mmf per leg between the leg pairs: the proprietary model, the base case, the fault is applied.

	Proprietary model, the base case: fault is applied	
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	70.3@103.5	16.9%@86.3%
$ AT_B - AT_C @ \text{angle difference}$	135.1@115	32.5%@95.8%
$ AT_C - AT_A @ \text{angle difference}$	64.8@11.5	15.6%@9.6%

L.3.2 The Coil Volume Internal Fault Leakage Model

Table L.19. Current phasors and mmf of each coil in three phases:
the coil volume internal fault leakage model, the base case, the fault is applied, X_{DNF} is not inserted.

Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is not inserted							
Coil	I, A_{peak}			N	$AT = I \times N, \text{Ampere-turns}_{peak}$		
	A	B	C		A	B	C
Series	638.3@-10.4°	371.1@-136.4°	402.2@125.1°	444	283,405.2@-10.4°	164,768.4@-136.4°	178,576.8@125.1°
Common	C1 38.9@-143.7°	310.6@70.8°	297.8@-77.5°	399	15,521@-143.7°	137,906.4@70.8°	132,223.2@-77.5°
	C2 347,250.4@167°			1	347,250.4@167°		
	C3 38.9@-143.7°			44	1,711.6@-143.7°		
Delta	517.7@-12.8°	517.7@-12.8°	517.7@-12.8°	140	72,478@-12.8°	72,478@-12.8°	72,478@-12.8°

Table L.20. The net mmf per leg and the mean net mmf per leg:
the coil volume internal fault leakage model, the base case, the fault is applied, X_{DNF} is not inserted.

	Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is not inserted	
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C1_A} + AT_{C2_A} + AT_{C3_A} + AT_{\Delta_A}$	2897.1@166.1°	3251.2
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	3336.7@-170.5°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	3519.8@164.2°	

Table L.21. Difference in the net mmf per leg between the leg pairs:
the coil volume internal fault leakage model, the base case, the fault is applied, X_{DNF} is not inserted.

	Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is not inserted	
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	439.6@23.4°	13.5%@19.5%
$ AT_B - AT_C @ \text{angle difference}$	183.1@25.4°	5.6%@21.2%
$ AT_C - AT_A @ \text{angle difference}$	622.7@1.9°	19.2%@1.6%

Table L.22. Current phasors and mmf of each coil in three phases:
the coil volume internal fault leakage model, the base case, the fault is applied, $X_{DNF} = 1.293 \text{ m}\Omega$.

Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is applied, $X_{DNF} = 1.293 \text{ m}\Omega$ is inserted to match the fault current with that of the proprietary model							
Coil	I, A_{peak}			N	$AT = I \times N, \text{Ampere-turns}_{\text{peak}}$		
	A	B	C		A	B	C
Series	361.3@-17°	321.2@-126.3°	359.9@115.1°	444	160,417.2@-17°	142,612.8@-126.3°	159,795.6@115.1°
Common	C1 320@-169.2°	353.3@59.2°	315.2@-62.9°	399	444	127,680@-169.2°	156,865.2@59.2°
	C2 95,073@101°			1		95,073@101°	
	C3 320@-169.2°			44		14,080@-169.2°	
Delta	138.8@-78.7°	138.8@-78.7°	138.8@-78.7°	140		19,432@-78.7°	19,432@-78.7°

Table L.23. The net mmf per leg and mean net mmf per leg:
the coil volume internal fault leakage model, the base case, the fault is applied, $X_{DNF} = 1.293 \text{ m}\Omega$.

Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is inserted to match the fault current		
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C1_A} + AT_{C2_A} + AT_{C3_A} + AT_{A_A}$	825@102.2°	907.4
$AT_B = AT_{S_B} + AT_{C_B} + AT_{A_B}$	807.5@111.8°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{A_C}$	1089.8@101.9°	

Table L.24. Difference in the net mmf per leg between the leg pairs:
the coil volume internal fault leakage model, the base case, the fault is applied, $X_{DNF} = 1.293 \text{ m}\Omega$.

Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is inserted to match the fault current		
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	17.5@9.6°	1.9%@8%
$ AT_B - AT_C @ \text{angle difference}$	282.3@9.9°	31.1%@8.3%
$ AT_C - AT_A @ \text{angle difference}$	264.8@0.3°	29.2%@0.25%

Table L.25. Current phasors and mmf of each coil in three phases:
the coil volume internal fault leakage model, the base case, the fault is applied, $X_{DNF} = 2.9 \text{ m}\Omega$.

	Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is applied, $X_{DNF} = 2.9 \text{ m}\Omega$ is inserted to match the delta current with that of the proprietary model						
	I, A_{peak}			N	$AT = I \times N, \text{Ampere-turns}_{\text{peak}}$		
Coil	A	B	C		A	B	C
Series	340.9@-10.4°	329.5@-124.7°	347.6@115.5°	444	151,359.6@-10.4°	146,298@-124.7°	154,334.4@115.5°
Common	C1 332.7@-176.9°	345.9@57.5°	327.9@-63.1°	399	132,747.3@-176.9°	153,579.6@57.5°	145587.6@-63.1°
	C2 44688.7@92.6°			1	44,688.7@92.6°		
	C3 332.7@-176.9°			44	14,638.8@-176.9°		
Delta	63.5@-86.9°	63.5@-86.9°	63.5@-86.9°	140	8,890@-86.9°	8,890@-86.9°	8,890@-86.9°

Table L.26. The net mmf per leg and mean net mmf per leg:
the coil volume internal fault leakage model, the base case, the fault is applied, $X_{DNF} = 2.9 \text{ m}\Omega$.

	Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is inserted to match the delta current	
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C1_A} + AT_{C2_A} + AT_{C3_A} + AT_{\Delta_A}$	497.1@71.7°	520.7
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	469.4@127.4°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	595.6@99°	

Table L.27. Difference in the net mmf per leg between the leg pairs:
the coil volume internal fault leakage model, the base case, the fault is applied, $X_{DNF} = 2.9 \text{ m}\Omega$.

	Coil Volume Internal Fault Leakage Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is inserted to match the delta current	
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	27.7@55.7°	5.3%@46.4%
$ AT_B - AT_C @ \text{angle difference}$	126.2@28.4°	24.2%@23.7%
$ AT_C - AT_A @ \text{angle difference}$	98.5@27.3°	18.9%@22.8%

Table L.28. Comparison of current phasors for the base case T2T fault:
the coil volume internal fault leakage model versus the proprietary model.

	Phase A				Phase B			Phase C		
	Series	Common		Delta	Series	Common	Delta	Series	Common	Delta
		Healthy portion	Shorted turn							
	Proprietary model									
	366.1 @-19°	324.9 @-167°	95,047 @99°	63.5 @-83°	330.1 @-124°	345.3 @58°	63.5 @-83°	341.3 @116°	328.4 @-63°	63.5 @-83°
	Coil Volume Internal Fault Leakage Model: options									
		Healthy portions C1 and C3	Shorted turn C2							
X_{DNF} omitted	638.3 @-10.4°	38.9 @-143.7°	347,250.4 @167°	517.7 @-12.8°	371.1 @-136.4	310.6 @70.8°	517.7 @-12.8°	402.2 @125.1°	297.8 @-77.5°	517.7 @-12.8
X_{DNF} 1.293 mΩ	361.3 @-17°	320 @-169.2°	95,073 @101°	138.8 @-78.7°	321.2 @-126.3°	353.3 @59.2°	138.8 @-78.7°	359.9 @115.1°	315.2 @-62.9°	138.8 @-78.7
X_{DNF} 2.9 mΩ	340.9 @-10.4°	332.7 @-176.9°	44,688.7 @92.6°	63.5 @-86.9°	329.5 @-124.7°	345.9 @57.5°	63.5 @-86.9°	347.6 @115.5°	327.9 @-63.1°	63.5 @-86.9

L.3.3 The Coil Volume Internal Fault Full Model

Table L.29. Current phasors and mmf of each coil in three phases:
the coil volume internal fault full model, the base case, the fault is applied, X_{DNF} is not inserted.

	Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is not inserted								
	I, A_{peak}				N		$AT = I \times N, \text{Ampere-turns}_{peak}$		
Coil	A		B	C			A	B	C
Series	638.4@-10.4°		371.2@-136.5°	402.3@125.1°	444		283,449.6@-10.4°	164,812.8@-136.5°	178,621.2@125.1°
Common	C1	39@-143.5°	310.5@70.8°	297.6@-77.5°	399	444	15,561@-143.5°	137,862@70.8°	132,134.4@-77.5°
	C2	347226.6@167°			1		347,226.6@167°		
	C3	39@-143.5°			44		1,716@-143.5°		
Delta	517.7@-12.8°		517.7@-12.8°	517.7@-12.8°	140		72,478@-12.8°	72,478@-12.8°	72,478@-12.8°

Table L.30. The net mmf per leg and the mean net mmf per leg:
the coil volume internal fault full model, the base case, the fault is applied, X_{DNF} is not inserted.

Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is not inserted		
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C1_A} + AT_{C2_A} + AT_{C3_A} + AT_{\Delta_A}$	2811.9@167.5°	3337.2
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	3601.9@169°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	3597.8@162.5°	

Table L.31. Difference in the net mmf per leg between the leg pairs:
the coil volume internal fault full model, the base case, the fault is applied, X_{DNF} is not inserted.

Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is not inserted		
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	790@1.5°	23.7%@1.3%
$ AT_B - AT_C @ \text{angle difference}$	4.1@6.5°	0.1%@5.4%
$ AT_C - AT_A @ \text{angle difference}$	785.9@5°	23.5%@4.2%

Table L.32. Current phasors and mmf of each coil in three phases:
the coil volume internal fault full model, the base case, the fault is applied, $X_{DNF} = 1.293 \text{ m}\Omega$.

Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is applied, $X_{DNF} = 1.293 \text{ m}\Omega$ is inserted to match the fault current with that of the proprietary model							
Coil	I, A_{peak}			N	$AT = I \times N, \text{Ampere-turns}_{\text{peak}}$		
	A	B	C		A	B	C
Series	361.7@-17°	321.3@-126.3°	360.1@115.1°	444	160,594.8@-17°	142,657.2@-126.3°	159,884.4@115.1°
Common	C1	319.9@-169.1°	353.2@59.2°	399	127,640.1@-169.1°	156,820.8@59.2°	139,860@-62.9°
	C2	95,067.1@101°		1	95,067.1@101°		
	C3	319.9@-169.1°		44	14,075.6@-169.1°		
Delta	138.8@-78.7°	138.8@-78.7°	138.8@-78.7°	140	19,432@-78.7°	19,432@-78.7°	19,432@-78.7°

Table L.33. The net mmf per leg and the mean net mmf per leg:
the coil volume internal fault full model, the base case, the fault is applied, $X_{DNF} = 1.293 \text{ m}\Omega$.

	Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is inserted to match the fault current	
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C1_A} + AT_{C2_A} + AT_{C3_A} + AT_{\Delta_A}$	521.2@80.4°	848.1
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	760.5@117.3°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	1262.7@103.9°	

Table L.34. Difference in the net mmf per leg between the leg pairs:
the coil volume internal fault full model, the base case, the fault is applied, $X_{DNF} = 1.293 \text{ m}\Omega$.

	Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is inserted to match the fault current	
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	239.3@36.9°	28.2%@30.8%
$ AT_B - AT_C @ \text{angle difference}$	502.2@13.4°	59.2%@11.2%
$ AT_C - AT_A @ \text{angle difference}$	741.5@23.5°	87.4@19.6%

Table L.35. Current phasors and mmf of each coil in three phases:
the coil volume internal fault full model, the base case, the fault is applied, $X_{DNF} = 2.9 \text{ m}\Omega$.

Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is applied, $X_{DNF} = 2.9 \text{ m}\Omega$ is inserted to match the delta current with that of the proprietary model							
Coil	I, A_{peak}			N	$AT = I \times N, \text{Ampere-turns}_{\text{peak}}$		
	A	B	C		A	B	C
Series	341@-10.4°	329.6@-124.7°	347.8@115.5°	444	151,404@-10.4°	146,342.4@-124.7°	154,423.2@115.5°
Common	C1	332.6@-176.8°	345.8@57.5°	399	444	132,707.4@-176.8°	153,535.2@57.5°
	C2	44685.9@92.6°		1		44,685.9@92.6°	
	C3	332.6@-176.8°		44		14,634.4@-176.8°	
Delta	63.5@-86.9°	63.5@-86.9°	63.5@-86.9°	140	8,890@-86.9°	8,890@-86.9°	8,890@-86.9°

Table L.36. The net mmf per leg and the mean net mmf per leg:
the coil volume internal fault full model, the base case, the fault is applied, $X_{DNF} = 2.9 \text{ m}\Omega$.

Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is inserted to match the delta current		
	Net mmf per leg	Mean net mmf per leg
$AT_A = AT_{S_A} + AT_{C1_A} + AT_{C2_A} + AT_{C3_A} + AT_{\Delta_A}$	330.8@38.7°	515.4
$AT_B = AT_{S_B} + AT_{C_B} + AT_{\Delta_B}$	448.5@138.2°	
$AT_C = AT_{S_C} + AT_{C_C} + AT_{\Delta_C}$	767@102.9°	

Table L.37. Difference in the net mmf per leg between the leg pairs:
the coil volume internal fault full model, the base case, the fault is applied, $X_{DNF} = 2.9 \text{ m}\Omega$.

Coil Volume Internal Fault Full Model, K_{cyl} neglected, the base case: fault is applied, X_{DNF} is inserted to match the delta current		
	Difference in the net mmf per leg between the leg pairs	The same difference in percent with respect to the magnitude of the mean mmf per leg and 120° balanced difference
$ AT_A - AT_B @ \text{angle difference}$	117.7@99.5°	22.8%@82.9%
$ AT_B - AT_C @ \text{angle difference}$	318.5@35.3°	61.8%@29.4%
$ AT_C - AT_A @ \text{angle difference}$	436.2@64.2°	84.6%@53.5%

Table L.38. Comparison of current phasors for the base case T2T fault:
the coil volume internal fault full model versus the proprietary model.

	Phase A				Phase B			Phase C		
	Series	Common		Delta	Series	Common	Delta	Series	Common	Delta
		Healthy portion	Shorted turn							
	Proprietary model									
	366.1 @-19°	324.9 @-167°	95,047 @99°	63.5 @-83°	330.1 @-124°	345.3 @58°	63.5 @-83°	341.3 @116°	328.4 @-63°	63.5 @-83°
	Coil Volume Internal Fault Full Model: options									
		Healthy portions C1 and C3	Shorted turn C2							
X_{DNF} omitted	638.4 @-10.4°	39 @-143.5°	347,226.6 @167°	517.7 @-12.8°	371.2 @-136.5	310.5 @70.8°	517.7 @-12.8°	402.3 @125.1°	297.6 @-77.5°	517.7 @-12.8
X_{DNF} 1.293 mΩ	361.7 @-17°	319.9 @-169.1°	95,067.1 @101°	138.8 @-78.7°	321.3 @-126.3°	353.2 @59.2°	138.8 @-78.7°	360.1 @115.1°	315 @-62.9°	138.8 @-78.7
X_{DNF} 2.9 mΩ	341 @-10.4°	332.6 @-176.8°	44,685.9 @92.6°	63.5 @-86.9°	329.6 @-124.7°	345.8 @57.5°	63.5 @-86.9°	347.8 @115.5°	327.7 @-63.1°	63.5 @-86.9

M Single T2T Fault at Different Locations of the Phase A Common Coil

Table M.1. The (base) case "bottom 10%":
the single T2T fault is located at 10% of the phase A common coil height counting from its bottom,
(C1: 89.865%, C2: 0.225%, C3: 9.910%).

I, A_{peak}		Proprietary model: core is included	Coil Volume Internal Fault Leakage Model: Cylindrical approach, K_{cyl} neglected			Coil Volume Internal Fault Full Model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH		
			X_{DNF} omitted	X_{DNF} 1.293 m Ω to match I_F	X_{DNF} 2.9 m Ω to match I_Δ	X_{DNF} omitted	X_{DNF} 1.293 m Ω to match I_F	X_{DNF} 2.9 m Ω to match I_Δ
Phase A: faulted	Series / H terminal	366.1@-19°	638.3@-10°	361.3@-17°	340.9@-10°	638.4@-10°	361.7@-17°	341@-10°
	Common	324.9@-167°	38.9@-144°	320@-169°	332.7@-177°	39@-144°	319.9@-169°	332.6@-177°
	X terminal	664.3@-4°	665.6@-8°	661.7@-4°	668.8@-4°	665.6@-8°	661.7@-4°	668.8@-4°
	Faulted turn	95,047@99°	347,250@167°	95,073@101°	44,689@93°	347,227@167°	95,067@101°	44,686@93°
	Delta	63.5@-83°	517.7@-13°	138.8@-79°	63.5@-87°	517.7@-13°	138.8@-79°	63.5@-87°
	Y terminal	N/A	0.012@-40°	0.011@-36°	0.011@-35°	0.012@-40°	0.011@-36°	0.011@-35°
Phase B	Series / H terminal	330.1@-124°	371.1@-136°	321.2@-126°	329.5@-125°	371.2@-137°	321.3@-126°	329.6@-125°
	Common	345.3@58°	310.6@71°	353.3@59°	345.9@58°	310.5@71°	353.2@59°	345.8@58°
	X terminal	675.3@-123°	662.6@-124°	673.7@-123°	675.3@-124°	662.6@-124°	673.7@-124°	675.3@-124°
	Delta	63.5@-83°	517.7@-13°	138.8@-79°	63.5@-87°	517.7@-13°	138.8@-79°	63.5@-87°
	Y terminal	N/A	0.011@-160°	0.011@-154°	0.011@-155°	0.011@-160°	0.011@-154°	0.011@-155°
Phase C	Series / H terminal	347.3@116°	402.2@125°	359.9@115°	347.6@116°	402.3@125°	360.1@115°	347.8@116°
	Common	328.4@-63°	297.8@-78°	315.2@-63°	327.9@-63°	297.6@-78°	315@-63°	327.7@-63°
	X terminal	675.6@116°	686.7@116°	675@116°	675.4@116°	686.7@115°	675@116°	675.4@116°
	Delta	63.5@-83°	517.7@-13°	138.8@-79°	63.5@-87°	517.7@-13°	138.8@-79°	63.5@-87°
	Y terminal	N/A	0.011@85°	0.011@82°	0.011@85°	0.011@85°	0.011@85°	0.011@85°

Table M.2. Percentage representation of the values from Table M.1 with respect to those of the proprietary model: the (base) case "bottom 10%".

I, A_{peak}		Proprietary model: core is included	Coil Volume Internal Fault Leakage Model: Cylindrical approach, K_{cyl} neglected			Coil Volume Internal Fault Full Model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH		
			X_{DNF} omitted	X_{DNF} 1.293 m Ω to match I_F	X_{DNF} 2.9 m Ω to match I_Δ	X_{DNF} omitted	X_{DNF} 1.293 m Ω to match I_F	X_{DNF} 2.9 m Ω to match I_Δ
Phase A: faulted	Series / H terminal	100%@0°	174.35%@-9°	98.69%@-2°	93.12%@-9°	174.38%@-9°	98.8%@-2°	93.14%@-9°
	Common	100%@0°	11.97%@-23°	98.49%@2°	102.4%@10°	12%@-23°	98.46%@2°	102.37%@10°
	X terminal	100%@0°	100.2%@4°	99.61%@0°	100.68%@0°	100.2%@4°	99.61%@0°	100.68%@0°
	Faulted turn	100%@0°	365.35%@68°	100.03%@2°	47.02%@-6°	365.32%@68°	100.02%@2°	47.01%@-6°
	Delta	100%@0°	815.28%@-70°	218.58%@-4°	100%@4°	815.28%@-70°	218.58%@-4°	100%@4°
	Max dev							
	amplitude	0	Out of comparison	-1.51%	-6.88%	Out of comparison	-1.54%	-6.86%
	angle			2°	10°		2°	10°
Phase B	Series / H terminal	100%@0°	112.42%@12°	97.3%@2°	99.82%@1°	112.45%@13°	97.33%@2°	99.85%@1°
	Common	100%@0°	89.95%@13°	102.32%@1°	100.17%@0°	89.92%@13°	102.29%@1°	100.14%@0°
	X terminal	100%@0°	98.12%@1°	99.76%@0°	100%@1°	98.12%@1°	99.76%@1°	100%@1°
	Delta	100%@0°	815.28%@-70°	218.58%@-4°	100%@4°	815.28%@-70°	218.58%@-4°	100%@4°
	Max dev							
	amplitude	0	Out of comparison	-2.7%	-0.18%	Out of comparison	-2.67%	-0.15%
	angle			2°	4°		2°	4°
Phase C	Series / H terminal	100%@0°	115.81%@9°	103.63%@-1°	100.09%@0°	115.84%@9°	103.69%@-1°	100.14%@0°
	Common	100%@0°	90.68%@15°	95.98%@0°	99.85%@0°	90.62%@15°	95.92%@0°	99.79%@0°
	X terminal	100%@0°	101.64%@0°	99.91%@0°	99.97%@0°	101.64%@-1°	99.91%@0°	99.97%@0°
	Delta	100%@0°	815.28%@-70°	218.58%@-4°	100%@4°	815.28%@-70°	218.58%@-4°	100%@4°
	Max dev							
	amplitude	0	Out of comparison	-4.02%	-0.15%	Out of comparison	-4.08%	-0.21%
	angle			-1°	4°		-1°	4°

Table M.3. The case "bottom 30%":
the single T2T fault is located at 30% of the phase A common coil height counting from its bottom,
(C1: 69.820%, C2: 0.225%, C3: 29.955%).

I, A_{peak}		Proprietary model: core is included	Coil Volume Internal Fault Leakage Model: Cylindrical approach, K_{cyl} neglected			Coil Volume Internal Fault Full Model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH		
			X_{DNF} omitted	X_{DNF} 0.954 m Ω to match I_F	X_{DNF} 1.321 m Ω to match I_Δ	X_{DNF} omitted	X_{DNF} 0.954 m Ω to match I_F	X_{DNF} 1.321 m Ω to match I_Δ
Phase A: faulted	Series / H terminal	383.5@-22°	661.3@-10°	385.8@-21°	363.7@-18°	661.4@-10°	385.9@-21°	363.9@-18°
	Common	317.6@-162°	19.8@-116°	308.4@-163°	320@-168°	19.9@-115°	308.4@-163°	319.9@-168°
	X terminal	658.7@-4°	667@-8°	657.7@-4°	661.7@-4°	667@-8°	657.6@-4°	661.7@-4°
	Faulted turn	124,491@103°	348,450@169°	124,475@106°	93,843@101°	348,432@169°	124,469@106°	93,838@101°
	Delta	102.6@-79°	393@-12°	137.6@-74°	102.9@-79°	393.1@-12°	137.6@-74°	102.9@-79°
	Y terminal	N/A	0.012@-39°	0.011@-36°	0.011@-36°	0.012@-39°	0.011@-36°	0.011@-36°
Phase B	Series / H terminal	325.9@-125°	362.5@-134°	322.8@-127°	325.6@126°	362.6@-134°	322.9@-127°	325.7@-126°
	Common	349.1@59°	314.3@67°	351.6@59°	349.2@58°	314.2@67°	351.5@59°	349.1@58°
	X terminal	674.6@-123°	665.9@-124°	673.5@-124°	674.4@-124°	665.9@-124°	673.5@-124°	674.4@-124°
	Delta	102.6@-79°	393@-12°	137.6@-74°	102.9@-79°	393.1@-12°	137.6@-74°	102.9@-79°
	Y terminal	N/A	0.011@-159°	0.011@-155°	0.011@-154°	0.011@-159°	0.011@-155°	0.011@-155°
Phase C	Series / H terminal	353.3@116°	385.5@123°	360.3@115°	354.3@115°	385.6@123°	360.4@115°	354.5@115°
	Common	322.1@-63°	306.6@-74°	315.1@-63°	321@-63°	306.4@-74°	314.9@-63°	320.9@-63°
	X terminal	675.4@116°	687.3@116°	675.3@116°	675.3@116°	684.3@116°	675.3@116°	675.3@116°
	Delta	102.6@-79°	393@-12°	137.6@-74°	102.9@-79°	393.1@-12°	137.6@-74°	102.9@-79°
	Y terminal	N/A	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°

Table M.4. Percentage representation of the values from Table M.3 with respect to those of the proprietary model: the case "bottom 30%".

I, A_{peak}		Proprietary model: core is included	Coil Volume Internal Fault Leakage Model: Cylindrical approach, K_{cyl} neglected			Coil Volume Internal Fault Full Model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750 \text{ mH}$		
			X_{DNF} omitted	X_{DNF} 0.954 m Ω to match I_F	X_{DNF} 1.321 m Ω to match I_Δ	X_{DNF} omitted	X_{DNF} 0.954 m Ω to match I_F	X_{DNF} 1.321 m Ω to match I_Δ
Phase A: faulted	Series / H terminal	100%@0°	172.44%@-12°	100.6%@-1°	94.84%@-4°	172.46%@12°	100.63%@-1°	94.89%@-4°
	Common	100%@0°	6.23%@-46°	97.1%@1°	100.76%@6°	6.27%@-47°	97.1%@1°	100.72%@6°
	X terminal	100%@0°	101.26%@4°	99.85%@0°	100.46%@0°	101.26%@4°	99.83%@0°	100.46%@0°
	Faulted turn	100%@0°	279.9%@66°	99.99%@3°	75.38%@-2°	279.89%@66°	99.98%@3°	75.38%@-2°
	Delta	100%@0°	383.04%@-67°	134.11%@-5°	100.29%@0°	383.14%@-67°	134.11%@-5°	100.29%@0°
	Max dev							
	amplitude	0	Out of comparison	-2.9%	-5.16%	Out of comparison	-2.9%	-5.11%
	angle			3°	6°		3°	6°
Phase B	Series / H terminal	100%@0°	111.23%@9°	99.05%@2°	99.91%@1°	111.26%@9°	99.08%@2°	99.94%@1°
	Common	100%@0°	90.03%@8°	100.72%@0°	100.03%@-1°	90%@8°	100.69%@0°	100%@-1°
	X terminal	100%@0°	98.71%@1°	99.84%@1°	99.97%@1°	98.71%@1°	99.84%@1°	99.97%@1°
	Delta	100%@0°	383.04%@-67°	134.11%@-5°	100.29%@0°	383.14%@-67°	134.11%@-5°	100.29%@0°
	Max dev							
	amplitude	0	Out of comparison	-0.95%	-0.09%	Out of comparison	-0.92%	-0.06%
	angle			2°	1°		2°	1°
Phase C	Series / H terminal	100%@0°	109.11%@7°	101.98%@-1°	100.28%@-1°	109.14%@7°	102.01%@-1°	100.34%@-1°
	Common	100%@0°	95.19%@11°	97.83%@0°	99.66%@0°	95.13%@11°	97.76%@0°	99.63%@0°
	X terminal	100%@0°	101.76%@0°	99.99%@0°	99.99%@0°	101.32%@0°	99.99%@0°	99.99%@0°
	Delta	100%@0°	383.04%@-67°	134.11%@-5°	100.29%@0°	383.14%@-67°	134.11%@-5°	100.29%@0°
	Max dev							
	amplitude	0	Out of comparison	-2.17%	-0.34%	Out of comparison	-2.24%	-0.37%
	angle			-1°	-1°		-1°	-1°

Table M.5. The case “bottom 50%”:
the single T2T fault is located at 50% of the phase A common coil height counting from its bottom,
(C1: 49.775%, C2: 0.225%, C3: 50%).

I, A_{peak}		Proprietary model: core is included	Coil Volume Internal Fault Leakage Model: Cylindrical approach, K_{cyl} neglected			Coil Volume Internal Fault Full Model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH		
			X_{DNF} omitted	X_{DNF} 0.857 m Ω to match I_F	X_{DNF} 0.75 m Ω to match I_Δ	X_{DNF} omitted	X_{DNF} 0.857 m Ω to match I_F	X_{DNF} 0.75 m Ω to match I_Δ
Phase A: faulted	Series / H terminal	392.6@-23°	683.1@-9°	402.9@-23°	417.9@-24°	683.2@-9°	403@-23°	418@-24°
	Common	314.1@-159°	17.9@-46°	302.3@-160°	294.7@-157°	18.1@-46°	302.2@-160°	294.6@-157°
	X terminal	656.8@-4°	668.8@-8°	657.5@-4°	655.8@-4°	668.8@-8°	657.4@-4°	655.8@-4°
	Faulted turn	136,959@104°	349,548@170°	136,933@109°	152,204@112°	349,535@170°	136,929@109°	152,199@112°
	Delta	117.3@-78°	275.3@-11°	105.1@-72°	117.2@-69°	275.4@-11°	105.1@-72°	117.2@-69°
	Y terminal	N/A	0.011@-38°	0.011@-36°	0.011@-36°	0.011@-38°	0.011@-36°	0.011@-36°
Phase B	Series / H terminal	324.5@-126°	354.7@-131°	326.9@-126°	326.3@-126°	354.8@-131°	327@-126°	326.3@-126°
	Common	350.3@59°	319.6@64°	347.8@59°	348.2@59°	319.6@64°	347.7@59°	348.1@59°
	X terminal	674.3@-123°	669@-124°	674.1@-124°	673.8@-124°	669@-124°	674.1@-124°	673.7@-124°
	Delta	117.3@-78°	275.3@-11°	105.1@-72°	117.2@-69°	275.4@-11°	105.1@-72°	117.2@-69°
	Y terminal	N/A	0.011@-157°	0.011@-155°	0.011@-155°	0.011@-157°	0.011@-155°	0.011@-155°
Phase C	Series / H terminal	355.7@116°	371@122°	355.2@116°	357.4@116°	370.6@121°	355.4@116°	357.5@116°
	Common	319.7@-63°	315.3@-71°	320.4@-64°	318.4@-64°	315.1@-71°	320.3@-64°	318.2@-64°
	X terminal	675.3@116°	681.9@116°	675.7@116°	675.7@116°	681.9@116°	675.6@116°	675.7@116°
	Delta	117.3@-78°	275.3@-11°	105.1@-72°	117.2@-69°	275.4@-11°	105.1@-72°	117.2@-69°
	Y terminal	N/A	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°

Table M.6. Percentage representation of the values from Table M.5 with respect to those of the proprietary model: the case "bottom 50%".

I, A_{peak}		Proprietary model: core is included	Coil Volume Internal Fault Leakage Model: Cylindrical approach, K_{cy} neglected			Coil Volume Internal Fault Full Model: Cylindrical approach, K_{cy} neglected, $L_0 = 750$ mH		
			X_{DNF} omitted	X_{DNF} 0.857 m Ω to match I_F	X_{DNF} 0.75 m Ω to match I_Δ	X_{DNF} omitted	X_{DNF} 0.857 m Ω to match I_F	X_{DNF} 0.75 m Ω to match I_Δ
Phase A: faulted	Series / H terminal	100%@0°	173.99%@-14°	102.62%@0°	106.44%@1°	174.02%@-14°	102.65%@0°	106.45%@1°
	Common	100%@0°	5.7%@-113°	96.24%@1°	93.82%@-2°	5.76%@-113°	96.21%@1°	93.79%@-2°
	X terminal	100%@0°	101.83%@4°	100.11%@0°	99.85%@0°	101.83%@4°	100.09%@0°	99.85%@0°
	Faulted turn	100%@0°	255.22%@66°	99.98%@5°	111.13%@8°	255.21%@66°	99.98%@5°	111.13%@8°
	Delta	100%@0°	234.7%@-67°	89.6%@-6°	99.91%@-9°	234.78%@-67°	89.6%@-6°	99.91%@-9°
	Max dev	amplitude	0	Out of comparison	-3.76%	Out of comparison	-3.79%	6.45%
		angle			5°			
Phase B	Series / H terminal	100%@0°	109.31%@5°	100.74%@0°	100.55%@0°	109.34%@5°	100.77%@0°	100.55%@0°
	Common	100%@0°	91.24%@5°	99.29%@0°	99.4%@0°	91.24%@5°	99.26%@0°	99.37%@0°
	X terminal	100%@0°	99.21%@1°	99.97%@1°	99.93%@1°	99.21%@1°	99.97%@1°	99.91%@1°
	Delta	100%@0°	234.7%@-67°	89.6%@-6°	99.91%@-9°	234.78%@-67°	89.6%@-6°	99.91%@-9°
	Max dev	amplitude	0	Out of comparison	0.74%	Out of comparison	0.77%	-0.63%
		angle			1°			
Phase C	Series / H terminal	100%@0°	104.3%@6°	99.86%@0°	100.48%@0°	104.19%@5°	99.92%@0°	100.51%@0°
	Common	100%@0°	98.62%@8°	100.22%@1°	99.59%@1°	98.56%@8°	100.19%@1°	99.53%@1°
	X terminal	100%@0°	100.98%@0°	100.06%@0°	100.06%@0°	100.98%@0°	100.04%@0°	100.06%@0°
	Delta	100%@0°	234.7%@-67°	89.6%@-6°	99.91%@-9°	234.78%@-67°	89.6%@-9°	99.91%@-9°
	Max dev	amplitude	0	Out of comparison	0.22%	Out of comparison	0.19%	0.51%
		angle			1°			

Table M.7. The case "top 30%":
the single T2T fault is located at 30% of the phase A common coil height counting from its top,
(C1: 29.955%, C2: 0.225%, C3: 69.820%).

I, A_{peak}		Proprietary model: core is included	Coil Volume Internal Fault Leakage Model: Cylindrical approach, K_{cyl} neglected			Coil Volume Internal Fault Full Model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH		
			X_{DNF} omitted	X_{DNF} 0.968 m Ω to match I_F	X_{DNF} 0.45 m Ω to match I_Δ	X_{DNF} omitted	X_{DNF} 0.968 m Ω to match I_F	X_{DNF} 0.45 m Ω to match I_Δ
Phase A: faulted	Series / H terminal	383.7@-22°	703.1@-8°	398.2@-22°	505.2@-25°	703.2@-8°	398.3@-22°	505.4@-25°
	Common	317.7@-162°	32.6@-16°	307.6@-161°	251.5@-141°	32.8@-16°	307.5@-161°	251.5@-141°
	X terminal	658.9@-4°	670.8@-7°	661.6@-4°	654.8@-5°	670.8@-7°	661.6@-4°	654.8@-5°
	Faulted turn	124,427@103°	350,568@172°	124,465@107°	217,125@125°	350,561@172°	124,463@107°	217,121@125°
	Delta	101.4@-79°	167.8@-11°	56.7@-74°	101.6@-58°	167.8@-11°	56.7@-74°	101.6@-58°
	Y terminal	N/A	0.011@-37°	0.011@-35°	0.011@-36°	0.011@-37°	0.011@-35°	0.011@-36°
Phase B	Series / H terminal	326@-125°	347.7@-128°	331.8@-125°	330.6@-126°	347.8@-128°	331.8@-125°	330.7@-126°
	Common	349@59°	326.1@61°	343.5@58°	343.8@59°	326@61°	343.4@58°	343.7@59°
	X terminal	674.6@-123°	671.9@-124°	675.1@-124°	673.8@-124°	671.8@-124°	675.1@-124°	673.8@-124°
	Delta	101.4@-79°	167.8@-11°	56.7@-74°	101.6@-58°	167.8@-11°	56.7@-74°	101.6@-58°
	Y terminal	N/A	0.011@-156°	0.011@-155°	0.011@-155°	0.011@-156°	0.011@-155°	0.011@-155°
Phase C	Series / H terminal	353.1@116°	357.5@120°	347.4@116°	355.1@116°	357.7@120°	347.5@116°	355.2@116°
	Common	322.3@-63°	323.6@-68°	328.5@-64°	321.3@-64°	323.4@-68°	328.3@-64°	321.1@-64°
	X terminal	675.4@116°	679.7@116°	675.9@116°	676.4@116°	679.7@116°	675.8@116°	676.3@116°
	Delta	101.4@-79°	167.8@-11°	56.7@-74°	101.6@-58°	167.8@-11°	56.7@-74°	101.6@-58°
	Y terminal	N/A	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°

Table M.8. Percentage representation of the values from Table M.7 with respect to those of the proprietary model: the case "top 30%".

I, A_{peak}		Proprietary model: core is included	Coil Volume Internal Fault Leakage Model: Cylindrical approach, K_{cyl} neglected			Coil Volume Internal Fault Full Model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH		
			X_{DNF} omitted	X_{DNF} 0.968 m Ω to match I_F	X_{DNF} 0.45 m Ω to match I_Δ	X_{DNF} omitted	X_{DNF} 0.968 m Ω to match I_F	X_{DNF} 0.45 m Ω to match I_Δ
Phase A: faulted	Series / H terminal	100%@0°	183.24%@-14°	103.78%@0°	131.67%@3°	183.27%@-14°	103.81%@0°	131.72%@3°
	Common	100%@0°	10.26%@-146°	96.82%@-1°	79.16%@-21°	10.32%@-146°	96.79%@-1°	79.16%@-21°
	X terminal	100%@0°	101.81%@3°	100.41%@0°	99.38%@1°	101.81%@3°	100.41%@0°	99.38%@1°
	Faulted turn	100%@0°	281.75%@69°	100.03%@4°	174.5%@12°	281.74%@69°	100.03%@4°	174.5%@12°
	Delta	100%@0°	165.48%@-68°	55.92%@-5°	100.2%@-21°	165.48%@-68°	55.92%@-5°	100.2%@-21°
	Max dev							
	amplitude	0	Out of comparison	3.78%	31.67%	Out of comparison	3.81%	31.72%
	angle			4°	-21°		4°	-21°
Phase B	Series / H terminal	100%@0°	106.66%@3°	101.78%@0°	101.41%@1°	106.69%@3°	101.78%@0°	101.44%@1°
	Common	100%@0°	93.44%@2°	98.42%@-1°	98.51%@0°	93.41%@2°	98.4%@-1°	98.48%@0°
	X terminal	100%@0°	99.6%@1°	100.07%@1°	99.88%@1°	99.58%@1°	100.07%@1°	99.88%@1°
	Delta	100%@0°	165.48%@-68°	55.92%@-5°	100.2%@-21°	165.48%@-68°	55.92%@-5°	100.2%@-21°
	Max dev							
	amplitude	0	Out of comparison	1.78%	-1.49%	Out of comparison	1.78%	-1.52%
	angle			1°	-21°		1°	-21°
Phase C	Series / H terminal	100%@0°	101.25%@4°	98.39%@0°	100.57%@0°	101.3%@4°	98.41%@0°	100.59%@0°
	Common	100%@0°	100.4%@5°	101.92%@1°	99.69%@1°	100.34%@5°	101.86%@1°	99.63%@1°
	X terminal	100%@0°	100.64%@0°	100.07%@0°	100.15%@0°	100.64%@0°	100.06%@0°	100.13%@0°
	Delta	100%@0°	165.48%@-68°	55.92%@-5°	100.2%@-21°	165.48%@-68°	55.92%@-5°	100.2%@-21°
	Max dev							
	amplitude	0	Out of comparison	1.92%	0.57%	Out of comparison	1.86%	0.59%
	angle			1°	-21°		1°	-21°

Table M.9. The case “top 10%”:
the single T2T fault is located at 10% of the phase A common coil height counting from its top,
($C1$: 9.91%, $C2$: 0.225%, $C3$: 89.865%).

I, A_{peak}		Proprietary model: core is included	Coil Volume Internal Fault Leakage Model: Cylindrical approach, K_{cyl} neglected			Coil Volume Internal Fault Full Model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH		
			X_{DNF} omitted	X_{DNF} 1.315 m Ω to match I_F	X_{DNF} 0.17 m Ω to match I_Δ	X_{DNF} omitted	X_{DNF} 1.315 m Ω to match I_F	X_{DNF} 0.17 m Ω to match I_Δ
Phase A: faulted	Series / H terminal	366.8@-19°	721.4@-6°	379@-19°	658.5@-18°	721.5@-6°	379.2@-19°	658.7@-18°
	Common	324.9@-166°	48.7@-8°	318.5@-166°	141.4@-104°	48.8@-8°	318.4@-166°	141.5@-104°
	X terminal	664.5@-4°	672.8@-6°	667.9@-4°	663.2@-6°	672.7@-6°	667.9@-4°	663.2@-6°
	Faulted turn	95,412@99°	351,530@173°	95,394@104°	311,892@150°	351,527@173°	95,393@104°	311,889@150°
	Delta	60.8@-83°	69.7@-14°	16@-84°	60.6@-37°	69.8@-14°	16@-84°	60.7@-37°
	Y terminal	N/A	0.011@-36°	0.011@-35°	0.011@-36°	0.011@-36°	0.011@-35°	0.011@-36°
Phase B	Series / H terminal	330.4@-124°	341.6@-126°	336.2@-124°	337.1@-125°	341.7@-126°	336.3@-124°	337.2@-125°
	Common	345@58°	333.2@58°	339.8@57°	337.8@58°	333.1@58°	339.7@57°	337.7@58°
	X terminal	675.3@-123°	674.4@-124°	675@-124°	674.6@-124°	674.4@-124°	676@-124°	674.6@-124°
	Delta	60.8@-83°	69.7@-14°	16@-84°	60.6@-37°	69.8@-14°	16@-84°	60.7@-37°
	Y terminal	N/A	0.011@-155°	0.011@-155°	0.011@-155°	0.011@-155°	0.011@-155°	0.011@-155°
Phase C	Series / H terminal	346.9@116°	346.5@118°	340.7@116°	347.6@117°	346.7@118°	340.9@116°	347.8@117°
	Common	328.8@-63°	331.3@-65°	335.3@-64°	329.3@-65°	331.2@-65°	335.2@-64°	329.2@-65°
	X terminal	675.6@116°	677.6@116°	676.1@116°	676.9@116°	677.6@116°	676@116°	676.9@116°
	Delta	60.8@-83°	69.7@-14°	16@-84°	60.6@-37°	69.8@-14°	16@-84°	60.7@-37°
	Y terminal	N/A	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°

Table M.10. Percentage representation of the values from Table M.9 with respect to those of the proprietary model: the case "top 10%".

I, A_{peak}		Proprietary model: core is included	Coil volume internal fault leakage model: Cylindrical approach, K_{cyl} neglected			Coil volume internal fault full model: Cylindrical approach, K_{cyl} neglected, $L_0 = 750$ mH		
			X_{DNF} omitted	X_{DNF} 1.315 mΩ to match I_F	X_{DNF} 0.17 mΩ to match I_Δ	X_{DNF} omitted	X_{DNF} 1.315 mΩ to match I_F	X_{DNF} 0.17 mΩ to match I_Δ
Phase A: faulted	Series / H terminal	100%@0°	196.67%@-13°	103.33%@0°	179.53%@-1°	196.7%@-13°	103.38%@0°	179.58%@-1°
	Common	100%@0°	14.99%@-158°	98.49%@0°	43.52%@-62°	15%@-158°	98%@0°	43.55%@-62°
	X terminal	100%@0°	101.25%@2°	99.96%@0°	99.8%@2°	101.23%@2°	100.51%@0°	99.8%@2°
	Faulted turn	100%@0°	368.43%@74°	99.98%@5°	326.89%@51°	368.43%@74°	99.98%@5°	326.89%@51°
	Delta	100%@0°	114.64%@-69°	26.32%@1°	99.67%@-46°	114.8%@-69°	26.32%@1°	99.84%@-46°
	Max dev							
Phase B	amplitude	0	Out of comparison	3.33%	79.53%	Out of comparison	3.38%	79.58%
	angle			5°	-62°		5°	-62°
	Series / H terminal	100%@0°	103.39%@2°	101.76%@0°	102.03%@1°	103.42%@2°	101.79%@0°	102.06%@1°
	Common	100%@0°	96.58%@0°	98.49%@-1°	97.91%@0°	96.55%@0°	98.46%@-1°	97.88%@0°
	X terminal	100%@0°	99.87%@1°	99.96%@1°	99.9%@1°	99.87%@1°	100.1%@1°	99.9%@1°
	Delta	100%@0°	114.64%@-69°	26.32%@1°	99.67%@-46°	114.8%@-69°	26.32%@1°	99.84%@-46°
Phase C	Max dev							
	amplitude	0	Out of comparison	1.76%	2.09%	Out of comparison	1.79%	2.12%
	angle			1°	-46°		1°	-46°
	Series / H terminal	100%@0°	99.88%@2°	98.21%@0°	100.2%@1°	99.94%@2°	98.27%@0°	100.26%@1°
	Common	100%@0°	100.76%@2°	98.49%@1°	100.15%@2°	100.73%@2°	101.95%@1°	100.12%@2°
	X terminal	100%@0°	100.3%@0°	100.07%@0°	100.19%@0°	100.3%@0°	100.06%@0°	100.19%@0°
	Delta	100%@0°	114.64%@-69°	26.32%@1°	99.67%@-46°	114.8%@-69°	26.32%@1°	99.84%@-46°
	Max dev							
	amplitude	0	Out of comparison	1.51%	0.2%	Out of comparison	1.95%	0.26%
	angle			1°	-46°		1°	-46°

Table M.11. Sets of the binary reactances estimated by the Cylindrical approach:
the single T2T fault at different locations on the phase A common coil, the Rogowski factors K_{cy} are neglected.

Binary reactances, Ohms		the (base) case "bottom 10%"	the case "bottom 30%"	the case "bottom 50%"	the case "top 30%"	the case "top 10%"
Phase A: faulted	X_{SC1}	21.945148	20.724288	19.427778	18.071456	16.624496
	X_{SC2}	34.086331	30.424603	26.535924	22.467797	18.127767
	X_{SC3}	34.688112	32.248093	29.656774	26.945810	24.053590
	X_{C1D}	10.790438	7.703825	4.557750	1.894110	0.235724
	X_{C2D}	3.688367e-05	5.545399e-05	7.517555e-05	9.580743e-05	0.000118
	X_{C3D}	0.059979	0.657488	2.155768	4.864352	9.226207
	X_{C1C2}	4.907789	2.367804	0.882388	0.197952	0.007463
	X_{C1C3}	5.393769	3.256721	1.655588	0.599764	0.065659
	X_{C2C3}	2.985215e-06	9.178196e-06	1.575492e-05	2.263506e-05	2.997497e-05
	X_{SD}	41.370719				
	X_{S-Core}	45.893869				
	$X_{C1-Core}$	14.443196	9.908773	5.678374	2.299972	0.280144
	$X_{C2-Core}$	5.982799e-05	7.839831e-05	9.811986e-05	0.000119	0.000141
	$X_{C3-Core}$	0.104399	1.063350	3.286555	7.069300	12.878965
	X_{D-Core}	0.333962				
Phases B and C: healthy	X_{SC}	22.533641				
	X_{SD}	41.370719				
	X_{CD}	12.184636				
	X_{S-Core}	45.893869				
	X_{C-Core}	16.707787				
	X_{D-Core}	0.333962				

Table M.12. Sets of the coil resistances:
the single T2T fault at different locations on the phase A common coil.

Coil resistances, Ohms		the (base) case "bottom 10%"	the case "bottom 30%"	the case "bottom 50%"	the case "top 30%"	the case "top 10%"
Phase A: faulted	R_S	0.292781				
	R_{C1}	0.149480	0.116138	0.082795	0.049827	0.016484
	R_{C2}	0.374637e-03				
	R_{C3}	0.016484	0.049827	0.083170	0.116138	0.149480
	R_D	0.03417				
Phases B and C: healthy	R_S	0.292781				
	R_C	0.166339				
	R_D	0.03417				

Table M.13. The proprietary internal fault model:
the single T2T fault at different locations on the phase A common coil.

I, A_{peak}		the (base) case "bottom 10%"	the case "bottom 30%"	the case "bottom 50%"	the case "top 30%"	the case "top 10%"
Phase A: faulted	Series / H terminal	366.1@-19°	383.5@-22°	392.6@-23°	383.7@-22°	366.8@-19°
	Common	324.9@-167°	317.6@-162°	314.1@-159°	317.7@-162°	324.9@-166°
	X terminal	664.3@-4°	658.7@-4°	656.8@-4°	658.9@-4°	664.5@-4°
	Faulted turn	95,047@99°	124,491@103°	136,959@104°	124,427@103°	95,412@99°
	Delta	63.5@-83°	102.6@-79°	117.3@-78°	101.4@-79°	60.8@-83°
Phase B	Series / H terminal	330.1@-124°	325.9@-125°	324.5@-126°	326@-125°	330.4@-124°
	Common	345.3@58°	349.1@59°	350.3@59°	349@59°	345@58°
	X terminal	675.3@-123°	674.6@-123°	674.3@-123°	674.6@-123°	675.3@-123°
	Delta	63.5@-83°	102.6@-79°	117.3@-78°	101.4@-79°	60.8@-83°
Phase C	Series / H terminal	347.3@116°	353.3@116°	355.7@116°	353.1@116°	346.9@116°
	Common	328.4@-63°	322.1@-63°	319.7@-63°	322.3@-63°	328.8@-63°
	X terminal	675.6@116°	675.4@116°	675.3@116°	675.4@116°	675.6@116°
	Delta	63.5@-83°	102.6@-79°	117.3@-78°	101.4@-79°	60.8@-83°

Table M.14. The coil volume internal fault full model with X_{DNF} omitted:
the single T2T fault at different locations on the phase A common coil.

I, A_{peak}		the (base) case "bottom 10%"	the case "bottom 30%"	the case "bottom 50%"	the case "top 30%"	the case "top 10%"
$X_{DNF}, m\Omega$		omitted				
Phase A: faulted	Series / H terminal	638.4@-10°	661.4@-10°	683.2@-9°	703.2@-8°	721.5@-6°
	Common	39@-144°	19.9@-115°	18.1@-46°	32.8@-16°	48.8@-8°
	X terminal	665.6@-8°	667@-8°	668.8@-8°	670.8@-7°	672.7@-6°
	Faulted turn	347,227@167°	348,432@169°	349,535@170°	350,561@172°	351,527@173°
	Delta	517.7@-13°	393.1@-12°	275.4@-11°	167.8@-11°	69.8@-14°
	Y terminal	0.012@-40°	0.012@-39°	0.011@-38°	0.011@-37°	0.011@-36°
Phase B	Series / H terminal	371.2@-137°	362.6@-134°	354.8@-131°	347.8@-128°	341.7@-126°
	Common	310.5@71°	314.2@67°	319.6@64°	326@61°	333.1@58°
	X terminal	662.6@-124°	665.9@-124°	669@-124°	671.8@-124°	674.4@-124°
	Delta	517.7@-13°	393.1@-12°	275.4@-11°	167.8@-11°	69.8@-14°
	Y terminal	0.011@-160°	0.011@-159°	0.011@-157°	0.011@-156°	0.011@-155°
Phase C	Series / H terminal	402.3@125°	385.6@123°	370.6@121°	357.7@120°	346.7@118°
	Common	297.6@-78°	306.4@-74°	315.1@-71°	323.4@-68°	331.2@-65°
	X terminal	686.7@115°	684.3@116°	681.9@116°	679.7@116°	677.6@116°
	Delta	517.7@-13°	393.1@-12°	275.4@-11°	167.8@-11°	69.8@-14°
	Y terminal	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°

Table M.15. The coil volume internal fault full model with X_{DNF} to match the fault current I_F :
the single T2T fault at different locations on the phase A common coil.

I, A_{peak}		the (base) case "bottom 10%"	the case "bottom 30%"	the case "bottom 50%"	the case "top 30%"	the case "top 10%"
$X_{DNF}, m\Omega$ for matching I_F		1.293	0.954	0.857	0.968	1.315
Phase A: faulted	Series / H terminal	361.3@-17°	385.9@-21°	403@-23°	398.3@-22°	379.2@-19°
	Common	320@-169°	308.4@-163°	302.2@-160°	307.5@-161°	318.4@-166°
	X terminal	661.7@-4°	657.6@-4°	657.4@-4°	661.6@-4°	667.9@-4°
	Faulted turn	95,073@101°	124,469@106°	136,929@109°	124,463@107°	95,393@104°
	Delta	138.8@-79°	137.6@-74°	105.1@-72°	56.7@-74°	16@-84°
	Y terminal	0.011@-36°	0.011@-36°	0.011@-36°	0.011@-35°	0.011@-35°
Phase B	Series / H terminal	321.2@-126°	322.9@-127°	327@-126°	331.8@-125°	336.3@-124°
	Common	353.3@59°	351.5@59°	347.7@59°	343.4@58°	339.7@57°
	X terminal	673.7@-123°	673.5@-124°	674.1@-124°	675.1@-124°	676@-124°
	Delta	138.8@-79°	137.6@-74°	105.1@-72°	56.7@-74°	16@-84°
	Y terminal	0.011@-154	0.011@-155°	0.011@-155°	0.011@-155°	0.011@-155°
Phase C	Series / H terminal	359.9@115°	360.4@115°	355.4@116°	347.5@116°	340.9@116°
	Common	315.2@-63°	314.9@-63°	320.3@-64°	328.3@-64°	335.2@-64°
	X terminal	675@116°	675.3@116°	675.6@116°	675.8@116°	676@116°
	Delta	138.8@-79°	137.6@-74°	105.1@-72°	56.7@-74°	16@-84°
	Y terminal	0.011@82°	0.011@85°	0.011@85°	0.011@85°	0.011@85°

Table M.16. The coil volume internal fault full model with X_{DNF} to match the delta current I_{Δ} :
the single T2T fault at different locations on the phase A common coil.

I, A_{peak}		the (base) case "bottom 10%"	the case "bottom 30%"	the case "bottom 50%"	the case "top 30%"	the case "top 10%"
$X_{DNF}, m\Omega$ for matching I_{Δ}		2.9	1.321	0.75	0.45	0.17
Phase A: faulted	Series / H terminal	341@-10°	363.9@-18°	418@-24°	505.4@-25°	658.7@-18°
	Common	332.6@-177°	319.9@-168°	294.6@-157°	251.5@-141°	141.5@-104°
	X terminal	668.8@-4°	661.7@-4°	655.8@-4°	654.8@-5°	663.2@-6°
	Faulted turn	44,686@93°	93,838@101°	152,199@112°	217,121@125°	311,889@150°
	Delta	63.5@-87°	102.9@-79°	117.2@-69°	101.6@-58°	60.7@-37°
	Y terminal	0.011@-35°	0.011@-36°	0.011@-36°	0.011@-36°	0.011@-36°
Phase B	Series / H terminal	329.6@-125°	325.7@-126°	326.3@-126°	330.7@-126°	337.2@-125°
	Common	345.8@58°	349.1@58°	348.1@59°	343.7@59°	337.7@58°
	X terminal	675.3@-124°	674.4@-124°	673.7@-124°	673.8@-124°	674.6@-124°
	Delta	63.5@-87°	102.9@-79°	117.2@-69°	101.6@-58°	60.7@-37°
	Y terminal	0.011@-155°	0.011@-155°	0.011@-155°	0.011@-155°	0.011@-155°
Phase C	Series / H terminal	347.8@116°	354.5@115°	357.5@116°	355.2@116°	347.8@117°
	Common	327.7@-63°	320.9@-63°	318.2@-64°	321.1@-64°	329.2@-65°
	X terminal	675.4@116°	675.3@116°	675.7@116°	676.3@116°	676.9@116°
	Delta	63.5@-87°	102.9@-79°	117.2@-69°	101.6@-58°	60.7@-37°
	Y terminal	0.011@85°	0.011@85°	0.011@85°	0.011@85°	0.011@85°

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