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Energy Storage Control and Requirements For Inverter-Based Microgrids

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ENERGY STORAGE CONTROL AND REQUIREMENTS FOR
INVERTER-BASED MICROGRIDS

By
Mehrzad Mohammadi Bijaieh

A DISSERTATION
Submitted in partial fulfillment of the requirements for the degree of
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<td>Battery Energy Storage System</td>
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<tr>
<td>DGU</td>
<td>Distributed Generation Unit</td>
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<tr>
<td>ES</td>
<td>Energy Storage</td>
</tr>
<tr>
<td>ESS</td>
<td>Energy Storage System</td>
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<tr>
<td>FESS</td>
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<td>FOLPF</td>
<td>First Order Low Pass Filter</td>
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<td>HSSPFC</td>
<td>Hamiltonian Surface Shaping and Power Flow Control</td>
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<tr>
<td>LPF</td>
<td>Low Pass Filter</td>
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<tr>
<td>PCC</td>
<td>Point of Common Coupling</td>
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<td>PLL</td>
<td>Phase Locked Loop</td>
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<td>RHS</td>
<td>Right Hand Side</td>
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Abstract

The intermittent nature of distributed renewable sources such as wind or solar requires integration of energy storage systems. In this dissertation a distributed form of the Hamiltonian Surface Shaping and Power Flow Control (HSSPFC) method is used to determine the energy storage requirements of three-phase inverter-based microgrids. The overall control is appropriate to be integrated into a hierarchical control system. As the primary control, a novel dq droop control sets the local references and is supported by a level-zero Hamiltonian controller which includes energy storage feed-forward and feedback, and an inverter feed-forward controls. Here, the energy storage element performs as the sole actuator of the system and enforces the references that are set by the droop control while the inverter feed-forward matches the voltage levels of the inverter to the local bus. The control method as well as the power flow and energy transfer model of the microgrid system enables the capacity and bandwidth of the storage system to be determined.

The Hamiltonian control is further derived for parallel operation of hybrid, band-limited and reduced-order battery and flywheel storage systems. Moreover, a control scheme is proposed to enable sharing of power between parallel battery and flywheel storage systems according to their bandwidth support capabilities. Here, battery
storage systems are considered as the primary storage elements while flywheel systems are controlled to complement the deficit for higher power fluctuations. Power and energy sizing guidelines are presented and relevant trade-offs are addressed in illustrative examples.

Energy storage baseline requirements for pulsed power loads are also presented in this work. Here, the energy storage system combination with the pulsed load is controlled to mimic a constant power load that can further be integrated into power buffer systems. Examples of control and requirements for ideal, band-limited and reduced-order battery and flywheel storage systems are given. By considering these requirements, a system designer can derive the specifications for source-side or load-side energy storage devices and their control systems.
Chapter 1

Introduction

Microgrids, first introduced in [1], are clusters of Distributed Generation Units (DGUs), loads and Energy Storage Systems (ESSs) [2]. The pervasive nature of electrical grids and emergence of distributed micro-sources such as wind and solar as well as advancement of ESSs require case-specific integration schemes.

The challenges for integration of microgrids has attracted a lot of research for conceptual/physical systems modeling and appropriate control strategies. Considering the distributed nature of microgrids, development of the controllers requires dispersed local or a unified global information about the system for effective management of power. A centralized scheme can achieve high levels of performance, however it limits expandability [3] and a risk for single points of failure which is intuitively an inherent
property of a non-dispersed control system. On the other hand, distributed control schemes add flexibility and a potential to adapt autonomously against failures.

1.1 Background

A typical distributed control approach is droop control in dc [3] or ac [4] microgrids. In dc microgrids, generally, droop control utilizes virtual impedances between the source and the bus to distribute the load power by creating a weighted sum of powers based on the criteria set by the droop settings. In a one common-bus system, typically, this power sharing reduces to simple sharing of load currents since the bus voltage is a common entity. Simplicity of the dc system analysis opens the path for its application in ac microgrids as well. [5] presents inverter-based three-phase ac microgrid model that makes use of dc system formulation and analysis for control system development.

Utilization of dc sources of intermittent nature such as solar or wind would require presence of ESS components. Microgrid systems with high penetration of these sources would potentially require a very large ESS capacity to alleviate the effects of this intermittent behavior. This intermittency or in general, variability, occurs at different time stamps or frequencies. With existence of various energy storage technologies, the system designer can determine the optimal energy storage combination based on the case-specific local and global constraints.
1.2 Hierarchical Control Method in Microgrids

Hierarchical control in microgrid systems typically consist of four levels [6, 7, 8]. Each level has supervisory control over lower-level sub-systems. To ensure a low impact for stability issue of the overall control system, a decrease in communication bandwidth from the highest to lowest levels is considered.

The level-zero control as the lowest control level regulates current and voltage levels of the local system. The controller may include multiple linear or non-linear control loops in the form of feed-forward or feedback loops. One of the main functionalities of this level is to ensure stability of the system while maintaining voltage and currents of the system. One level higher in the hierarchy, the primary controller is typically a droop control scheme. As the slowest local control system, this controller uses local information to operate the local system autonomously in case of loss or limitations of communication systems. A secondary controller maintains the electrical levels of the microgrid and may include control loops to ensure the synchronism of the system for cases such as seamless connection to another microgrid or grid. The highest control level, is a tertiary controller which supervises the local grid in the energy production level and controls the power flow from the local microgrid to a larger grid.
1.3 Decentralized Hamiltonian Surface Shaping and Power Flow Control

Hamiltonian Surface Shaping and Power Flow Control (HSSPFC) method [9] provides a powerful tool to determine the local and global system stability and performance [10]. The HSSPFC modeling of the global system, centralized in its unified form, has been developed in recent works [11, 12, 13, 14, 15] to incorporate the constraints that decentralized or distributed approaches impose on the system. HSSPFC provides an alternative approach to determine ESS capacity and bandwidth requirements [16, 17]. It uses the principle of conservation of energy as a core concept for control development of the microgrid system to ensure stability and performance. In this method, the microgrid system is modeled as a power-flow model and control laws are developed to control the ESS in a way that the storage requirements, specifically capacity and bandwidth can be obtained [11, 12].

For ac and dc systems, in [11, 12, 18] HSSPFC has been used to determine storage requirements for a series combination of the ESS and the renewable source. This series form enables the utilization of the HSSFC method. However, in practical implementations the series form approach might not provide the best solutions since typical problems such as source current limitations, necessity of costly high-side driving dc-dc
converters and hybrid storage limitations may occur. Hence, control development of parallel ESS and source combination seems appropriate choice to make use of to more conventional topologies.

1.4 Contributions of This Dissertation

In the first section of this dissertation a new distributed form of the HSSPFC for three-phase inverter-based microgrids is developed. The goal is to obtain a tool to derive energy storage requirements versus variable sources and loads. In this section, the HSSPFC method has been modified to support a novel dq droop control method so that the requirements for the ESSs can be obtained. As a result a zero-output storage element is achieved. Here, the significance is that it can be used as a baseline for further study of storage energy requirements versus additional constraints. These constraints can be of the storage element itself or enforced by the topology of the control system or even the mode of operation of the local DGU. Ideally the capacity and bandwidth specifications of each constraint can be individually obtained and further stacked to aid microgrid system designers.

The second part of this work makes use of the zero-output storage element and derives the requirements versus additional constraints. The work in this section aims to bridge the gap between analytical and practical implementations of HSSPFC for sizing ESSs.
To this end, the Hamiltonian controller is further modified to support hybrid parallel ESSs. It is shown that the individual operation bandwidths of batteries and flywheels can be defined by low-pass band-limited storage elements and the power and energy capacity can be specified for each.

The last part of this work will demonstrate the ES baseline requirements for pulsed power loads. Here, the energy storage is controlled to maintain the voltage and current levels of a local load with variable transient contents. As a result the storage requirements versus period and duty cycle of the pulsed load are obtained for hybrid ESSs of batteries and flywheels.

Herewith, systems designers can use the tools in this dissertation to specify ESSs and to design microgrid systems. The overall power flow and energy transfer approach of the work here is suitable to further be used in relevant studies such as probabilistic analysis or optimization of microgrid systems.
Chapter 2

Three-Phase Inverter-Based Microgrid in DQ coordinates

2.1 Introduction

The DGU model is transformed into a 0-d-q equivalent model based on [5] which provides a valuable catalog for modeling and control of distributed microgrids in 0-d-q coordinates. 0-d-q transform has been widely used to enable the implementation of dc control schemes on dc-to-ac inverters. In this section, the microgrid is developed based on ideal reduced-order model and any residual losses/effects caused by switching or component value uncertainties are neglected. The inverter, dc source, ESS and the
2.2 Three-Phase DC-AC Inverter Model

Assuming a balanced system, the voltage in the output point of the inverter presented in Fig. 2.2 can be shown as

\[
V_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \cos (\omega t + \phi_v) \\ \cos (\omega t + \phi_v - \frac{2\pi}{3}) \\ \cos (\omega t + \phi_v + \frac{2\pi}{3}) \end{bmatrix} \lambda \frac{v_{dc}}{2},
\]

where, \(\lambda\) is known as the depth of modulation and is limited to \([0, 1]\) range, \(v_{dc}\) is the voltage across positive and negative dc ports of the inverter and \(\phi_v\) is the voltage phase angle. The fixed frame abc time domain can be transformed to spinning reference frame, using Park’s power invariant matrix

\[
\Gamma_{0dq} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix},
\]

where \(\theta = \omega t + \varphi\), and \(\varphi\) is the arbitrarily chosen angle between the fixed and rotating frames. It is notable that in a multi-inverter microgrid with decentralized or
distributed control, the synchronization of inverters fall into choosing the appropriate variables \( \varphi \) and \( t \) which ideally need to be a global variables accessible by any inverter controller. However, geographical distribution of DGUs limits the choice of communication means. Here, the Global Positioning System (GPS) and GPS disciplined oscillators are considered for individual inverter rotating frames synchronization. Indeed, conventional Phase Locked Loops (PLLs) can also be used for these purposes.

From (2.2), the 0-d-q voltage components corresponding to \( V_{abc} \) in (2.1) can be obtained as

\[
\begin{bmatrix}
\Gamma_{0dq} \cdot V_{abc} = \\
\begin{bmatrix}
0 \\
v_d \\
v_q
\end{bmatrix}
\end{bmatrix} = \beta \lambda \nu_{dc}
\begin{bmatrix}
0 \\
\cos(\phi) \\
\sin(\phi)
\end{bmatrix}
\]

(2.3)

where \( \beta = \frac{1}{2} \sqrt{\frac{3}{2}} \). Here, \( \lambda \) and \( \phi \) are the inverter voltage command inputs.

Considering the dc-link of the inverter, \( i_{dc} \) can be obtained from the power of the load side

\[
P = v_d i_d + v_q i_q, \quad (2.4)
\]

where \( i_d \) and \( i_q \) are the dq current components. Additionally, the reactive power can
be obtained as

\[ Q = v_q i_d - v_d i_q. \]  \hspace{1cm} (2.5)

\( P \) and \( Q \) are inverter output active and reactive powers injected into the \( RL \) line/filter. dc-side power is determined as

\[ P_{dc} = v_{dc} i_{dc}. \]  \hspace{1cm} (2.6)

and it should match the expression in \( (2.4) \). Hence, solving for the inverter input current \( i_{dc} \) yields

\[ i_{dc} = \beta \lambda (\cos(\phi) i_d + \sin(\phi) i_q), \]  \hspace{1cm} (2.7)

where the multipliers of \( i_d \) and \( i_q \) are known as the dq switching vectors.

### 2.3 DC Source and Energy Storage Model

The DC source parallel to series equivalent model is shown in Fig. 2.1 which consists of two components: a renewable or fossil fuel source and an energy storage voltage source. The series model of the two sources is obtained by Norton/Thevenin equivalent calculations and variable change to
Figure 2.1: Input source and energy storage conversion of parallel to series equivalent model.

\[ v = \frac{R_u}{R_u + R_v} v_v \]  

\[ u = \frac{R_v}{R_u + R_v} u_u \]  

\[ R_{dc} = \frac{R_u R_v}{R_u + R_v} \]

where \( v, u \) and \( R_{dc} \) represent the energy source, storage device and series dc resistance.

Here, it is important to consider the properties for each component; component \( u \) has bi-directional power flow ability and may contain internal bi-directional dc-dc converters as opposed to \( v \) that can be a uni-directional fossil fuel or solar panel source. The overall series form of this hybrid source model allows utilization of HSSPFC method.
2.4 Microgrid Model in DQ Coordinates

The microgrid system demonstrated in Fig. 2.2 comprise a Distributed Generation Unit (DGU) connected to an RLC load. Below is the state-space representation of the system demonstrated in Fig. 2.2.

\[
\begin{align*}
L \frac{di_d}{dt} &= -Ri_d + \omega Li_q + v_d - v_{db} \quad (2.9a) \\
L \frac{di_q}{dt} &= -Ri_q - \omega Li_d + v_q - v_{qb} \quad (2.9b) \\
C_{dc} \frac{dv_{dc}}{dt} &= \frac{(v + u - v_{dc})}{R_{dc}} - \beta \lambda \left( \cos(\phi) i_d + \sin(\phi) i_q \right) \quad (2.9c) \\
C_b \frac{dv_{db}}{dt} &= i_d - \frac{v_{db}}{R_b} + \omega C_b v_{qb} - i_{dLb} \quad (2.9d) \\
C_b \frac{dv_{qb}}{dt} &= i_q - \frac{v_{qb}}{R_b} - \omega C_b v_{db} - i_{qLb} \quad (2.9e) \\
L_b \frac{di_{dLb}}{dt} &= -R_b i_{dLb} + \omega L_b i_{qLb} + v_{db} \quad (2.9f) \\
L_b \frac{di_{qLb}}{dt} &= -R_b i_{qLb} - \omega L_b i_{dLb} + v_{qb} \quad (2.9g)
\end{align*}
\]
where,
\[
\begin{bmatrix}
v_d \\
v_q
\end{bmatrix} = \beta \lambda v_{dc} \begin{bmatrix}
cos(\phi) \\
sin(\phi)
\end{bmatrix}
\]  \hspace{1cm} (2.10)

Hence, the compact HSSPFC representation is,

\[
M \dot{x} = Rx + B^T u + D^T v,
\] \hspace{1cm} (2.11)

where,
\[
x = \begin{bmatrix}
i_d \\
i_q \\
v_{dc} \\
v_{db} \\
v_{qb} \\
i_{dLb} \\
i_{qLb}
\end{bmatrix}^T \hspace{1cm} (2.12a)
\]
\[
u = u \hspace{1cm} (2.12b)
\]
\[
v = v \hspace{1cm} (2.12c)
\]

and,
\[
M = \begin{bmatrix}
L & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{dc} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & C_b & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C_b & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & L_b & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & L_b
\end{bmatrix}, \hspace{1cm} (2.13)
\[
R = \begin{bmatrix}
-R & \omega L & \beta \lambda \cos \phi & -1 & 0 & 0 & 0 \\
-\omega L & -R & \beta \lambda \sin \phi & 0 & -1 & 0 & 0 \\
-\beta \lambda \cos \phi & -\beta \lambda \sin \phi & -1/R_{dc} & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1/R_b & \omega C_b & -1 & 0 \\
0 & 1 & 0 & -\omega C_b & -1/R_b & 0 & -1 \\
0 & 0 & 0 & 1 & 0 & -R_b & \omega L_b \\
0 & 0 & 0 & 0 & 1 & -\omega L_b & -R_b \\
\end{bmatrix}, \quad (2.14)
\]

\[
B^T = \begin{bmatrix} 0 & 0 & \frac{1}{R_{dc}} & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad (2.15)
\]

\[
D^T = \begin{bmatrix} 0 & 0 & \frac{1}{R_{dc}} & 0 & 0 & 0 & 0 \end{bmatrix}^T. \quad (2.16)
\]

The above system can further be analyzed for stability and performance using the HSSPFC [12, 16, 17].

### 2.5 Summary and Conclusions

In this chapter, the state-space model of the reduced-order microgrid with its inverter and ESSs was reformed according to the HSSPFC method. For the rest of this dissertation, the combination of source, ESS, inverter and its filter/transmission line is referred to as the Distributed Generation Unit (DGU) or the local microgrid system.
Here, a single DGU connected to a load was presented for the sake of simplicity, however, there are typically multiple DGUs in a microgrid system. In the next chapter, a droop control is developed to be used for sharing the load power between DGUs. In chapters 4 and 6, the overall control systems will be derived based on the model that was presented in this chapter.
Chapter 3

DC Droop control and Droop Control in DQ Coordinates for Three-Phase Distributed Systems

3.1 Introduction

Distributed operation of DGUs is possible using droop control methods. For ac systems, the conventional PQ droop control scheme allows sharing of active and reactive powers by manipulating frequency and amplitude respectively [4, 19]. This method takes advantage of the fact that system frequency is a global entity. While PQ control
is effective and well-implemented, it might not provide the best solution for inverter-based microgrids where no inertia is present.

In this chapter a novel dq droop for three-phase ac systems is presented. The overall droop scheme enables active and reactive power sharing without altering the system frequency at control system level. The control is based on the dc droop method which will be reviewed in the next section.
3.2 DC droop control

Considering Fig. 3.1, droop control utilizes virtual impedances between the source and the bus to distribute the load power by creating a weighted sum of powers based on the criteria set by the droop settings. In a one common-bus system, typically, this power sharing reduces to simple sharing of load currents since the bus voltage is a common entity. Hence, the currents are shared according to,

\[ I_{d,i} = \frac{(V_{d,i} - V_b)}{R_{d,i}} = \gamma_i I_b, \quad (3.1) \]
where, \( V_{d,i} \) and \( R_{d,i} \) are droop settings. \( \gamma_i \) represents the shared current weightings and should meet (0,1) criteria. Hence, the powers can be shared according to,

\[
P_i = \gamma_i V_i I_b = \gamma_i P_b, \quad \text{where} \quad \sum_{i} \gamma_i = 1.
\] 

(3.2)

### 3.3 DQ droop control

Droop control in dq coordinates is shown in Fig. 3.3 with droop settings specified in Fig. 3.4. Here, the currents corresponding to dq components are shared according to,

\[
I_{d,i} = \frac{V_{d,i} - V_{db}}{R_{d,i}} = \gamma_i I_{db} \tag{3.3a}
\]

\[
I_{q,i} = \frac{V_{q,i} - V_{qb}}{R_{q,i}} = \gamma_i I_{qb}. \tag{3.3b}
\]

Where, \( V_{d,i} \) and \( V_{q,i} \) are the d and q components droop voltage settings. \( R_{d,i} \) and \( R_{q,i} \) are the droop slope settings. \( I_{d,i} \) and \( I_{q,i} \) are the corresponding currents injected into dq buses. \( I_{db} \) and \( I_{qb} \) are the load d and q currents respectively. \( V_{db} \) and \( V_{qb} \) are the measured bus voltages. Here, the synchronization can be done using conventional PLL systems or GPS and GPS disciplined oscillators [20][21].

Considering Fig. 3.4, each d and q subsystems can perform dc droop control [3]. If
$I_d$ and $I_q$ are determined, the real and reactive powers, $P$ and $Q$ can be obtained according to,

\begin{align}
  P_i &= V_{db}I_{d,i} + V_{qb}I_{q,i} = \gamma_i P_b \\  Q_i &= V_{qb}I_{d,i} - V_{db}I_{q,i} = \gamma_i Q_b.
\end{align}  \tag{3.4a}  \tag{3.4b}

In a hierarchical control system, generally, the droop settings $V_{d,i}$, $V_{q,i}$, $R_{d,i}$ and $R_{q,i}$ can be obtained from a secondary controller to improve regulation of $V_{db}$ and $V_{qb}$ which represent the three-phase bus voltage.
3.4 Droop Parameter Setting using Curve Shift and Slope Change

Considering droop control for a parallel two source system as shown in Fig. 3.4, if \( V_{db}^* \) and \( V_{qb}^* \) denote the desired d and q bus voltages, droop settings can be obtained from,

\[
\begin{align*}
V_{d,\text{ref},1} &= R_{d,1} \gamma_1 \frac{V_{db}}{R_B} + V_{db}^* \\
V_{d,\text{ref},2} &= R_{d,2} \gamma_2 \frac{V_{db}}{R_B} + V_{db}^* \\
V_{q,\text{ref},1} &= R_{q,1} \gamma_1 \frac{V_{qb}}{R_B} + V_{qb}^* \\
V_{q,\text{ref},2} &= R_{q,2} \gamma_2 \frac{V_{qb}}{R_B} + V_{qb}^*,
\end{align*}
\]

where, \( V_{db} \) and \( V_{qb} \) are obtained from local bus measurements. Generally, there are two approaches to weight individual d and q currents; curve shifting and slope changing [22]. In this dissertation, curve shifting droop is implemented where droop slope settings \( R_{d,1}, R_{d,2}, R_{q,1} \) and \( R_{q,2} \), and weightings \( \gamma_1 \) and \( \gamma_2 \) are chosen as constant values. This turns the system in (3.5) into a four equation four variable problem to obtain \( V_{d,\text{ref},1}, V_{d,\text{ref},2}, V_{q,\text{ref},1} \) and \( V_{q,\text{ref},2} \).
In slope changing method, droop voltage setting relationships can be defined as,

\[ V_{d,\text{ref},1} = V_{d,\text{ref},2} = \hat{V}_{d,\text{ref}} \]  
\[ V_{q,\text{ref},1} = V_{q,\text{ref},2} = \hat{V}_{q,\text{ref}}, \]  
\[ (3.6a) \]
\[ (3.6b) \]

where, \( \hat{V}_{d,\text{ref}} \) and \( \hat{V}_{q,\text{ref}} \) are constant values. By substituting (3.6) into (3.5), a four equation four variable system is obtained where,

\[ R_{d,1}\gamma_1 = R_{d,2}\gamma_2 \]  
\[ (3.7a) \]
\[ R_{q,1}\gamma_1 = R_{q,2}\gamma_2. \]  
\[ (3.7b) \]

By choosing \( R_{d,1}, R_{d,2}, R_{q,1} \) and \( R_{q,2} \), the current and power weightings \( \gamma_1 \) and \( \gamma_2 \) are obtained accordingly.

### 3.5 Summary and Conclusions

In this chapter, the dq droop control method was presented. It was shown that the control mimics the behavior of the dc droop for each d and q subsystems. As a result, active and reactive powers were shared according to the droop settings. In chapters 4 and 6, the system shown in 2.2 will be controlled by using the dq droop as the primary control system.
Chapter 4

Energy Storage Requirements for Inverter-Based Microgrids Under Droop Control in DQ Coordinates

This chapter will investigate the ES requirements for ac microgrid systems that utilize the dq droop control method. dq droop control uses a fixed frequency 0-d-q transformation to allow accurate sharing of active/reactive power in decentralized or distributed control schemes. The ES combination with the microgrid system is modeled in the form of HSSPFC. Here, the system references are obtained through the dq droop control and the ES devices behave as the sole actuators of the system to enforce reference points. Simulation examples of a common-bus system as well as a
looped WSCC, 9-bus microgrid system are presented.

4.1 Introduction

The next sections present requirements for a combination of ES technologies that are used in inverter-based three-phase renewable source systems. Here, the control of the inverters are performed by implementing the dq droop control method presented in the previous chapter. In conventional ac droop control method, the active and reactive powers are shared between generation units by altering frequency and voltage level respectively [23]. In contrast, dq voltage droop control method makes use of a dc approach to accurately share active and reactive powers without altering the system frequency. In this approach the system frequency is determined by an externally generated reference signal that maintains the DGU synchronization by using the Global Positioning System (GPS) and GPS-disciplined oscillators which produce accurate time references [20, 21, 24].

To implement the dq droop control method the DGU model is transformed [25] into a 0dq equivalent model based on [5] which provides a valuable catalog for modeling and control of distributed microgrids in dq coordinates. For control system development, [11] suggests that the distributed generation can be obtained by making use of control architecture that includes a feed-forward, a Hamiltonian-based feedback and a servo...
control that supports the Hamiltonian-based control. In this chapter, the design of the controller includes ES feed-forward and feedback control laws and an inverter feed-forward control law. Here, droop control sets the DGU current references which are enforced through inverter feed-forward control of inverter command inputs. The ES feed-forward control is chosen so that the ES is requested in a minimal form. The feedback control law consists of a proportional and an integral action control to maintain the voltage in the input point of the inverter.

This chapter expands on [26] and is organized as follows. First, the Hamiltonian control is developed for the inverter and ES element u. Then, the droop control method is discussed. The performance of the overall control combination with the dq droop control is demonstrated in a load step-change simulation for parallel DGU systems. ES requirements versus the communication network update-rate for a WSCC 9-bus microgrid system is also presented.
4.2 Hamiltonian Control Development

4.2.1 Three-Phase Inverter Feed-forward Control

For the system presented in (2.9), the reference state space expression is defined by

\[ M \dot{x}_{ref} = Rx_{ref} + B^T u_{ref} + D^T v. \]  \hspace{1cm} (4.1)

Considering the steady-state solution, the above expression defines system constraints for specific nominal values from which, \( i_{d,ref}/i_{q,ref} \) are set by the dq droop control and \( v_{ds,ref}/v_{qs,ref} \) are obtained from droop settings calculations. In expanded systems that utilize multi-level hierarchical control schemes, the latter is typically set from a secondary controller which maintains the voltage and current levels of the microgrid network [6].

From (4.1), the steady-state feed-forward expression for the dc-side reference voltage, \( v_{dc,ref} \) is obtained as

\[ v_{dc,ref} = v - R_{dc} \beta \lambda(\cos(\phi)i_{d,ref} + \sin(\phi)i_{q,ref}), \]  \hspace{1cm} (4.2)

so that the ES operates at zero output condition. It is notable that the overall value for
dc-side reference voltage can be tuned by considering the ES and system limitations presented in (4.18). $v_{dc,ref}$ is enforced through the ES Proportional-Integral (PI) feedback control and a steady-state feed-forward operation which will be defined in the next sub-section.

The inverter output feed-forward expression is defined as

$$v_d = R_i d, ref - \omega L_i q, ref + v_{ds}$$  
$$(4.3a)$$

$$v_q = R_i q, ref + \omega L_i d, ref + v_{qs}. \quad (4.3b)$$

The RHS of the expression includes a coupled expression for the $RL$ filter/transmission line and a point of measurement which is represented by $v_{ds}$ and $v_{qs}$. Here, the control expression is derived according to the exact model of the system. In practical implementation of dq controlled inverters, to compensate for uncertainty for component values of the ac-side, PI feedback control loops can be designed to enforce $i_{d,ref}/i_{q,ref}$ for the dq sub-circuits through control of $v_d/v_q$. In a feedforward-only operation however, the inverter input commands, $\lambda$ and $\phi$ appear in the skew symmetric matrix and are eventually removed from the system stability criteria [12].

The feedforward-only operation prevents possible cross-talk between multiple parallel sources. Furthermore, the appropriate use of a higher order compensator such as PI feedback controller would decrease the energy trade with the ES elements. Here, to keep the stress on the ES element, no feedback loops are used for any controller other
than the ES element.

From (4.3), Substituting \( v_d \) and \( v_q \) according to (2.3) yields

\[
\beta \lambda v_{dc} \cos(\phi) = R_{i,d,ref} - \omega L_{i,q,ref} + v_{ds} \quad (4.4a)
\]

\[
\beta \lambda v_{dc} \sin(\phi) = R_{i,q,ref} + \omega L_{i,d,ref} + v_{qs}, \quad (4.4b)
\]

hence, \( \phi \) is obtained from

\[
\tan(\phi) = \frac{R_{i,q,ref} + \omega L_{i,d,ref} + v_{qs}}{R_{i,d,ref} - \omega L_{i,q,ref} + v_{ds}}. \quad (4.5)
\]

\( \lambda \) is obtained by substituting \( \phi \) in any of the sub-equations of (4.4) that meets \((0,1)\) range.

4.2.2 Energy Storage Feed-forward and Feedback Control

In order to define a PI loop for the ES feedback control, the error state for the microgrid system is defined as

\[
\tilde{x} = x_{ref} - x. \quad (4.6)
\]
As mentioned earlier, it is assumed that the ES element is the only actuator of the system. Therefore, the error state of interest is

\[ e = v_{dc,ref} - v_{dc}, \]  

(4.7)

and the PI feedback for \( u \) is

\[ \Delta u = -k_pe - k_i \int_0^t e d\tau, \]  

(4.8)

where \( k_p \) and \( k_i \) are gains of the PI feedback controller. The feed-forward control is obtained from the last row of the steady-state solution of (4.1),

\[ u_{ref} = R_{dc} \beta \lambda (\cos(\phi) i_{d,ref} + \sin(\phi) i_{q,ref}) - v + v_{dc,ref}, \]  

(4.9)

where, \( i_{d,ref} \) and \( i_{q,ref} \) are set by the droop control. The overall feed-forward and feedback expression for ES is

\[ u = u_{ref} - \Delta u \]

\[ = R_{dc} \beta \lambda (\cos(\phi) i_{d,ref} + \sin(\phi) i_{q,ref}) - v + v_{dc,ref} \]

\[ + k_pe + k_i \int_0^t e d\tau. \]  

(4.10)
Defining the dc-link current as,

\[ i_{dc-link} = \frac{v + u - v_{dc}}{R_{dc}}, \]  

(4.11)

the energy discharge of the ES component is

\[ W_u = \frac{1}{3.6 \times 10^6} \int_0^t i_{dc-link} d\tau u, \]  

(4.12)

where, the unit for \( W_u \) is kWh. The dc power of the ES component, \( u \) is obtained as

\[ P_u = u i_{dc-link}. \]  

(4.13)

### 4.3 Energy Storage and Inverter Control

The control law for ES feedback and inverter \( \lambda \) and \( \phi \) feed-forward is developed for the model demonstrated in Fig. 2.2. In this chapter, the control law is formulated so that ES element \( u \) is kept at a steady-state value of zero at different levels of renewable voltage source.
Considering (2.9) or the dc source loop of Fig. 2.2,

\[ v + u - iR_{dc} = v_{dc}, \]  

(4.14)

and defining a nominal voltage \( v^* \) for the renewable source \( v \), the deviation from this nominal value is

\[ e = v - v^*. \]  

(4.15)

One can eliminate the effects of \( e \) by changing the variable \( u \) to

\[ u = u^* + e, \]  

(4.16)

or change the variable \( v_{dc} \) to

\[ v_{dc} = v_{dc}^* - e, \]  

(4.17)

or both so that (4.14) becomes

\[ v^* + u^* + pe - iR_{dc} = v_{dc}^* - (1 - p)e, \]  

(4.18)

where, \( p \) is a probabilistic entity. This expression facilitates the implementation of optimization schemes with constraints that are defined by voltage set point limitations.
at each node of the dc-side system of the model in Fig. 2.2. According to (4.18), $u$ and $v_{dc}$ set points can be changed so that the unwanted deviation in the renewable input source is eliminated however, $p$ as a weighting parameter can determine the priority for each set point. With $p = 1$, the traded energy with the ES component is maximum while a system with zero value for $p$ is subjected to the limitation of the dc port of the inverter. However, the latter minimizes the energy trade with the ES element. Indeed a dc-dc power conversion stage with its input/output filters between the source/ESS link and the dc port of the inverter can be added but it may affect the frequency response requirements of the ES element. Here, to keep the concentration on ES element, a wide voltage range in input port of the inverter is assumed for development of the inverter control law.

In this section, first, the ES control law is developed using the steady-state reference state space for (7.16), then a $(\lambda, \phi)$ feed-forward control expression is presented.

### 4.4 Droop Control and Settings

#### 4.4.1 Droop Control

dq transformation enables the utilization of dc droop technique for control of an ac microgrid. The dc droop control implementation is explained in detail in [12].
Considering the system demonstrated in Fig. 3.3, the following holds

\[ I_{d,i} = \frac{V_{d,i} - V_{ds}}{R_{d,i}} \]  
\[ I_{q,i} = \frac{V_{q,i} - V_{qs}}{R_{q,i}} \]  

(4.19a)  
(4.19b)

where, \( V_{d,i} / V_{q,i} \) are the droop voltage settings, \( V_{ds} / V_{qs} \) are the common-bus dq voltages, \( I_{d,i} / I_{q,i} \) are the dq currents injected into dq buses and \( R_{d,i} / R_{q,i} \) are the droop virtual resistances. \( V_{d,i} \) and \( V_{q,i} \) can be chosen so that \( I_d \) and \( I_q \) components are shared between the sources hence, active and reactive powers, \( P_i \) and \( Q_i \) obtained from (2.4) and (2.5) are shared accordingly.

### 4.4.2 Droop Settings

Assuming that the load information is known, considering Fig. 3.3, the dq load currents can be shared according to

\[ I_{d,i} = \gamma_i I_{db} = \frac{V_{d,i} - V_{ds}}{R_{d,i}} \]  
\[ I_{q,i} = \gamma_i I_{qb} = \frac{V_{q,i} - V_{qs}}{R_{q,i}} \]  

(4.20a)  
(4.20b)
where, $\gamma$ is the weighting parameter. Considering a nominal bus voltage represented by $V_{ds}^*$ and $V_{qs}^*$, then the droop voltage setting $V_{d,i}$ and $V_{q,i}$ are obtained as

\begin{align}
V_{d,i} &= I_{d,i} R_{d,i} + V_{ds}^* \\
V_{q,i} &= I_{q,i} R_{q,i} + V_{qs}^*.
\end{align}

The choice of substituted expressions for $I_{d,i}$ and $I_{q,i}$ depends on the available information about the local or global system. Here, the deviation between implementing a centralized, decentralized or distributed control scheme is highlighted. Considering this expression for Fig. 3.3, a system designed with an accurate load current information would result in an accurate current and power sharing between the DGUs. On the other hand, in a case that partial load information is available, the designer can implement local tracking controllers to set $V_{d,i}$ and $V_{q,i}$ so that the bus voltages represented by $V_{ds}$ and $V_{qs}$ are in acceptable preset ranges. In this case, the local tracking needs to be set carefully since KVL problems and stability issues may occur when voltage regulation is performed through parallel controllers. Here, the stability issues are handled by the HSSPFC and ES element and with inappropriate control, performance may suffer and the ES may not always return to zero level.

In this work, the choice of $V_{ds}^*/V_{qs}^*$ and the substituted expressions for $I_{d,i}/I_{q,i}$ are obtained from a simplified secondary controller which solves a power flow problem of the system and provides constant nominal values for each DGU. The design of
a secondary controller with dynamic action capability is out of the scope of this dissertation.

The controller parameters such as PI gains or virtual resistances, $R_{d,i}$ and $R_{q,i}$ should be carefully set as several local or global constraints may exist for various system configurations [27]. For example, in (4.21), considering the implementation of two local parallel inverters, each with equal values of $V'_d$'s and $V'_q$'s, alongside weighted $R_d$ and $R_q$ sets, will result in an effective current sharing which will require minimum load information or communication infrastructure, however, such droop settings may compromise the full-rate dispatch capability of the inverter systems.

4.5 Simulation Results

In this section, two test cases are simulated in Modelica [28]. First, the effects of a load step-change on two inverter-based sources with constant dc inputs are presented. This case would highlight the ES requirement of the system while a perturbation is introduced in the inverter ac-side where the steady-state behavior explicitly depends on the dq droop control method and its settings.

The second case investigates the effects of feed-forward update rate values for a WSCC 9-bus looped microgrid system. For this case, the system has three ideal voltage
sources of wind and solar trend and three local ES elements for the dq-controlled inverters. Here, an update rate sweep of the control network is performed to obtain the overall traded energy with the storage components for a WSCC, 9-bus microgrid system.

For all the cases, for each inverter source, the systems subjected to test events, are allowed to reach steady-state before introducing the subsequent events. For each case, simplified secondary controllers have been used to supply nominal bus values for each DGU. For the first case, the nominal values are set so that a nominal voltage at load bus is maintained and for the second case, values are set so that active and reactive powers are shared according to an arbitrary ratio. It is assumed that the aforementioned controllers have same update rate as the feed-forward control networks.

Table 4.1
Load Step Case Microgrid Parameters

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<td>$L(\mu H)$</td>
<td>$R(\Omega)$</td>
<td>$C_{dc}(mF)$</td>
<td>$R_{dc}(\Omega)$</td>
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<tr>
<td>DGU 1</td>
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<td>$C_B(\mu F)$</td>
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<td>$L_{line}(mH)$</td>
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<td>2.1</td>
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<tr>
<td>$RL\ Line_2$</td>
<td>0.19</td>
<td>2</td>
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</table>

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4.5.1 Load Step-Change Response

The variable load test case is implemented on the expanded system demonstrated in Fig. 4.1 with the parameters shown in Table 4.1 where both inverters are connected to a common bus represented by $v_{ds}$ and $v_{qs}$. $v_{db}$ and $v_{qb}$ represent the inverter-based source bus voltages which are the droop control local points of measurement.

Fig. 4.2 shows the step-change from 2 $\Omega$ to 5 $\Omega$ and the ES components $u_1$ and $u_2$ in a window of 1 s. During the simulation time, the droop settings are chosen so that a 1 : 2 current sharing ratio is maintained between inverter 1 and 2 respectively.

The feed-forward update rate of the control network for the ES element and the
Figure 4.2: (a) Load is changed from $2 \, \Omega$ to $5 \, \Omega$ at $t = 0.5 \, s$. Responses of energy storage elements for (b) $u_1$ and (c) $u_2$.

Figure 4.3: Current Share for (a) DGU 1, (b) DGU 2 and (c) DGU 3. The load is stepped up from $2 \, \Omega$ to $5 \, \Omega$ at $t = 0.5 \, s$. 
Figure 4.4: Response for update rate of 40ms of control network of inverters 1 and 2, (a) Inverter dc voltages, (b) Input voltage amplitude commands and (c) Input voltage phase commands.

Figure 4.5: (a) Inverter DC-side current. (b) Load side bus voltage.
inverter command inputs are 40 ms. It can be seen that the decrease in load power momentarily supplies power into the ES elements, then the periodically updated ES and inverter feed-forward alongside the ES feedback controller recover the ES voltage to the zero reference level.

The shared three-phase currents are shown in Fig. 4.3 Fig. 4.4 presents the change in the inverter command inputs and dc input voltages while maintaining the current sharing demonstrated in Fig. 4.3 Fig. 4.5a shows the dc currents for each inverter and Fig. 4.5b demonstrates the maintained load voltage for phase a.

4.5.2 WSCC 9-bus DGU Control Network Update-Rate Sweep

This test case demonstrates the ES requirements of the droop controlled system against a sweep of feed-forward update rates for three control networks of three DGUs. A WSCC system, shown in Fig. 4.6 has two solar sources, a wind energy source, six transmission lines that supply power to three RC loads. Here, it is assumed that the control networks for all of the DGUs are operating at the same update rate.

The system subject to the three emulated solar and wind sources that are shown in Fig. 4.7 is simulated under the feed-forward update rate sweep of 10 ms to 80 ms.
Figure 4.6: WSCC 9-bus test system single-line diagram. It consists of three sources, three loads and six transmission lines. The transformer ratios are chosen as 1:1 and tap changes and impedances are neglected; buses 7, 8 and 9 are ignored.

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<th>( R(\Omega) )</th>
<th>( C_{dc}(\mu F) )</th>
<th>( R_{dc}(\Omega) )</th>
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<td>100</td>
<td>0.1</td>
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<td>DGU 3</td>
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<table>
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<th>( C(\mu F) )</th>
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</thead>
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<td>72</td>
</tr>
<tr>
<td>Load b</td>
<td>5</td>
<td>56</td>
</tr>
<tr>
<td>Load c</td>
<td>4</td>
<td>47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line Parameters</th>
<th>( R_{line}(\Omega) )</th>
<th>( L_{line}(mH) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.4 1.6 3.6 3.5 2.4 2.5</td>
<td>4.05 4.15 4.03 3.95 4.2 4</td>
</tr>
</tbody>
</table>

The load buses A, B and C are stepped every two hours as shown in Fig. 4.7:

The droop settings for all the local microgrids are set so that a 2 : 3 : 5 ratio for both active and reactive powers is maintained from 8 a.m. to 6 p.m. Fig. 4.8 demonstrates the ES currents for each inverter for the update rate of 80 ms.
Figure 4.7: Wind and solar emulated voltages from 8 a.m. to 6 p.m. for three source buses, a) Bus 1, b) Bus 2 and c) Bus 3 and d) load step-changes. To investigate the performance of the system, additional stochastic step signals are added to the overall trend of the dc sources.

The supplied energy by each ES component is shown in Fig. 4.9. Here, for the three cases, the trend of supplied energy is as expected; as the renewable input voltage increases, the ES elements store the excess energy and supply it back when the input voltages decrease.

Fig. 4.10 shows the real power sharing among the inverter-based sources. For first two hours, the obtained results are $P_1 \approx 5.2\ kW$, $P_2 \approx 13\ kW$ and $P_3 \approx 7.8\ kW$ which demonstrate the effective performance of the dq droop control method. Fig. 4.11 also, shows the reactive power sharing with $Q_1 \approx 0.48\ kVar$, $Q_2 \approx 1.2\ kVar$ and $Q_3 \approx 0.72\ kVar$ which meet the assigned ratio for reactive power sharing.
Figure 4.8: WSCC test system energy storage currents for the case when the update rate is 80 ms. (a) Wind DGU at bus 1, (b) Solar DGU at 2, (c) Solar DGU at bus 3.

Figure 4.9: WSCC system supplied energy, when the update rate is 80 ms. (a) Wind DGU at bus 1, (b) Solar DGU at bus 2, (c) Solar DGU at bus 3.
Figure 4.10: WSCC test system real power sharing for the case when the update rate is 80 ms. (a) Wind DGU at bus 1, (b) Solar DGU at bus 2, (c) Solar DGU at bus 3.

Figure 4.11: Reactive power sharing for when the update rate is 80 ms. (a) DGU 1, (b) DGU 2, (c) DGU 3.
Fig. 4.12 shows the overall traded energy with each storage component against the control network feed-forward update rate time. It is observed that as the update rate time increases, the requested energy increases in a concave curve form. This concave increase is observed in all the three ESSs. It is also verified that any addition of further inverter-based sources with their own filter/transmission lines follow this trend. Fig. 4.12 also shows that the overall energy requirement increases by 12 times, from 1 kWh to 12 kWh for update rate of 20 ms and 70 ms respectively. These requirements can be tuned by changing parameters such as the ES feedback gains. The variation in PI gains will also change the ES frequency response requirements.

The WSCC storage requirements in the form of Power Spectral Densities (PSD) are shown in Fig. 4.13. It can be observed that the periodically updated control networks
Figure 4.13: The Power Spectral Densities (PSDs) for (a) $u_1$, (b) $u_2$ and (c) $u_3$, when the update rate is 80 ms.

(at the rate of 80 ms) create harmonics that manifest in the form of repetitive modes of 12.5 Hz apart. This shows that the update rate has significant effect on the ES frequency response which should be taken into consideration when choosing the most appropriate ES technology.

4.6 Summary and Conclusions

In this chapter, the energy storage requirements for a dq modeled microgrid that makes use of the dq droop controller is demonstrated. The control network consists of four parts. First, the dq controller that sets local system references. Second, the inverter ($\lambda, \phi$) feed-forward control that enforces the set references. Third, the
ES feed-forward control that is formulated so that the traded energy is of minimal amount and Fourth, the ES feedback proportional and integral action control that maintains the inverter input voltage levels and ensures stability of the network. To verify the performance of the system, two test cases were considered. First, a simple load step-change was devised to demonstrate the storage requirements against the DGU load-side perturbations. The second case demonstrated the energy trade with the ES elements against the sweep of the control network update rate.

The future work will include a more complete hierarchical controller, ES coordination based on renewable technology or ES technology and implementation of optimization schemes to improve the transient behavior.
Chapter 5

Energy Storage Systems and Control: Band-limited, Battery and Flywheel Models

5.1 Introduction

There are two main factors of consideration for ES devices. One is the rate of charge or discharge of energy and the other is the energy amount that can be stored. The state of art technologies store energy by conversion of electrochemical, kinetic, potential and electromagnetic energies \[29\]. The power density (in W/kg), energy density (in
Wh/kg), cycle efficiency, lifetime, cycle life and self-discharge are the characteristics with which the specific storage technologies can be described \cite{30}. In this dissertation, battery and flywheel storage systems are considered to fulfill the energy density and power density requirements respectively.

In this chapter, it will be shown that ESSs such as batteries and flywheels can be represented as band-limited filters. The objective is that the band-limited storage element can then be used to define power and energy requirements for the DGU presented in \hyperref[2.2]{2.2}.

\section{Band-limited Storage}

The operation bandwidth of ES devices is limited. Generally, this limitation can be shown in the form of a Low Pass Filter (LPF) \cite{31, 32}. The cut-off frequency of this LPF can depend on various aspects; the ES technology, control and specifications can define the overall frequency response. Here, \( u \) represents the ESS control command while the injected current into the bus is defined by \( u_f \) as,

\[
\frac{du_f}{dt} = \omega_{cut-off}(u - u_f)
\]  

Fig. \ref{fig:5.1} demonstrates the gain versus the frequency for (5.1) when \( \omega_{cut-off} = \)
Figure 5.1: First-order LPF when $\omega_{\text{cut-off}} = 100000 \text{ rad/s}$.

Figure 5.2: Overall form of flywheel and battery models

100000 rad/s. In this section, a flywheel and a battery system model with their overall frequency responses are presented. For both devices, the overall topology is as shown in Fig. 5.2.
5.3 Flywheel System and Control

A general reduced-order flywheel energy storage model is shown in Fig. 5.3. The flywheel system descriptions and parameters for the reduced-order flywheel device are given in Table. 5.1. This model contains a spinning mass flywheel, Permanent Magnet (PM) dc machine and a dc-dc converter to interface with the load bus.

There are several assumptions for this model. Switching effects are ignored and converter model is average mode with control input duty cycle $\lambda_u$. The electrical machine here is a permanent magnet dc machine in reduced-order for simplicity. Typically this machine would be a 3-phase induction machine or switched reluctance...
Table 5.1
Flywheel Cell System and Control Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_f$</td>
<td>Moment of Inertia</td>
<td>0.018 kg m²</td>
</tr>
<tr>
<td>$k_t$</td>
<td>Torque Constant</td>
<td>1 Nm/A</td>
</tr>
<tr>
<td>$R_{pm}$</td>
<td>Armature Resistance</td>
<td>0.05 Ω</td>
</tr>
<tr>
<td>$L_{pm}$</td>
<td>Armature Inductance</td>
<td>10 mH</td>
</tr>
<tr>
<td>$C_u$</td>
<td>Converter Capacitance</td>
<td>1000 μF</td>
</tr>
<tr>
<td>$R_{Cu}$</td>
<td>Converter Resistance</td>
<td>10 kΩ</td>
</tr>
<tr>
<td>$L_u$</td>
<td>Line Inductance</td>
<td>10 mH</td>
</tr>
<tr>
<td>$R_u$</td>
<td>Line Resistance</td>
<td>0.01 Ω</td>
</tr>
<tr>
<td>$B$</td>
<td>Friction Coefficient</td>
<td>0.001 Nm/rad/s</td>
</tr>
</tbody>
</table>

Control Gains

| $k_i$     | Bus current integral gain | 10            |
| $k_p$     | Bus current proportional gain | 1            |

The minimum speed of the flywheel to support a voltage yields,

$$e_{pm} = k_t \omega_f(t) \geq v_{bus}, \forall t.$$  \hspace{1cm} (5.2)

Therefore a buck converter in source mode as shown in Fig. 5.3 is used as the bus interface. The energy stored is

$$W_f = \frac{1}{2} J_f \omega_f(t)^2.$$  \hspace{1cm} (5.3)
Hence, the minimum energy stored in the device is,

\[ W_{f,\text{min}} = \frac{1}{2} J_f \left( \frac{v_{\text{bus}}}{k_t} \right)^2. \]  \hspace{1cm} (5.4)

The overall power losses in the device is,

\[ P_{\text{loss}} = R_{pm} i_{pm}^2(t) + R_{Lu} i_u^2(t) + v_u^2(t) \frac{1}{R_{Cu}} + B\omega_f^2(t). \]  \hspace{1cm} (5.5)
Electrical torque from the PM machine is \( \tau_{pm} = k_t i_{pm}(t) \) and \( e_{pm} = k_t w_f(t) \) is the speed voltage generated by the mechanical moment. The overall flywheel model is,

\[
J_f \frac{d\omega_f}{dt} = -B\omega_f(t) - k_t i_{pm}(t) \tag{5.6a}
\]

\[
L_{pm} \frac{di_{pm}}{dt} = k_t \omega_f(t) - R_{pm} i_{pm}(t) - v_u(t) \tag{5.6b}
\]

\[
C_u \frac{dv_u}{dt} = -\frac{v_u(t)}{R_{Cu}} + i_{pm}(t) - \lambda_u i_u(t) \tag{5.6c}
\]

\[
L_u \frac{di_u}{dt} = -v_{bus} - R_{Lu} i_u(t) + \lambda_u v_u(t), \tag{5.6d}
\]

and the converter control and constraints are,

\[
e_p = i_{u,ref}(t) - i_u(t) \tag{5.7a}
\]

\[
\frac{de_i}{dt} = e_p \tag{5.7b}
\]

\[
\lambda_u = k_i e_i + k_p e_p \tag{5.7c}
\]

\[
0 \leq \lambda_u \leq 1, \tag{5.7d}
\]

where, \( \lambda_u \) is the converter control command under a PI control to enforce \( i_{u,ref} \), the reference current for the front-side injected current. The effectiveness of tracking the reference current however, depends on the response of the system. The overall frequency response for the band-limited flywheel storage system in [5.6] with its control in [5.7] is shown in Fig. 5.4.
5.4 Battery System and Control

A generalized reduced-order battery and converter model is shown in Fig. 5.5. Relevant system parameter descriptions are presented in Table 5.2. In this model, switching effects are ignored and converter model is the average mode with control input duty cycle $\lambda_u$, and $v_{batt} < v_{bus}$. $R_{c1}$ is very large, $R_{c2}$ is small.

We can measure the energy discharged from the battery in terms of the sum of charge provided over some period of time,

$$Ah = \frac{\int_{0}^{t} i_{batt}(\tau) d\tau}{3600 \frac{sec}{hr}}.$$  \hspace{1cm} (5.8)

A battery will have a maximum amount of charge storage capacity $(Ah)_{capacity}$. A
Table 5.2
Battery Cell System and Control Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{oc}$</td>
<td>Open Circuit Voltage</td>
<td>48 V</td>
</tr>
<tr>
<td>$Q$</td>
<td>Max charge capacity</td>
<td>10 $A.Hr$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>Electrochemical Polarization Capacitance</td>
<td>750 $F$</td>
</tr>
<tr>
<td>$R_{c1}$</td>
<td>Electrochemical Polarization Resistance</td>
<td>10 $K\Omega$</td>
</tr>
<tr>
<td>$L$</td>
<td>Equivalent Series Inductance</td>
<td>0.17 $\mu H$</td>
</tr>
<tr>
<td>$R$</td>
<td>Equivalent Series Resistance</td>
<td>0.31 $\Omega$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Concentration Polarization Capacitance</td>
<td>400 $F$</td>
</tr>
<tr>
<td>$R_{c2}$</td>
<td>Concentration Polarization Resistance</td>
<td>0.24 $m\Omega$</td>
</tr>
<tr>
<td>$C_u$</td>
<td>Converter Capacitance</td>
<td>10 $\mu F$</td>
</tr>
<tr>
<td>$R_{Cu}$</td>
<td>Converter Resistance</td>
<td>1 $K\Omega$</td>
</tr>
<tr>
<td>$L_u$</td>
<td>Line Inductance</td>
<td>10 $mH$</td>
</tr>
<tr>
<td>$R_u$</td>
<td>Line Resistance</td>
<td>0.1 $\Omega$</td>
</tr>
</tbody>
</table>

Control Gains

<table>
<thead>
<tr>
<th>Gain</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{i,u}$</td>
<td>Bus current integral gain</td>
<td>300</td>
</tr>
<tr>
<td>$k_{p,u}$</td>
<td>Bus current proportional gain</td>
<td>20</td>
</tr>
<tr>
<td>$k_{i,batt}$</td>
<td>Battery current integral gain</td>
<td>1000</td>
</tr>
<tr>
<td>$k_{p,batt}$</td>
<td>Battery current proportional gain</td>
<td>100</td>
</tr>
</tbody>
</table>

Metric relative to the battery available energy is the State of Charge (SOC),

$$SOC(\%) = 100 \frac{(Ah)_{capacity} - Ah}{(Ah)_{capacity}},$$

(5.9)

where, SOC of 100% and 0% denote fully charged and fully discharged battery storage respectively.
The Energy in the battery is,

\[ W_c(t) = \frac{1}{2} C v_c^2(t). \]  

(5.10)

The Ah available in the battery is,

\[ Q = \frac{C v_c}{3600} = \frac{1}{3600} \int i_{batt}(t) dt. \]  

(5.11)
Hence, the SOC of the battery is found from,

\[ SOC = Q - \frac{1}{3600} \int i_{\text{batt}}(t) dt. \]  

(5.12)

The battery losses are,

\[ P_{\text{loss}} = i_{\text{batt}}^2(t) R_{\text{batt}} + \frac{v_{c1}^2(t)}{R_{c1}} + \frac{v_{c2}^2(t)}{R_{c2}}. \]  

(5.13)

The state-space model of the battery storage system in Fig. 5.5 is,

\begin{align*}
C_1 \frac{dv_{c1}}{dt} &= -i_{\text{batt}} - \frac{v_{c1}}{R_{c1}} \quad \text{(5.14a)} \\
L_1 \frac{di_{\text{batt}}}{dt} &= -R_1 i_{\text{batt}}(t) + v_{c1}(t) + v_{c2}(t) + V_{oc} - \lambda u v_u(t) \quad \text{(5.14b)} \\
C_2 \frac{dv_{c2}}{dt} &= -i_{\text{batt}}(t) - \frac{v_{c2}}{R_2} \quad \text{(5.14c)} \\
C_u \frac{dv_u}{dt} &= \lambda i_{\text{batt}}(t) - \frac{v_u(t)}{R_{Cu}} - i_u(t) \quad \text{(5.14d)} \\
L_u \frac{di_u}{dt} &= -v_{bus} - R_{Lu} i_u(t) + v_u(t). \quad \text{(5.14e)}
\end{align*}
The control of the boost converter is obtained from two nested PI loops as,

\[ e_p = i_{u,ref}(t) - i_u(t) \]  \hspace{1cm} (5.15a)

\[ \frac{de_i}{dt} = e_p \]  \hspace{1cm} (5.15b)

\[ i_{batt,ref} = k_{i,u}i_i + k_{p,u}e_p \]  \hspace{1cm} (5.15c)

\[ e_{p,batt} = i_{batt,ref}(t) - i_{batt}(t) \]  \hspace{1cm} (5.15d)

\[ \frac{de_{i,batt}}{dt} = e_{p,batt} \]  \hspace{1cm} (5.15e)

\[ \lambda_u = -k_{i,batt}i_{i,batt} - k_{p,batt}e_{p,batt} + 1 \]  \hspace{1cm} (5.15f)

\[ 0 \leq \lambda_u \leq 1, \]  \hspace{1cm} (5.15g)

where, the inner loop controls the battery current \( i_{batt} \), then the outer loop controls the bus injection current \( i_u \). The low pass filter representation of the battery model with its control is demonstrated in Fig. 5.6.

### 5.5 Summary and Conclusions

In this chapter, the state-space models of flywheel and battery systems and their control were presented. The frequency responses demonstrate that they can be represented as band-limited filters. Hence, for the sake of simplicity, these systems are estimated as First-Order Low-Pass Filters (FOLPFs). The next chapter will include
matching the frequency response of the battery model and its control with a FOLPF by optimizing the control gains.

Hereafter, the combinations of the battery and flywheel models with their corresponding control systems are referred to as Battery Energy Storage System (BESS) and Flywheel Energy Storage Systems (FESSs) respectively. Therefore, by addressing BESS frequency response, the intent is the frequency response of the lumped battery with its control rather than the battery itself. Here, the main focus is on the control approach; optimization for obtaining exact component values is out of the capacity of this dissertation and is left for future iterations of his work.

In the next chapter, BESS and FESSs are represented as simple FOLPFs for control system development. The simplicity of this approach allows integration of various sources such as Supercapacitors (SCs), Pumped-Hydro Storage (PHS) and Compressed Air Storage (CAES) systems [32]. For more detailed studies, higher-order estimates can be integrated into the analysis.
Chapter 6

Energy Storage Power and Energy Sizing and Specification Using HSSPFC

6.1 Introduction

Appropriate technology needs to be the chosen so that a suitable bandwidth of operation versus system transients is provided. Long term transients such as generation fluctuations over time span of hours or days can be alleviated with storage systems
with high energy density such as batteries [34]. On the other hand, short term transients such as load step changes may specifically require storage systems with high power density such as flywheels or super-capacitors [35]. The inherent benefits of these technologies can be combined to create a more capable hybrid storage system [36].

The overall approach of this chapter is to preserve the series form control from Chapter 4 and define electrical levels for ES and the source systems so that the control laws for equivalent parallel form are derived. The series and parallel forms are equivalent in the power flow model defined by the HSSPFC [12].

Here, the control laws for parallel source and hybrid storage elements are derived to achieve two main objectives: First, to regulate the inverter dc voltage and reduce its variations versus source fluctuations. And second, to have an overall zero energy trade for ES elements after a specific amount of time. It will be shown that the corresponding effects of these two objectives on ESS can be defined individually and stacked using superposition to form the overall storage control law.

One purpose of this chapter is to demonstrate the efficacy of the HSSPFC method to control and specify a battery and flywheel hybrid storage system while maintaining electrical levels of the system. Here, to reduce the complexity of the overall system, reduced-order models for microgrid, battery and flywheel systems are used according to Chapters 4 and 5. The renewable source is modeled as an ideal variable voltage
source which can represent a solar or wind source. It is also assumed that the average value of the variable source voltage is known. This is a major simplification of control design based on forecast. The purpose here is to demonstrate the functionality and capability of the developed control scheme for control design and to find the minimum ESS capacity rather than finding the exact or optimum sizing value. Hence, the control scenarios are developed to serve the requirements of the control scheme. Integration of solar or wind systems as well as control scenarios such as power curtailment, MPPT, peak shaving and load shifting are left for future iterations of this work.

In this chapter, first the Hamiltonian control is derived for the parallel source and ESSs, then, the ESS sizing based on power and energy requirements are presented. Here, the minimum power and energy requirements of the overall hybrid ESS as well as battery and flywheel storage subsystems are derived.
6.2 Hamiltonian Control Derivations for Parallel Hybrid Storage Systems

From Chapter 2, The reference state-space of the system in Fig. 2.2 is obtained as below,

\[ M\dot{x}_{ref} = R x_{ref} + B^T u_{ref} + D^T v. \]  

(6.1)

The above system consists of system reference, nominal and measured values. The feed-forward control expressions for ES and inverter command controls are obtained from the steady-state solution of the reference state-space system. In this section, the ES feed-forward and feedback and inverter feed-forward control laws are obtained.

From Chapter 4, the series combination of the ES \( u \) with the source voltage \( v \) might not be the best representation since at some operation points internal currents may flow between the two sources. Moreover, in series topology ES element might limit the current supply of the combined source and introduce more complexity such as requirement of high-side driving for converter control. In this section, the parallel form of the combined source is realized from the baseline series form.

Here, the general aim is to develop control laws for ES element so that it can satisfy
specific constraints. Ideally, the effects of each constraint on ES element can be identified and eventually stacked using superposition principle to obtain the overall ES requirements. In this section, the constraints comprise regulation of the inverter dc-input voltage $v_{dc}$ in Fig. 2.2 and also to have zero steady-state response and zero energy trade for the ES element $i_u$ in Fig. 6.1. It will be shown that the super-position can be used in developing the inverter dc-input reference voltage denoted by $v_{dc,ref}$ to enable the Hamiltonian controller to be used to determine the ES requirements.

Considering (6.1), corresponding error state is,

$$\tilde{x} = x_{ref} - x.$$  \hfill (6.2)

For the system in (2.9) the above is defined as,

$$e = v_{dc,ref} - v_{dc}.$$  \hfill (6.3)

Then the ES PI feedback is,

$$\Delta u = -k_p e - k_i \int_0^t e d\tau,$$  \hfill (6.4)

and the ES feed-forward is,

$$u_{ref} = R_{dc} \beta \lambda (\cos(\phi) i_{d,ref} + \sin(\phi) i_{q,ref}) - v + v_{dc,ref},$$  \hfill (6.5)
Hence, the ES feed-forward and feedback control law is,

\[ u = u_{\text{ref}} - \Delta u \]

\[ = R_{dc} \beta \lambda (\cos(\phi) i_{d,\text{ref}} + \sin(\phi) i_{q,\text{ref}}) - v + v_{dc,\text{ref}} \]

\[ + k_p e + k_i \int_0^t e d\tau. \]  

(6.6)

The series topology of \( v \) and \( u \) in can be converted to parallel form and vice versa according to Fig. 6.1 so that,

\[ v = \frac{R_u}{R_u + R_v} v_v \]  

(6.7a)

\[ u = \frac{R_v}{R_u + R_v} u_u \]  

(6.7b)

\[ R_{dc} = \frac{R_u R_v}{R_u + R_v} \]  

(6.7c)

\[ i_u = \frac{u_u - v_{dc}}{R_u} \]  

(6.7d)

\[ i_v = \frac{u_v - v_{dc}}{R_v}, \]  

(6.7e)
where $v_v$, $u_u$ represent the energy source, storage device. $i_v$, $i_u$, $R_v$ and $R_u$ are the corresponding dc currents and resistances.

The objective is to control $i_u$ to regulate $v_{dc}$ to satisfy two constraints: Firstly, $v_{dc}$ to remain constant versus the change in the input source $v_v$ and secondly, the steady-state value for $i_{u,u}$ is zero. Assuming $v_{v,avg}$ is the average value of the source voltage $v_v$, the source nominal value can be set so that the dc error, $e_{dc}$ is defined as,

$$e_{dc,v} = v_v - v_{v,avg}. \tag{6.8}$$

Considering, (6.7), in series form, this error can be scaled to

$$e_{dc} = \frac{R_u}{R_u + R_v} e_{dc,v} = v(t) - v_{avg}. \tag{6.9}$$

To remove and compensate for $e_{dc}$, the ES $u$ in (6.6) can be set to,

$$u_1(t) = -e_{dc}. \tag{6.10}$$

Also, to have a zero $i_u$, $u_u$ should satisfy,

$$\frac{u_u - v_{dc}}{R_{dc}} = i_u = 0, \tag{6.11}$$
from which the response for $u$ can be obtained as,

$$u_2 = \frac{R_v}{R_u + R_v} v_{dc}. \quad (6.12)$$

From above, $u_1$ and $u_2$ are the required responses for $u$ to satisfy the constraints in (6.9) and (6.11). Using superposition, from (6.10) and (6.12), the overall ES $u$ should satisfy,

$$u_{1,2} = \frac{R_v}{R_u + R_v} v_{dc} - e_{dc}. \quad (6.13)$$

To enforce the above equation, from (6.5), $v_{dc,ref}$ can be set to,

$$v_{dc,ref} = v + u_{1,2} - R_{dc} \beta \lambda (\cos(\phi) i_{d,ref} + \sin(\phi) i_{q,ref})$$

$$= v_{avg} + \frac{R_v}{R_u + R_v} v_{dc} - R_{dc} \beta \lambda (\cos(\phi) i_{d,ref} + \sin(\phi) i_{q,ref}) \quad (6.14)$$

For overall ES control, from (6.7), $u_u$ is controlled according to,

$$u_u = \frac{R_u + R_v}{R_v} u, \quad (6.15)$$

or $i_u$ can be controlled according to,

$$i_u = \frac{u_u - v_{dc}}{R_{dc}}. \quad (6.16)$$
Hence, for a current-controlled storage system the overall ES command is,

\[ i_u = \frac{(R_u + R_v)^2}{R_u^2} u - \frac{R_u + R_v}{R_v R_u} v_{dc} \]  

(6.17)

where, \( u \) is obtained from (6.6) and \( v_{dc} \) is the measured dc bus voltage.

### 6.3 Primary and Secondary Control Derivations

#### 6.3.1 Primary Control System: DQ Droop Control

The droop control as the primary controller for the source microgrid in Fig. 2.2 is given as,

\[ I_{d,i} = \frac{V_{d,i}^* - V_{db}}{R_{d,i}} \]  

(6.18a)

\[ I_{q,i} = \frac{V_{q,i}^* - V_{qb}}{R_{q,i}}. \]  

(6.18b)

Where, \( V_{d,i}^* \) and \( V_{q,i}^* \) are the droop voltage settings. \( R_{d,i} \) and \( R_{q,i} \) are the droop slope settings. \( I_{d,i} \) and \( I_{q,i} \) are the corresponding currents injected into dq buses. \( V_{db} \) and \( V_{qb} \) are the measured bus dq voltages.

If \( I_d \) and \( I_q \) are determined, the real and reactive powers, \( P \) and \( Q \) can be obtained
according to,

\[ P_i = V_{db} I_{d,i} + V_{qb} I_{q,i} \]  \hspace{1cm} (6.19a)

\[ Q_i = V_{qb} I_{d,i} - V_{db} I_{q,i} \]  \hspace{1cm} (6.19b)

The droop settings \( V_{d,i}^*, V_{q,i}^*, R_{d,i} \) and \( R_{q,i} \) are obtained from a secondary controller to improve regulation of \( V_{db} \) and \( V_{qb} \) which represent the three-phase bus voltage.

### 6.3.2 Secondary Control System: Power Flow Calculations

Droop control in (6.18) can operate without communication and load information however, the bus voltage may suffer for lack of communication for long duration of time. A secondary controller can update the droop settings in (6.18) periodically to adjust the bus voltage level with respect to the load.

If \( I_{db} \) and \( I_{qb} \) represent the load currents, the injected currents from DGUs are shared according to,

\[ I_{d,i} = \gamma_i I_{db} \]  \hspace{1cm} (6.20a)

\[ I_{q,i} = \gamma_i I_{qb} \]  \hspace{1cm} (6.20b)

Considering (6.18), for a nominal bus voltage represented by \( V_{db}^* \) and \( V_{qb}^* \), \( V_{d,i} \) and \( V_{q,i} \)
can be set as,

\[ V_{d,i} = I_{d,i} R_{d,i} + V_{db}^* \quad (6.21a) \]

\[ V_{q,i} = I_{q,i} R_{q,i} + V_{qb}^* \quad (6.21b) \]

so that \( P \) and \( Q \) are shared according to,

\[ P_i = \gamma_i (V_{db} I_{db} + V_{qb} I_{qb}) \quad (6.22a) \]

\[ Q_i = \gamma_i (V_{qb} I_{db} - V_{db} I_{qb}). \quad (6.22b) \]

For larger microgrid systems with more lines and loads, the droop settings can be obtained from the solution of conventional power flow calculations. In this case, the aim of the power flow problem would be to set nominal values for local bus electrical levels so that a certain power sharing is achieved.

It is important to note that the secondary control for Fig. 2.2 is significantly simple since the distributed DGUs measure the same local bus. However, for more complex systems such as Fig. 4.6, power flow calculations are typically required. For the expanded system in Fig. 6.2, the simplified secondary control calculations in dq
coordinates are,

\[ I_{db} = I_{dLb} - \omega C_b V_{qb}^* \] \hfill (6.23a)
\[ I_{qb} = I_{qLb} + \omega C_b V_{db}^* \] \hfill (6.23b)
\[ I_{dLb} = \frac{(V_{db}^* + \omega L_b i_{qLb})}{R_b} \] \hfill (6.23c)
\[ I_{qLb} = \frac{(V_{qb}^* - \omega L_b i_{dLb})}{R_b} \] \hfill (6.23d)
\[ I_{d,i} = \gamma_i I_{db} \] \hfill (6.23e)
\[ I_{q,i} = \gamma_i I_{qb} \] \hfill (6.23f)
\[ V_{d,i} = V_{db}^* + I_{d,i} R_{d,i} + I_{d,i} R_{line,i} - \omega L_{line,i} i_{q,i} \] \hfill (6.23g)
\[ V_{q,i} = V_{qb}^* + I_{q,i} R_{q,i} + I_{q,i} R_{line,i} + \omega L_{line,i} i_{d,i} \] \hfill (6.23h)

From above, (6.23a) to (6.23d) represent a four equation and four variable system which can be solved to obtain \( I_{db} \) and \( I_{qb} \). From which \( I_{d,i} \) and \( I_{q,i} \) can be calculated to obtain \( V_{d,i}, V_{q,i}, R_{d,i} \) and \( R_{q,i} \). It is important to note that two other equations need to be added to (6.23) so that the specific droop control method such as slope changing or curve shifting or both are determined [22]. Applying additional constraints will open the grounds for optimization and augmentation of additional loops [24] to the secondary controller which is out of the scope of this work. Here, it is assumed that the secondary controller is always present hence the values of two of the four droop settings can be chosen arbitrarily.
6.4 Hybrid Battery and Flywheel System Control and Specification

A hybrid storage system, with its series and parallel battery and flywheel cells requires some form of power sharing scheme. Here, the hybrid system consists of parallel battery and flywheel systems where each have their respective series and parallel cells. The battery system is considered as the primary storage system and the flywheel system compensates for when the battery cannot effectively track the reference control signal in 6.16. The reference signals for individual battery and flywheels cells are,

\[ i_{\text{batt}, \text{ref}} = \frac{i_{u, \text{ref}, \text{total}}}{N_{p, \text{batt}}} \]  \hspace{1cm} (6.24a)

\[ i_{\text{fw}, \text{ref}} = \frac{i_{u, \text{ref}, \text{total}} - i_{u, \text{batt}, \text{meas}} * N_{p, \text{batt}}}{N_{p, \text{fw}}} \]  \hspace{1cm} (6.24b)

where, \( N_{p, \text{batt}} \) and \( N_{p, \text{fw}} \) are the number of parallel cells for battery and flywheel systems respectively. \( i_{u, \text{ref}, \text{total}} \) is the reference current for the overall hybrid system and \( i_{u, \text{batt}, \text{meas}} \) is the measured current injected by the overall BESS.
The hybrid ESS can be represented as,

\[
\frac{di_{\text{batt}}}{dt} = \omega_{\text{cut-off,batt}} (i_{\text{batt,ref}} - i_{\text{batt}}) \quad (6.25a)
\]

\[
\frac{di_{fW}}{dt} = \omega_{\text{cut-off,fW}} (i_{fW,ref} - i_{fW}) \quad (6.25b)
\]

where, \(i_{\text{batt}}\) and \(i_{fW}\) represent the injected currents by the BESS and FESS respectively. \(\omega_{\text{cut-off,batt}}\) and \(\omega_{\text{cut-off,fW}}\) represent the estimated minimum cut-off frequencies of the BESS and FESS.

To avoid undesired excursions for the BESS, an additional filter can be added to the system in (6.25) and its control in (6.24). Hence, the overall system is represented as,

\[
\frac{di_{\text{batt}}}{dt} = \omega_{\text{cut-off,batt}} (i_{\text{filter,ref}} - i_{\text{batt}}) \quad (6.26a)
\]

\[
\frac{di_{fW}}{dt} = \omega_{\text{cut-off,fW}} (i_{fW,ref} - i_{fW}) \quad (6.26b)
\]

\[
\frac{di_{\text{filter,ref}}}{dt} = \omega_{\text{cut-off,filter}} (i_{\text{batt,ref}} - i_{\text{filter,ref}}) \quad (6.26c)
\]

\[
i_{\text{batt,ref}} = \frac{i_{u,ref,total}}{N_{p,batt}} \quad (6.26d)
\]

\[
i_{fW,ref} = \frac{i_{u,ref,total} - i_{\text{batt}} \ast N_{p,batt}}{N_{p,fW}} \quad (6.26e)
\]

\[
i_{\text{ESS}} = i_{\text{batt}} + i_{fW} \quad (6.26f)
\]

\[
\omega_{\text{cut-off,batt,filter}} \leq \omega_{\text{cut-off,batt}} \quad (6.26g)
\]
where, $\omega_{\text{cut-off,filter}}$ and $i_{\text{filter,ref}}$ are the cut-off frequency and the output of the LPF respectively. $i_{\text{ESS}}$ is the overall hybrid ESS supplied current. For this system, the overall input is $i_{u,\text{ref,total}}$.

Considering (6.26), for the overall reference current $i_{u,\text{ref,total}}$, a system designer can choose the cut-off frequencies $\omega_{\text{cut-off,batt}}$ and $\omega_{\text{cut-off,FW}}$ according to the accepted system tolerance for the dc-link voltage $v_{dc}$. With the cut-off frequencies identified, one can use it to estimate the BESS and FESS systems and their controller parameters. Since ESS devices and the sources are in parallel and $v_{dc}$ is a common entity, the $i_{\text{ref}} = i_{u,\text{ref,total}}$ command can be substituted with $\frac{P_{\text{ref}}}{v_{dc}}$. In the next section, $P_{\text{ref}}$ will be used in order to determine power and energy capacities of ESSs.

### 6.5 ES Power and Energy Sizing

A measure for power and energy capacity for the ES is needed. There are two aspects to consider; power and energy density requirements. For a single ES element, the ESS should meet both of these requirements that is having high power and energy densities simultaneously. Should the single ESS fail to meet these requirements, power quality issues may occur and the local microgrid system may fail. A single device with both high power and energy characteristics may be very expensive compared to a hybrid ESS.
In this work a parallel battery and flywheel hybrid ESS is used. Generally, a hybrid ESS is more flexible in the sense that they can be controlled according to their merits. The disadvantage in this case would be the complexity of the control system compared to a single ESS operation however, there is more potential for further optimization. A hybrid storage is capable to reduce battery degradation since faster power fluctuations can be compensated by the flywheel system.

Here, it is assumed that the average source power minus the average losses meets the load demands. In other words, the net of power and energy of the ESS over the operation cycle is considered to be zero. It is also assumed that the average power for individual BESS and FESSs over one cycle is zero. Losses are ignored and relevant investigations for losses in the system are left for future iterations of this work.

In this section, the overall approach is to share the ESS power according to BESS and FESS bandwidths. Here, individual reference power commands for BESS and FESS are obtained from the overall ESS reference power command. The power sizing of the ESSs is determined from the power spectrum of the reference power signal using the Power Spectral Density (PSD) of the ESS response. The energy sizing on the other hand is performed by analysis of the time-domain requirements.
6.5.1 Power Requirements

If $P_{ref}$ is the reference ESS command signal with an average value of zero, the required RMS value of the continuous power signal is,

$$P_{ref,RMS} = \sqrt{\frac{1}{T} \int_{0}^{T} P_{ref}^2(t) dt}, \quad (6.27)$$

and for the discrete time signal it is,

$$P_{ref,RMS} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} P_{ref}^2[n].} \quad (6.28)$$

The PSD is generally defined as,

$$S_r(\omega) = \lim_{T \to \infty} \frac{E[|F_r(\omega)|^2]}{T} d\omega, \quad (6.29)$$

where, $F_r(\omega)$ is the Fourier transform of the truncated signal $P_{ref}(t)$. According to the conservation of energy principle, Parseval’s theorem states,

$$\frac{1}{T} \int_{0}^{T} P_{ref}^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_r(\omega) d\omega. \quad (6.30)$$
Assuming that the ESS can match the control signal, from (6.27) and (6.30), the minimum RMS power can be defined as,

\[ P_{\text{ESS,RMS}} \geq \sqrt{\int_{-\infty}^{+\infty} S_r(\omega)df}, \]  

(6.31)

which provides a measure for sizing the power of the ESS. For when a filter is added,

\[ S_{rf}(\omega) = S_r(\omega)|H(j\omega)|^2, \]  

(6.32)

Hence,

\[ P_{rf,RMS} \geq \sqrt{\int_{-\infty}^{+\infty} S_{rf}(\omega)df}, \]  

(6.33)

Considering that the BESS represents a LPF, the above expression yields an estimate for bandwidth support of the battery. For discrete, one-sided power spectrum representation, assuming the magnitude of each frequency content is the corresponding RMS value, the expression in (6.31) can be re-written as,

\[ P_{\text{ref,RMS}} = \sqrt{\sum_{n=1}^{N} 2(P_S[n])^2} \]  

(6.34)

where, \( P_S[n] \) is the power spectrum corresponding to frequency component \( n \). Here, \( N \) is the length of the spectrum.
Ideally, one can split the power spectrum by choosing $N_0$ such that,

$$P_{ref,RMS,ESS} = \sqrt{\sum_{n=1}^{N_0} 2(P_S[n])^2} \approx \gamma \sqrt{\sum_{n=1}^{N} 2(P_S[n])^2} \quad (6.35)$$

where, $P_{ref,RMS,ESS}$ is the required RMS power of the ES device for the case the overall PSD is portioned. $N_0(\leq N)$ represents the maximum frequency that the device can support and it is directly related to the ESS cut-off frequency. $\gamma$ is a probabilistic entity that should meet (0,1). $N_0$ and $\gamma$ are design variables and can be chosen based on the capability, topology, size and bandwidth of the ES system. Considering a system with only one ES element, for a small value of $N_0$ (or $\gamma$), a smaller portion of the overall power can be supplied. On the other hand, for a larger value of $N_0$(or $\gamma$), more capable ESS is required. In the case that the device cannot sufficiently support a large $N_0$ (and the corresponding $\gamma$), electrical levels may degrade and the power quality may suffer. This will be shown in the next sections and also in Chapter 7.

LPFs split the power around a cut-off frequency in a weighted manner rather than a clean split as $P_{ref,RMS,ESS}$. Hence $(6.35)$ can be re-written versus the weighting applied by the filter transfer function as,

$$P_{ref,RMS,ESS} = \sqrt{\sum_{n=1}^{N} 2(P_S[n]|H(jn)|)^2} \approx \gamma \sqrt{\sum_{n=1}^{N} 2(P_S[n])^2}, \quad (6.36)$$

where, $H(jn)$ is the filter transfer function. Here, the cut-off frequency of the filter
\(w_{\text{cut-off, filter}}\), and \(\gamma\) are the design variables. Hence, if an ESS with a specific cut-off frequency is present, assuming \(H(j\omega)\) is an accurate estimation of the system, \(\gamma\) can be found. On the other hand, if a power sharing of say 90\% is desired, \ref{6.36} can be used to calculate the required ESS cut-off frequency.

For a hybrid ESS, it is important to note that the above expression deals with the power sharing based on the bandwidth of the ESS rather than sharing based on the overall energy. The energy sharing however, is dictated by this power sharing. Hence, energy is allocated by the control system topology based on the priority that is given to each storage device according to their bandwidth. For a battery/flywheel hybrid storage the power sharing can be as,

\[
P_{\text{ref,RMS,Batt}} = \sqrt{\sum_{n=1}^{N} 2(P_S[n]|H(jn)|)^2} \tag{6.37a}
\]

\[
P_{\text{ref,RMS,fw}} = \sqrt{P_{\text{ref,RMS,ESS}}^2 - P_{\text{ref,RMS,Batt}}^2} = \sqrt{\sum_{n=1}^{N} 2(P_S[n])^2 - \sum_{n=1}^{N_{\text{batt}}} 2(P_S[n]|H(jn)|)^2}. \tag{6.37b}
\]

where, \(P_{\text{ref,RMS,Batt}}\) and \(P_{\text{ref,RMS,fw}}\) represent the power requirement from the battery and flywheel devices respectively. Here, respective to the BESS, flywheel device may have higher bandwidth of operation but it is yet band-limited and may fail to support the full requested power. Hence, the overall ESS must meet \ref{6.36} with a sufficiently high \(\gamma\).
A FOLPF might not be an accurate representation of the BESS. As a result a filter
matching leakage may occur. In the next sections, it will be shown that, rather than
to ignore the filter leakage, BESS control gains can be optimized to fit to a FOLPF.
It is important to note that while this approach is suitable for this work, the general
approach is to accurately design a higher order LPF to represent each ESS. Hence,
for the sake of simplicity, and keep the generality of this work, BESSs are estimated
as FOLPFs.

6.5.2 Energy Requirements

Energy of the overall ESS can be obtained from,

\[ E_{ESS}(t) = \frac{1}{T} \int_0^t P_{ref}(t)dt. \]  \( (6.38) \)

The range of the processed energy for a cycle is hence,

\[ \Delta E_{ESS} = max(E_{ESS}) - min(E_{ESS}). \]  \( (6.39) \)

The required ES range is,

\[ E_r = |max(E_{ESS}) - min(E_{ESS})|. \]  \( (6.40) \)
This expression represents the required capacity of the hybrid ESS. This value is eventually shared when the control system shares the overall power between the battery and flywheel sub-systems according to their bandwidth of operation. Hence, the above expression can be re-written for each ESS sub-system.

Considering a battery system, required energy range is,

\[ E_{r,batt} = |max(E_{batt}) - min(E_{batt})|. \]  \hspace{1cm} (6.41)

Assuming the initial energy of the battery is known, keeping the battery charge and discharge range between 20% and 80% of the overall SOC yields,

\[ E_{r,batt} = 0.8(Ah)_{capacity} - 0.2(Ah)_{capacity} \]  \hspace{1cm} (6.42)

where, \((Ah)_{capacity}\) is the overall energy capacity for a battery. Hence, the minimum overall capacity of the battery can be obtained from,

\[ (Ah)_{capacity} \geq \frac{1}{0.6} E_{r,batt}. \]  \hspace{1cm} (6.43)

The above can then be used for comparison with the overall capacity in (5.9).
Considering a flywheel system, the required energy range is,

\[ E_{r, fw} = |\max(E_{fw}) - \min(E_{fw})|. \]  \hspace{1cm} (6.44)

Assuming the Depth of Discharge (DOD) for the flywheel system over a cycle is 75%,

\[ E_{r, fw} = \frac{1}{2} J_f \omega_{f,max}^2(t) - 0.25(\frac{1}{2} J_f \omega_{f,max}^2(t)). \]  \hspace{1cm} (6.45)

Hence, the flywheel energy capacity should meet,

\[ W_{cap, fw} \geq \frac{1}{0.75} E_{r, fw}. \]  \hspace{1cm} (6.46)

### 6.6 Battery Energy Storage System and First-Order LPF Matching and Estimation

In previous sections, BESS and FESS systems were described as FOLPFs such as in 6.26. However, the BESS in Chapter 5 is clearly a higher order system than the FOLPF. Mismatching the frequency responses of both systems may lead to substantial inaccuracies in power sharing based on frequency response. Moreover, choosing a large ESS cut-off frequency may lead to an over-sized BESS from the power density point of view.
The general approach is to choose the order of the LPF system in 6.26 so that the filter can be a more accurate representation of the BESS. Here, the filter is kept as a FOLPF but the BESS control is optimized to mimic a FOLPF. In this case, the battery control system PI gains can be considered as optimization variables. The optimization problem is set up as,

\[
\text{Minimize} \quad F(k_{p,\text{batt}}, k_{i,\text{batt}}, k_{p,u}, k_{i,u}) : \\
|H_{\text{BESS}}(j\omega_{\text{cut-off}})| - |H_{\text{filter}}(j\omega_{\text{cut-off}})|^2 \\
+ \sum_{k=1}^{k_{\text{max}}} (|H_{\text{BESS}}(j(\omega_0 + k\delta\omega))| - |H_{\text{filter}}(j\omega_0 + k\delta\omega)|)^2 \tag{6.47a}
\]

\text{Subject to :} \quad k_{i,\text{batt}} > k_{i,u} > 0, \quad k_{p,\text{batt}} > k_{p,u} > 0, \tag{6.47b}

where, \(H_{\text{BESS}}(jw)\) and \(H_{\text{filter}}(jw)\) represent the frequency responses of BESS and the FOLPF respectively. \(H_{\text{BESS}}(jw)\) is obtained from battery system in [5.14] with control system in [5.15]. \(\omega_{\text{cut-off}}\) represents the cut off frequency for both BESS and LPF. \(\omega_0\) and \(\delta\omega\) represent initial frequency and the frequency deviation respectively. \(k\delta\omega\) represent the step in frequencies for which the frequency responses are matched. \(6.47b\) shows the inequality constraints for the PI gains of the nested loops in (5.15).

From, \(6.47\), the objective function \(F\) can be minimized so that an accurate estimate of the LPF is obtained. The overall approach from above is equivalent to fitting the BESS and LPF Bode amplitude plots. It is important to note that the BESS
cut-off frequency can be chosen relatively larger than the LPF so that the minimum power contribution of the BESS shown in (6.37) is ensured however, (6.47) yields an accurate estimation of the LPF. The design of higher order LPFs are left for future iterations of this work.

6.7 Hybrid Battery and Flywheel Storage Discussion

Considering ideal first-order band-limited representations of BESS and FESS, hybrid ESS of each DGU in (6.2) can be sized and specified by following the below steps,

1. The system with an ideal ESS controlled under (6.17) is run and overall $P_{\text{ref}}$ and voltage and current levels are obtained. Time domain analysis are performed to obtain the overall energy capacity.

2. From voltage and current levels, the number of series and parallel battery and flywheel cells and individual battery $P_{\text{ref}}$’s are obtained.

3. (6.34) is used to declare RMS of the power signal in frequency domain as one-sided power spectrum.

4. If $H(j\omega)$ represents the frequency response of the FOLPF, its cut-off frequency is obtained according to (6.37); for a chosen $\gamma$, the BESS cut-off frequency

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\( \omega_{\text{cut-off,BESS}} \), is calculated.

5. BESS frequency response should match the LPF according to \(6.26\). Ideally LPF is designed to match BESS frequency response. Alternatively, BESS parameters can be optimized to mimic the LPF.

6. The system with the designed band-limited BESS with control specification in \(6.26\) is evaluated and compared with the results from step 1. The required energy is calculated using \(6.43\) and \(6.46\).

It is important to note, for reduced-order systems as shown in Chapter 5, the overall energy trade with BESS and FESS will not be zero due to losses. However, generally, in the sixth step, \(6.43\) and \(6.46\) should match \(5.12\) and \(5.3\) respectively. Compensation versus system losses can be done by modifying \(6.26\) however, it is out of the scope of this dissertation and is left for future iterations of this work.
6.8 Illustrative Examples

The overall system in Fig. 6.2 is simulated in Wolfram Mathematica and SystemModeler [28] and Modelica [38]. The system comprise two DGUs and transmission...
<table>
<thead>
<tr>
<th>BESS Parameters</th>
<th>FESS Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_1(F))</td>
<td>(J_f(Kgm^2))</td>
</tr>
<tr>
<td>750</td>
<td>0.91</td>
</tr>
<tr>
<td>(C_2(F))</td>
<td>(k_i(Nm/A))</td>
</tr>
<tr>
<td>400</td>
<td>47</td>
</tr>
<tr>
<td>(C_u(uF))</td>
<td>(R_{pm}(m\Omega))</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>(L_1(uH))</td>
<td>(L_{pm}(mH))</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>(L_u(mH))</td>
<td>(C_u(mF))</td>
</tr>
<tr>
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<td>20</td>
</tr>
<tr>
<td>(R_1(m\Omega))</td>
<td>(R_{cu}(K\Omega))</td>
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<td>1</td>
</tr>
<tr>
<td>(R_{c,1}(\Omega))</td>
<td>(L_u(mH))</td>
</tr>
<tr>
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<td>20</td>
</tr>
<tr>
<td>(R_{c,2}(m\Omega))</td>
<td>(R_u(m\Omega))</td>
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</tr>
<tr>
<td>(R_{cu}(\Omega))</td>
<td>(B(Nm/rad/s))</td>
</tr>
<tr>
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<td>0.0019</td>
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<tr>
<td>(R_{Lu}(mH))</td>
<td>(N_{s,fw})</td>
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<tr>
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<tr>
<td>(V_{oc}(V))</td>
<td>(N_{p,fw})</td>
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</tr>
<tr>
<td>(Q(Ah))</td>
<td>(N_{s,batt})</td>
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<td>2</td>
</tr>
<tr>
<td>(N_{p,batt})</td>
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<table>
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<tr>
<th>BESS Control Parameters</th>
<th>FESS Control Parameters</th>
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<td>(k_{p,u})</td>
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<tr>
<td>(k_{i,batt})</td>
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<tr>
<td>(k_{p,batt})</td>
<td>(k_{p,batt})</td>
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<td>3.73</td>
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The 120/208V corresponding values for \(V_{db}^*\) and \(V_{qb}^*\) are 147V and 0V respectively.
Voltage sources $v_{v,1}$ and $v_{v,2}$ have a randomly chosen dc value around 300V with a superimposed uniform random white noise. A practical constraint to consider is that the source voltages cannot have values less than the dc bus voltages $v_{dc,1}$ and $v_{dc,2}$. The random noise component aims to introduce fluctuations so that the contributions of ESSs with different bandwidths are presented for a worst-case scenario for a field deployed microgrid.

Two examples are presented in this section. For the first case, load power is solely supported by DGU 1 and droop control is set so that DGU 2 does not contribute to the load power. The electrical levels of the system is presented versus the ideal and band-limited ESSs. The second case performs the simulation of the overall system in Fig. 6.2 for when the hybrid BESSs and FESSs are present. For both cases, the ESS aims to compensate for source fluctuations rather than the load. The ESS requirements for load-side deployments are shown in part in Chapter 7.

### 6.8.1 Single DGU with Constant Load Example

A 6 s simulation of the system is performed for a fixed load value of $R_b = 2 \Omega$. The results are shown in Figs. 6.3 to 6.12. The results for a band-limited ES element with cut-off frequency of $\omega_{cut-off} = 1000 \frac{rad}{s}$ is presented in Figs. 6.4 to 6.8. The source
voltage with superimposed sampled noise is shown in Fig. 6.3. The maintained three-phase bus voltages and the injected load currents are shown in Fig. 6.4a and 6.4b respectively. Fig. 6.5 shows the dc bus voltage and Fig. 6.6 demonstrates the current

\[ v_{v,1} \] and its average value.

\[ i_{abc} \] (a) voltage, and (b) current.

\[ v_{abc} \] (V)

\[ t_{abc} \] (V)

\[ v_{abc} \] (a)

\[ i_{abc} \] (b)

\[ v_{abc} \] (V)

\[ i_{abc} \] (V)

\[ t_{abc} \] (s)

\[ i_{abc} \] (s)

\[ v_{abc} \] (V)

\[ i_{abc} \] (V)

\[ t_{abc} \] (s)

\[ v_{abc} \] (V)

\[ i_{abc} \] (V)

\[ t_{abc} \] (s)
of the band-limited ES element. Figs. 6.7a and 6.7b demonstrate the bus voltage and injected load current amplitude fluctuations respectively. direct and quadrature current components, \( i_d \) and \( i_q \), represent the injected current to the bus/load and are shown in Figs. 6.7c and 6.7d.

The overall dc power of the source and the ESS is presented in Fig. 6.8a. Fig. 6.8b shows dc power is delivered to the load with some reduction due to the transmission
Figure 6.7: Load (a) phase a voltage and (b) current amplitudes, (c) d and (d) q current components.

A sweep of the ESS bandwidth is implemented. Figs. 6.9, 6.10, and 6.11 represent the dc bus voltage for when the ESS cutoff frequency $\omega_{\text{cut-off}}$ is $10\ \frac{rad}{s}$, $100\ \frac{rad}{s}$ and $10000\ \frac{rad}{s}$ respectively. It can be seen that there is a trade-off between the ESS size and bandwidth of operation and the voltage fluctuations in dc bus. The overall sweep of the ESS cut-off frequency versus dc voltage quality is demonstrated in Fig. 6.12. Fig. 6.13 demonstrates the required normalized capacity against the bandwidth.
Figure 6.8: (a) power injected into inverter dc port $P_{dc}$, (b) load ac real power $P_{ac,b}$ and (c) ESS power injection.

Figure 6.9: dc-link voltage for $w_{cut-off} = 10 \frac{rad}{s}$.

sweep. The normalized capacity is obtained by dividing the required capacity at specific frequency support over the maximum required capacity when an ideal ES element is present.

Here, the aim is to put more stress on the ESS. Typically, filters such as dc reservoir
Figure 6.10: dc-link voltage for $w_{\text{cut-off}} = 100 \frac{\text{rad}}{s}$.

Figure 6.11: dc-link voltage for $w_{\text{cut-off}} = 10000 \frac{\text{rad}}{s}$.

capacitors are chosen significantly larger than for the system in Fig. 6.2. The bus fluctuations that have higher frequency contents can generally be compensated with such capacitors.
Figure 6.12: dc voltage variation against ESS cut-off frequency sweep.

Figure 6.13: ESS energy capacity requirement against ESS cut-off frequency.

6.8.2 Parallel DGUs Example

In this example, the two DGU system in (6.2) with parameters in Table 6.1 and 6.2 is considered. Here, the overall results for the system with hybrid reduced-order BESS and FESS are presented.
First, the system is run with an ideal ESS model. The sources and load profile are as shown in Fig. [6.14]. The obtained reference powers, $P_{ESS,1}$ and $P_{ESS,2}$ are shown in Fig. [6.15]. Using Section 6.5, the overall RMS of the power signals are 2042$RMS$ and 2408$RMS$ for $DGU_1$ and $DGU_2$ respectively. The overall required energy, from (6.44), for $DGU_1$ and $DGU_2$ are 5159.9$J$ and 4529.9$J$ respectively.
Here, the aim is to define the control system so that BESSs contribute to more than 95% of the RMS of the power signals $P_{ESS,1}$ and $P_{ESS,2}$. Hence, according to (6.36), with $\gamma = 95\%$, the cut-off frequencies are found to be $12.56 \, \frac{rad}{s}$ and $18.84 \, \frac{rad}{s}$ for filters corresponding to $BESS_1$ and $BESS_2$. With the individual cut-off frequencies known, the control gains of BESSs are optimized to mimic FOLPFs as shown in Figs.

**Figure 6.16:** $DGU_1$ filter system response estimate.

**Figure 6.17:** $DGU_2$ filter system response estimate.
The obtained gain values are shown in Table 6.2.

The obtained BESS RMS contributions are 1957.3\( RMS \) and 2306.6\( RMS \) corresponding to 95.84\% and 95.78\% of the total RMS of the power signals. The total required energy for BESS\textsubscript{1} and BESS\textsubscript{2} are 4836.1\( J \) and 4372.7\( J \) respectively.
Figure 6.19: Source and ESS dc power contributions for (a) DGU₁ and (b) DGU₂.

The system performance under the above power sharing is shown in Figs. 6.18 to 6.22. Fig. 6.18 demonstrates individual hybrid current contributions for BESSs and FESSs. It can be seen that significant low-frequency portions of the currents are allocated to BESSs and the FESSs are requested only for high power and fast fluctuations.

Fig. 6.19 presents the source powers and overall ESSs dc powers. Individual dc-link voltages, $v_{dc,1}$ and $v_{dc,2}$ are shown in Fig. 6.20a and 6.20b respectively. It can be seen that ESSs regulate dc voltages versus the sources rather than the load. However, in constant load periods, $v_{dc}$ variations are small which demonstrates the efficacy of the hybrid ESSs for power bandwidth support. Figs. 6.20c and 6.20d demonstrate
individual inverter commands \((\lambda, \phi)\) for the two DGUs.

Considering the load-side bus, Fig. 6.21 shows real and reactive power sharing under the droop control. \(a\) and \(b\) from (6.23) are chosen to be 0.33 and 0.66 respectively. It can be seen that the power sharing is effectively maintained versus the load changes. The maintained three-phase bus voltage is demonstrated in Fig. 6.22a. It can be seen that as the sources and load fluctuate, the three-phase ac bus amplitude (denoted by phase a voltage \(v_a\)) is kept at 120/208V. Fig. 6.22b shows the overall load current while 6.22c demonstrates the shared three-phase currents.

**Figure 6.20:** dc-link voltages for (a) \(DGU_1\) and (b) \(DGU_2\). Individual inverter commands (c) \(\lambda_1\) and \(\lambda_2\), (d) \(\phi_1\) and \(\phi_2\).
Figure 6.21: Individual DGU and overall load ac (a) Real and (b) Reactive powers. Real and reactive powers sharing is maintained according to a 1:2 ratio.

Figure 6.22: Load phase a, (a) voltage and (b) overall current amplitudes. (c) three-phase ac currents sharing between $DGU_1$ and $DGU_2$. 
6.9 Summary and Conclusions

In this chapter, the HSSPFC method derivations for parallel topology of source and ESS was presented. A control law was developed in order to enable power sharing between hybrid ESSs such as batteries and flywheels based on their capabilities. Here, BESS was considered to be the primary ESS while the FESS was requested only to complement the power injection at high bandwidths. It was shown that the hybrid power can be shared by splitting the overall power spectrum of the reference power signal utilizing a first-order LPF. The results attest if the LPF and the band-limited BESS frequency responses are matched, the proposed power sharing based on power spectrum yields accurate results. The performance of a microgrid system with hybrid ESSs and two DGUs under dq droop control versus variable sources and load was also demonstrated.

In this chapter, the ESS control aimed to compensate for source variations however, the system electrical levels may suffer when load variations are present. The next chapter is dedicated to developing the ESS requirements for load-side variations for pulsed power loads.
Chapter 7

Energy Storage Baseline

Requirements for Pulsed Power Loads

7.1 Introduction

Microgrids with new designs and implementations are growing to integrate various local generation capacities as well as various types of loads into the power systems. One existing problem is the rapid variations in the load power demands which can
add unwanted frequency content to the bus voltage of the microgrid [39]. These fluc-
tuations can cause the collapse of voltage and system-wide performance degradation
and affect the power and energy transfer quality of the network. The fluctuations can
lead to tripping of other sensitive loads which may cause power outages. In an ac
or dc microgrid system, the existence of loads of nonlinear characteristics may com-
promise the stability of the system during the transients. One such loads are pulsed
power loads.

Pulsed power loads draw very high currents in a short time span which can vary with
periods of seconds to minutes [40]. These highly variable loads typically operate as
pulse train sequences with duty cycles and magnitudes. Considering the peak and
average power demand of the load, different ES technologies with different capacities
and bandwidths of operation are needed to complement the system to fulfill system
control objectives. This can be the quality or maintenance of voltage, current, power
and energy. Supercapacitors, flywheels and batteries have already been used for these
purposes [41, 42, 43]. In DC microgrid systems that have pulsed loads, the general
approach is to decouple the load from the source by using appropriately large ES [44].
The ES can mitigate instability of the system in a constant power approach [45].

In a constant power load, the drawn current changes inversely with respect to the
voltage. This creates a negative incremental impedance and can lead to instability
[46, 47, 48]. Power buffers [49], are proposed to decouple the load from the grid
and to compensate for non-linear load transients \cite{50, 51}. Load characteristics can be controlled to mimic a linear behavior versus the grid-side bus voltage transients hence providing support when grid-side current changes inversely with the grid-side voltage. Ideally, the power buffer filters the fast dynamics of the load and decouples the load-side system from the grid-side. Power buffers need large storage systems to sustain loads with extended transient time \cite{52}.

ES devices with various energy capacities and responses are widely used to improve the quality of power and energy transfer \cite{32}. Typically, to compensate for the slow change of load power such as in hourly variations, storage elements with high energy densities are required. In the contrast, for faster variations, high power density devices are needed. For this bandwidth of operation various devices are considered. While supercapacitors are suitable for high power bandwidth operations \cite{53}, batteries with lower bandwidths and more energy densities are available to alleviate the power and energy deficiencies and to extend the operating time \cite{54}.

In this chapter, an analysis and control strategy for a baseline energy storage element is presented. The objective of the baseline is to maintain the bus voltage while the ideal storage element supplies all of the frequency content of the load. Then, from this baseline a designer can then better understand the trade-offs in bus harmonic content and the storage element design. The storage system design could range from a single battery to a combination of storage technologies that will cover a greater
The pulse load power is defined as a PWM waveform with a duty cycle $D_p$, the period $T_p$ and the peak value $P_{peak}$ as shown in Fig. 7.1. An average load power is defined to provide a constraint for overall ES power flow. The ES control objective is to maintain the load voltage and the grid-side current flow. The combination of information of load power characteristics and the defined average power enables the required capacity of the ESS to be determined. The quality of the maintained voltage and current however, depends on the ES bandwidth of operation. The operation of the system under ideal, band-limited storage systems as well as reduced order flywheel and battery systems are also presented.

This chapter is organized as follows. First, in section 7.2 the pulse load system is modeled and the ES control law is developed. Next, the ES overall frequency content is defined in section 7.2.3. In section 7.2.4, the band-limited ES element is integrated into the system. Then, general reduced order flywheel and battery storage systems are modeled to illustrate the ideal and band-limited storage. Finally, in section 7.3, illustrative examples are presented for pulsed load systems with ideal, band-limited and hybrid flywheel and battery models of the ESS.
7.2 Pulsed Load System and Energy Storage Control

The reduced-order model of the pulse load [40] is shown in Fig. 7.2. The state-space representation is,

\[ L \frac{di(t)}{dt} = -R_L i(t) - v(t) + \lambda v_b \]  \hspace{1cm} (7.1a)

\[ C \frac{dv(t)}{dt} = i(t) - \frac{P(t)}{v(t)} - \frac{v(t)}{R} + u. \]  \hspace{1cm} (7.1b)
The bus voltage, $v_b$ and the front-end converter command $\lambda$ are assumed to be constant. To keep the concentration on ES element $u$, a sufficiently small value is considered for shunt capacitor $C$ so that it does not significantly affect the ES requirements. Then, $u$ can represent a single or several ESSs.

### 7.2.1 Energy Storage Control

For the baseline, the objective for the storage element $u$ is to supply all the necessary energy so that $i$ and $v$ be constants. Therefore, the steady-state average of (7.1) is

$$0 = -R_L \tilde{i} - \tilde{v} + \lambda \tilde{v}_b$$  \hspace{1cm} (7.2a)

$$0 = \tilde{i} - \frac{\tilde{P}}{\tilde{v}} - \frac{\tilde{v}}{R} + u,$$  \hspace{1cm} (7.2b)

where the average load power is,

$$\tilde{P} = D_p P_{peak}.$$  \hspace{1cm} (7.3)

Solving (7.2) for the average voltage and current, $\tilde{v}$ and $\tilde{i}$ yields,

$$\tilde{v} = \sqrt{R(\lambda^2 \tilde{v}_b^2 - 4R_L D_p P_{peak}(R + R_L)) + \lambda R \tilde{v}_b}$$  \hspace{1cm} (7.4a)

$$\tilde{i} = \frac{\lambda \tilde{v}_b (R + 2R_L) - \sqrt{R(\lambda^2 \tilde{v}_b^2 - 4R_L D_p P_{peak}(R + R_L))}}{2R_L (R + R_L)},$$  \hspace{1cm} (7.4b)

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Then, the storage device has to be,

\[ u = -\frac{2(R + R_L)(D_pP_{peak} - P(t))}{\sqrt{R(\lambda^2 R v_b^2 - 4R_L D_p P_{peak}(R + R_L)) + \lambda R v_b}}. \] (7.5)

The power from the storage device \( P_u = v u \), is,

\[ P_u(t) = P(t) - D_pP_{peak}. \] (7.6)

Integrating the storage power over the period of positive power output yields,

\[
\begin{align*}
W_u &= \int_0^{T_p} P_u(t) dt \\
&= \int_0^{D_p T_p} (P(t) - D_pP_{peak}) dt \\
&= \int_0^{D_p T_p} (P_{peak} - D_pP_{peak}) dt \\
&= \int_0^{D_p T_p} (P_{peak}(1 - D_p)) dt, \tag{7.7}
\end{align*}
\]

which gives a total ES in Joules as,

\[ W_u = -(D_p - 1)D_p T_p P_{peak}. \] (7.8)

The overall energy trade of the ES element \( u \) over the period of \( T_p \) is zero. The ES control law in (7.5) is derived considering the average power in (7.3). In the case that the storage element \( u \) has internal losses, (7.6) can be modified and combined with
(7.3) to compensate for the ES losses. However, for the rest of this chapter, such losses are neglected and storage devices are sized to sustain the load for a sufficiently long amount of time.

The maximum of (7.8) over one load cycle is found from

\[
\frac{dW_u}{dD_p} = T_p P_{peak} - 2D_p T_p P_{peak}
\]

\[
= T_p P_{peak} (1 - 2D_p)
\]

\[
= 0.
\]  

(7.9)

Hence, for \( D_p = \frac{1}{2} \) the required ES is the maximum. This is a useful measure for appropriate sizing of the ESS.

### 7.2.2 Linear Methods for Stability Bounds

Small-signal analysis is performed for the system in (7.1). The linearized model is in the form,

\[
\dot{x} = Ax + Bu,
\]  

(7.10)
where the linearized A matrix is,

\[
A = \begin{bmatrix}
  -\frac{R_L}{L} & \frac{1}{L} \\
  \frac{1}{C} & \frac{D_p P_{peak}}{v^2} - \frac{1}{R}
\end{bmatrix}.
\]  

(7.11)

Hence, the characteristic equation is,

\[
s^2 + s\left(-\frac{D_p P_{peak}}{C v^2} + \frac{1}{C R} + \frac{R_L}{L}\right) - \frac{R_L D_p P_{peak}}{C L v^2} + \frac{R_L}{C L R} + \frac{1}{C L} = 0
\]  

(7.12)

For stability the following should hold,

\[
-\frac{R_L D_p P_{peak}}{C L v^2} + \frac{R_L}{C L R} + \frac{1}{C L} = \frac{R_L}{C L} \left(1 - \frac{D_p P_{peak}}{v^2} \right) + 1 > 0
\]  

(7.13a)

\[
-\frac{D_p P_{peak}}{C v^2} + \frac{1}{C R} + \frac{R_L}{L} = \frac{1}{R} - \frac{D_p P_{peak}}{v^2} + \frac{R_L}{L} > 0
\]  

(7.13b)

The system is stable, no matter other variable values, if

\[
0 < R \leq \frac{v^2}{D_p P_{peak}}.
\]  

(7.14)
where, \( \frac{v^2}{D_p P_{peak}} \) is the equivalent average impedance of the pulse load. The above inequality implies that if the resistive load \( R \) dissipates more power than the average pulse load, then it is stable. However, if this is not the case and \( R \) is,

\[
R > \frac{v^2}{D_p P_{peak}}. \tag{7.15}
\]

We can still stabilize the system if the inductance and inductor resistance are chosen such that,

\[
0 < L < \frac{CR^2 v^4}{(v^2 - RD_p P_{peak})^2}, \tag{7.16a}
\]

\[
\frac{L(RD_p P_{peak} - v^2)}{CRv^2} < R_L < \frac{Rv^2}{RD_p P_{peak} - v^2}. \tag{7.16b}
\]

In the above inequality the series resistance \( R_L \) has to be less than the load impedance to enable to be less than the maximum power transfer (impedance matching). The equivalent parallel impedance is,

\[
\frac{v^2}{D_p P_{peak}} \parallel R = \frac{Rv^2}{RD_p P_{peak} + v^2}, \tag{7.17}
\]

which is the upper constraint on \( R_L \).
7.2.3 Energy Storage Frequency Content

Generally, the Fourier series of a PWM signal is,

\[ d_{PWM} = D + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi D)}{n} \cos(n \frac{2\pi}{T} t), \]  \hspace{1cm} (7.18) \]

where D is the duty cycle and T is the period. The frequency content of the pulse load is,

\[ P(t) = P_{peak}D_p + P_{peak} \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi D)}{n} \cos(n \frac{2\pi}{T_p} t). \]  \hspace{1cm} (7.19) \]

And the frequency content of storage device power is

\[ P_u(t) = P(t) - D_pP_{peak} \]
\[ = P_{peak} \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi D)}{n} \cos(n \frac{2\pi}{T_p} t). \]  \hspace{1cm} (7.20) \]

From (7.20) and (7.5), The frequency content of the storage device current is,

\[ u = \frac{2(R + R_L)(P_{peak} \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi D)}{n} \cos(n \frac{2\pi}{T_p} t))}{\sqrt{R(\lambda^2 R v_b^2 - 4R_LD_p^2P_{peak}(R + R_L)) + \lambda R v_b}}. \]  \hspace{1cm} (7.21) \]
For the pulse train in Fig. 7.1, the ES device should be able to supply large amounts of power in appropriately short time. It should also be able to meet the sustaining time \[50\] and have enough capacity. The bandwidth of operation for ESSs is limited. If the storage system does not track the above control input appropriately, the quality of electrical level support may suffer and the performance may degrade. In the next section, band-limited storage devices in ideal form as well as reduced order flywheel and battery models are specified for the system.

### 7.2.4 Hybrid Battery and Flywheel System

Battery and flywheel hybrid storage systems has been widely used to take advantage of the battery energy density and flywheel response and power density \[55, 56\]. A hybrid storage system, with its series and parallel battery and flywheel cells requires some form of power sharing scheme. Here, the hybrid system consists of parallel battery and flywheel systems where each have their respective series and parallel cells. The battery system is considered as the primary storage system and the flywheel system
compensates for when the battery cannot effectively track the reference control signal in (7.5). The reference signals for individual battery and flywheels cells are,

\[ i_{\text{batt,ref}} = \frac{i_{u,\text{ref,total}}}{N_{p,\text{batt}}} \]

\[ i_{\text{fw,ref}} = \frac{i_{u,\text{ref,total}} - i_{u,\text{batt,meas}} \ast N_{p,\text{batt}}}{N_{p,\text{fw}}} \]

where, \( N_{p,\text{batt}} \) and \( N_{p,\text{fw}} \) are the number parallel cells for battery and flywheel systems respectively. \( i_{u,\text{ref,total}} \) is the reference current for the overall hybrid system and \( i_{u,\text{batt,meas}} \) is the measured current injected by the overall battery storage system.

### 7.3 Illustrative Examples

In this section, three illustrative examples are presented. First, a numeric example presents the behavior of the pulse load system in Fig. 7.2 when the ES is controlled according to (7.21). The next example presents the case when the storage system is band-limited as shown in Fig. 7.3. The third example demonstrates the pulse load system behavior when a hybrid of battery and flywheel storage systems are present. For this case, the battery and flywheel systems each comprise series and parallel cells so that they can support the load voltage level as well as the requested current. For this case, the parameters are given in Tables 5.1 and 5.2 corresponding to Fig. 5.3 and Fig. 5.5 are used.
Figure 7.4: Pulsed load and ES powers in time and frequency domains for when duty cycle is a) 5%, b) 50% and c) 90%.

The parameters for the hybrid storage are chosen so that the overall storage meets the minimum requirements given in (7.8). As mentioned in previous sections, the control law in (7.5) accounts only for lossless storage elements hence it is expected that the overall traded energy with the hybrid storage system will not be zero. This means if an auxiliary energy source is not available, over a finite amount of time, the battery and flywheel elements will lose energy (proportional to (5.5) and (5.13)) to a point that they cannot support the system current request in (7.5). The considerations for control of lossy storage systems can bring about several optimization paths which is
Figure 7.5: Energy storage power surface versus frequency and duty cycle

![Energy storage power surface](image)

*Figure 7.5:* Energy storage power surface versus frequency and duty cycle

Figure 7.6: Pulse load and energy storage currents and the regulated load voltage when $\omega_{cut-off}$ is a) $100000 \frac{rad}{s}$, b) $100 \frac{rad}{s}$ and c) $10 \frac{rad}{s}$.

out of the scope of this work. Here, the capacity of the storage systems are chosen so that the storage system can sustain the load for sufficiently long amount of time.

The bandwidths of operation for battery and flywheel systems also depend on their respective control gains. For this example, some reasonable control gains (shown in Tables 5.1 and 5.2) are chosen so that the inherent bandwidths of each storage type are not significantly affected.

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7.3.1 Case 1: Frequency Content Numeric Example

The time domain signal for the pulse load and the device power for up to 30 harmonics are presented in Fig. 7.4. The peak power here is chosen as 700 kW, R is 100 Ω, $R_L$ is 2 Ω, $\lambda$ is 0.5 and $v_b$ is 6 kV.

In Figs. 7.4a, 7.4b and 7.4c, the corresponding load and ES powers are demonstrated for when duty cycle is 5%, 50% and 90% respectively. For each case, the relevant frequency content of the load and storage power are also demonstrated.

In Fig. 7.4 as the load duty cycle increases, there are more shares of power of low frequency contents. This is expected since as the duty cycle $D_p$ increases, the overall waveform tends more and more to a constant value. The most significant feature of
the ES control is that the overall energy trade with the ES element is zero. When the
duty cycle is 0.5, it can be observed that the maximum energy is requested from the
storage system hence, verifies (7.9). Fig. 7.5 presents the overall ES power surface
versus the duty cycle and the frequency.

7.3.2 Case 2: Pulse Load System with Band-limited Storage

Figs. 7.6a, 7.6b and 7.6c present the load current and the ES injected current for
when the cut-off frequency is 100000(rad/s), 100(rad/s) and 10(rad/s). It can be seen
that as the storage element becomes more limited in frequency response, the voltage
regulation suffers. This is because the system with lower $\omega_{\text{cut-off}}$ is not able to track
the control signal as effective as the system with higher bandwidth of operation. The
voltage variation versus the storage cut-off frequency $\omega_{\text{cut-off}}$ is shown in Fig. 7.7

7.3.3 Case 3: Pulse Load with Battery and Flywheel Hybrid
Storage

In this case, series and parallel battery and flywheel systems replace the band-limited
storage in Fig. 7.3. To support the load current and voltage, the battery system com-
prise 10 parallel and 12 series identical cells. Similarly, the flywheel system comprise
Figure 7.8: The overall injected currents for a) battery and b) flywheel systems. c) pulsed load and hybrid storage system currents.

3 and 8, parallel and series identical cells. Figs. 7.8a and 7.8b present the overall injected current by the hybrid battery and flywheel systems. Here, the battery supplies the majority of the power. On the other hand, the flywheel injects current (or power) when the current (or power) deviation exceeds a certain amount. This sharing of power is set by (7.22). The current is requested from flywheel system only to compensate for the current deficit between the control signal and the injected current of the battery system. Fig. 7.8c demonstrates the overall current for the hybrid storage and the pulse load systems.

Figs. 7.9a and 7.9b show individual battery SOC and flywheel RPM respectively. Here, the overall energy of all individual cells decrease however this change is not monotonic and the cells recharge when the instantaneous load power is more than
Figure 7.9: Individual cell a) battery SOC, and b) flywheel RPM. c) pulse load voltage

its average. Fig. 7.9c shows the maintained load voltage. The amount of voltage variation is comparable to the results obtained in Fig. 7.6 and 7.7.

7.4 Summary and Conclusions

This work aimed to provide a baseline for ES control and specification for pulsed power loads. A local ES control scheme was proposed to maintain the voltage and currents of a pulsed power load system. The ideal, band-limited and reduced order hybrid battery and flywheel storage systems were simulated and compared. It was shown that for the ideal lossless system, the ES could achieve zero energy trade over each cycle of the pulsed load duty cycle. On the other hand, the internal losses in the
simulated battery and flywheel systems led to an overall decrease in the energy of the battery and flywheel systems. For accurate sizing of the storage systems, it would be useful to account for losses. Optimization schemes can be used in various ways such as to determine the optimal power flow or the optimal amount of series and parallel cells to reduce losses. The future iterations of this work will include the design of a unified power buffer with its combined converter and ES controllers.
Chapter 8

Conclusions and Future Work

8.1 Conclusions

In this dissertation, a new distributed form of the HSSPFC method was developed for three-phase inverter-based microgrids. The control hierarchy was as follows. A secondary controller, that periodically provides the droop settings to a novel local droop controller in order to maintain the bus voltage at PCC. dq droop controller, that enables distributed operation for the local system and provides local system current references. An inverter feed-forward control, that enforces the references from the droop controller. And finally, an ES element with feedback and feed-forward control, that enables determining the capacity and bandwidth of the ES element. In
In this work, the control was set in a way that ES was requested in a minimal manner. It was shown that ES can operate on zero-output conditions at steady-state and the control system can be used to obtain the capacity and bandwidth requirements versus the control network update-rate. Hence, a tool was created to obtain ES requirements versus additional constraints.

The control system was further expanded to integrate parallel hybrid battery and flywheel ESSs. It was shown that ES requirements versus individual and superimposed constraints can be obtained. It was further demonstrated that power can be shared between BESS and FESSs according to their frequency response capabilities. The overall sharing scheme included band-limited BESS and FESS models and a filter. It was shown that filter can be designed to eventually split the overall power spectrum with respect to filter cut-off frequency. Frequency responses of the filter and BESS were matched, and power sharing was achieved with acceptable accuracy.

In the last portion of this dissertation, the baseline requirements for ES control and specification for pulsed loads was provided. A load-side ES control scheme was used to maintain the electrical levels of the pulsed load. It was shown that ES can be controlled in a way that its combination with the pulsed load mimics a constant power load which can be further integrated into power buffer systems.
8.2 Future Work

Most of the assumptions through this dissertation can be challenged for further work. Moreover, there are several ways to expand this work. In chapter 3, slope change and curve shift can be investigated to improve the transient response of the dq droop control. Since, this control scheme is based on conventional dc droop, it can further be investigated to highlight its advantages or disadvantages in comparison to similar methods such as angle and voltage droop. It might be useful to move the analysis of the system to frequency domain since a bulk of literature is dedicated to relevant concepts.

In chapter 4, the ES capacity requirements versus the communication network update rate was significant. This shows that lack of information of source or load for extended amount of time can cost energy processing through ES. However, a majority of the requested energy can be avoided by simply adding additional filtering. Hence, the developed method in this chapter can be used to design and size filters to reduce the processed energy of the ES element. This is specially useful for sizing the dc-link capacitor as in this work the reservoir capacitor value was chosen in $\mu F$ which is very small for typical bulk capacitors. Cost analysis can also be performed for this case.

In this work, nearly all of the proposed methods were verified through simulation.
However, for many cases such as investigating the synchronism of DGUs under droop control, it is necessary to employ separate control environments. An HIL implementation where each DGU is implemented in separate devices will be a suitable setting to verify the control efficacy for further practical and distributed implementations.

In chapter 6, BESS frequency response was matched with that of a FOLPF. Also, this estimation was performed by adjusting the control system parameters rather than specifying the $RC$ stages of the battery system. Hence, future work includes a more detailed modeling, sizing and matching of BESS and the employed filter. Other ES technologies such as Supercapacitors, can also be considered to be augmented to the power sharing that was given in (6.37). Subsequently, the control system in (6.26) can be expanded for more parallel devices.

In chapter 7, It was shown that the ES combination with the pulsed load imitates constant power loads which face negative incremental impedance problems. Hence, the developed control system can further be integrated into a detailed power buffer system. Furthermore, the power buffer system can then substitute the load system in chapter 6 to enable analysis of source-side and load-side ESSs simultaneously.
References


[10] R. C. Matthews, W. W. Weaver, R. D. Robinett, and D. G. Wilson, “Hamiltonian methods of modeling and control of ac microgrids with spinning machines and


