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POWER FLOW CONTROL IN HYBRID AC/DC
MICROGRIDS

By

Ronald C. Matthews

A DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

In Electrical Engineering

MICHIGAN TECHNOLOGICAL UNIVERSITY

2018

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This dissertation has been approved in partial fulfillment of the requirements for the Degree of DOCTOR OF PHILOSOPHY in Electrical Engineering.

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Abstract

Microgrid structures allow for more efficient utilization of renewable resources as well as autonomous operation. Ideally, a centralized controller would be available to allow for an optimizer to take all components into account so that they may collaboratively work towards a shared goal. To this end, a centralized optimization method was developed called the squared slack interior point method. The novelty of this method is that it incorporates the fraction to bound rule to alleviate the known ill-conditioning introduced by utilizing squared slack variables to handle inequality constraints. In addition, this method also allows for inequality constraint violations to be quantified in the same manner that equality constraints are quantified. The proposed method is found to quickly and accurately calculate the optimal power flow and reject solutions that violate the inequality constraints beyond some specified tolerance.

Where centralized information is not available, a decentralized method is required. In this method, constrained game theoretical optimization is utilized. However, due to unknown information about remote loads, inconsistent solution among players result in overloaded generators. To alleviate this issue, two perturbation methods are introduced. The first is overload feedback and the second is the perturb and observe squeeze method. In both methods, the goal is to adjust voltage angles and magnitudes to locally manage generator output. Both methods are found to rapidly drive overloaded sources back within their desired tolerances. Moreover, the game theoretical approach is found to have poor performance in the absence of shared load information among players. It is determined that the localized optimizers should be removed to reduce cost and that the operating condition should be perturb starting from the most recently available power flow calculation or starting from the nominal value.

Also, to manage voltage stability in the absence of communication, a Hamiltonian approach is implemented for the voltage source rectifier. This approach resulted in a highly stable voltage and

a fast response to large step changes. The method was able to maintain the reference dc output at unity power factor while not requiring any information about loading or interconnection.

Chapter 1

Introduction

In recent years, the environmental impact of the world's energy infrastructure has become an increasing concern. Therefore, much research has been dedicated to increasing the efficiency of energy resources [1]. The majority of energy is extracted from burning hydrocarbon fuels [2]. In terms of environmental impact, a major downside to burning hydrocarbon fuels is that the direct byproduct of the chemical reaction is CO_2 [3]. Therefore, unlike NO_x and CO emissions which are not present under ideal conditions, the only way to decrease CO_2 emissions is to burn less fuel. Thus, reducing environmental impact is directly dependent upon consuming less hydrocarbon fuel. As a result, renewable sources such as photovoltaic (PV) energy and wind energy have become increasingly popular alternatives to hydrocarbon fuels [4]. These renewable resources also require storage due to their intermittent nature [5]. Both PV and battery storage are dc in nature. Also, wind turbines operate at a wind-dependent frequency; therefore, a dc link is required. Thus, renewable sources and storage are often connected in parallel through their power electronics interfaces (PEIs) on a common dc bus forming a dc microgrid.

1.1 Background

Microgrids are useful structures in allowing for efficient management of distributed resources such as small scale distributed generators and renewable energy resources [6]. The microgrid structure allows for more optimal operation with potential results being reduced emissions, and fewer power interruptions [7]. Microgrids can be comprised a complex combination of ac and dc sources; ac and dc

loads; and power electronics interfaces. As [8] discusses, a microgrid is essentially a smaller, localized version of the grid. Microgrids integrate various renewable and non-renewable power sources along with storage. The microgrid can operate in parallel with the main grid in grid-connected mode, but must also be able to operate in a standalone or islanded mode [8]. The definition of a microgrid is relatively standard, but can vary slightly. In [9], three basic properties of a microgrid are discussed. The first property is that there must be clearly defined electrical boundaries. These electrical boundaries are defined such that the microgrid is capable of operating as an isolated system. The second characteristic is that there must be a master controller to manage distributed energy resources (DERs) and loads as a single controllable entity within the defined electrical boundary. Lastly, the total generation capacity should exceed the peak load capacity so that the system can operate in islanded mode. That is, the microgrid is capable of operating independent of the utility grid [9]. Ideally, an optimal power flow algorithm should take into account all ac and dc components in order to arrive at a solution that is as representative of the actual system as possible.

The goal of optimal power flow is to obtain an optimal operating point given all operational constraints are satisfied. The ac optimal power flow (ACOPF) problem was first posed in 1962 by Carpentier [10] and first solved in by Dommel & Tinney [11] in 1968. Dommel & Tinney utilized Newton's method along with quadratic penalty functions to enforce box inequality constraints. For the ACOPF problem, the formulation has not changed much since it was originally posed [12]. Historically, the power grid has been purely ac with transformers being used to step up voltages in order to transmit electrical energy over long distances with minimal losses and to allow for thinner, lower cost transmission lines. However, in recent years, high voltage power electronics interfaces have become more cost effective. These developments have made high voltage direct current (HVDC) transmission a more viable option for energy transport. HVDC transmission offers increased stability; low energy loss; and economical setup and maintenance cost [13]. Renewable energy sources such as photovoltaic (PV) arrays and wind farms have increasingly penetrated the power grid in recent years due to increased environmental concerns [14]. Devices such as PV arrays, wind turbines, electric vehicles, and fuel cells are easier integrated into dc networks since they are either dc in nature or operate at a frequency different from that of the main grid [15]. With the increased penetration of renewable energy sources, energy storage systems are also required in order to compensate for inherent power volatility due to the intermittent nature of these sources [5].

The American Electric Power test systems (IEEE 14-,57-,and 118-bus) have been standard test

cases since the 1960s [16]. With these standard test cases, researchers have been able to work independently on ACOPF algorithms while at the same time being able to compare results with other researchers who have worked on the same identical system. This has aided in the acceleration of research in the area of ACOPF. However, no widely accepted standard exist for hybrid ac/dc systems. Therefore the research that does exist in the area of hybrid ac/dc network optimization has been disjointed with each independent researchers defining different grids, constraints, and optimization goals. Typical methods are to either modify standard IEEE ac test systems by adding dc components as in [17] and [18] or to apply the proposed methodology to a specific system which the author has been working on directly as in [19]. The goal of this dissertation is not to put forth such a standard; however, it is worth emphasizing the need for this standard for the advancement of research in the area of hybrid ac/dc optimal power flow (HACDCOPF).

1.2 Power Management In Grids and Microgrids

Power management is typically broken apart into 3 levels [20]. The primary controller is the fastest controller allowing for rapid dynamic response to load changes. The method utilized for this control is typically droop control. The secondary control, is a longer time scale on the order of seconds to minutes. This level is the main focus of this dissertation. At this level, power flow calculations are computed and new set point are determined. The tertiary control is on the order of hours to days or longer. At this level, an attempt is made to predict long term behavior of the system based upon past data. For example, meteorological data may be utilized as well as historical load data over long time intervals in order to predict future system behavior.

As an example, consider a common power management method, multi-agent based hierarchical control. For this method, each autonomous entity is assigned an agent classification. For example, there are storage unit agents which are devices tasked with managing energy storage; renewable resource agents which are responsible for managing renewable energy resources; microresource agents which are responsible for managing small scale, low-inertia generators; and load agents which are responsible for managing load shedding and restoration. There are 3 level in the hierarchy. The upper level is concerned with energy management of the system as a whole with the goal of optimizing economical and environmental impact. The middle level is concerned with managing the operating modes of individual agents. For example, this agent may manage when a battery changes from

discharge to charge mode or when a PV array is enabled or disabled. At the lower level, frequency and voltage regulation are managed along with load sharing among parallel sources. The typical method at this level is droop control [21].

1.3 Decentralized Optimal Power Flow

In this dissertation, one goal is to develop a decentralized algorithm for maintaining voltage and frequency stability while at the same time avoiding prolonged overloading of individual components. Research into decentralized control of microgrids has been limited to the case in which microgrids are radially connected. This dissertation focuses on the case where loops in the grid structure are present to allow for contingency. Decentralized methods for control of radially connected microgrids have been published in [22], [23], [24], and [25] with the commonality that all focus on some variation of droop control of parallel sources.

Both centralized and decentralized control have drawbacks. For centralized controls, the main issue is that a communication infrastructure is required. This poses a risk to grid security from both natural causes such as trees falling on communication lines in a storm, or more malicious cyber attacks can occur in digital networks. Also such communication networks add additional cost in the network setup. However, centralized networks allow for all components to work collaboratively towards a common goal resulting in a more optimal operating point. Decentralized control is less susceptible to network security problems; however, it does not allow for collaboration among resources which can result in a sub-optimal operating point.

1.4 Contributions of This Dissertation

There are three contributions to the field of network optimization introduced in this dissertation. The first contribution is the control method proposed in section 4.2 for voltage source rectifiers with unknown load information. The goal is to develop a controller for a voltage source rectifier such that load information such as local load power and power sent to the grid are not required for stable operation or maintaining unity power factor. In this section, the Hamiltonian surface shaping and power flow control (HSSPFC) method is applied to the VSR with unknown load information and stability criteria are established analytically with the goals of maintaining dc output voltage stability and maintaining unity power factor.

The second contribution is the squared slack interior point method introduced in Chapter 6. In this chapter, a deterministic method for constrained optimization is introduced. Penalty functions are removed from the optimization process in favor of a direct method for enforcing inequality constraints. This method is found to achieve results that are locally optimal and sufficiently fast from a controls standpoint. A soft constraint is one which should not be violated if at all possible. For example, if the upper bound on a per-unit voltage is 1.05, operating at 1.06 may still be permissible. If a constraint is hard (or rigid), it cannot be violated [11, 26]. Since, in general, nonlinear optimization methods are numerical, a rigid constraint is effectively one that cannot be violated beyond some specified error tolerance. The squared slack interior point method also allows inequality constraint violations to be quantified directly and treats all constraints as rigid. Chapter 6 specifically focuses on hybrid ac/dc grids; however, the squared slack interior point method can be applied to any nonlinear constrained optimization problem.

The third contribution of this dissertation is the game theoretical methods developed in Chapter 7. A localized constrained game theoretic optimization approach is applied at each bus using the squared slack interior point method developed in Chapter 6. Each local optimization is working independently of others since there is no communication infrastructure present. However, since the all variables are coupled, these differing solutions can have adverse affects on each other causing overloading of some components. Therefore perturbation methods are introduced to drive overloaded devices back within their desired operating ranges. A perturb and observe method is introduced and applied to an algebraically solved steady-state ac network model . Also, a closed loop feedback control is applied to a dynamic hybrid ac/dc network model for the same purpose. Both are able to successfully drive overloaded components into their desired ranges without communication with other parts of the system.

1.5 Structure of This Dissertation

The structure of this dissertation is outlined as follows. In Chapter 2, power converter and grid component models are detailed. The Electric Grid Builder (EGB) for MATLAB/Simulink and RT-LAB used to construct general hybrid ac/dc grid models is introduced. EGB automatically pulls blocks from built-in Simulink libraries as well as specialized libraries and automatically places the blocks into a blank Simulink model file to construct a hybrid ac/dc grid model. The model is

constructed based upon system parameters that are stored in an Excel file. This specialized software is further detailed in Appendix A. Chapter 2 introduces the mathematical models of electrical and mechanical components which will be referenced through this dissertation. In Chapter 3, Hamiltonian surface shaping and power flow control design is introduced and applied using power balance to control spinning machines and power inverters. In Chapter 4, Hamiltonian surface shaping and power flow control (HSSPFC) is expanded upon to be applied to various power electronics interfaces. Also, a HSSPFC method for controlling the voltage source rectifier (VSR) in the absence of load information is introduced. In Chapter 5, standard constrained optimization algorithms are discussed. In Chapter 6, the squared slack interior point method for constrained optimization is introduced and applied to the hybrid ac/dc grid. The method is applied to a dynamic system with the goal of driving the steady-state values of the system to those calculated by the optimizer. Convergence and quality of solution is also discussed based on computational trials. In Chapter 7, a decentralized algorithm using constrained game theory is proposed. The method introduced in Chapter 6 is again utilized in solving the constrained game theory problems of this chapter. Dynamic and algebraic simulations are presented in Chapter 7 where feedback control is implemented for the purpose of avoiding overload of components due to inconsistent solutions among competing, but coupled players. In Chapter 8, the conclusions are summarized and plans for future expansion of this work are detailed.

Chapter 2

Power Converter and Grid Models

2.1 Introduction

With significant advancements in semiconductor devices over the past few decades, an increased interest in renewable energy sources, and increased demand for interconnection among ac and high voltage dc networks, there has been heightened interest in power electronic converters. Power converters allow for interconnections to be formed between ac and dc networks and between dc components operating at different voltage levels [27, 28, 29]. In a broad sense, power electronics refers to the part of a system that controls and regulates power flow. Power electronics encompasses various types of converters such as ac to dc, dc to ac, and dc to dc converters. The energy consumption of a power converters should be small relative to the power that it is controlling [30]. The 4 basic types of power converters utilized in this dissertation are buck converters, boost converters, rectifiers, and inverters. These converters are either voltage source or current source converters. Buck converters and boost converters are used to step down and step up dc voltages respectively. The two devices are structurally identical since one is the reverse of the other. The only difference between the two is the controls. That is, for the buck converter, the low voltage side output is to be controlled and for the boost converter, the high voltage side output is to be controlled. A rectifier is used to convert an ac voltage to a dc voltage and its reverse is an inverter. Analogous to the buck and boost converters, the only difference between the rectifier and the inverter is the control scheme. For the rectifier, the dc output is to be controlled and for the inverter the ac output is to be controlled.

For all models in this dissertation, it is assumed that the inductors have both a resistive and

inductive component acting in series. Resistances of inductors are not depicted in figures, but they are always assumed to be present. Also, throughout this dissertation it is assumed that switching is ideal so that there are no frequency-dependent switching losses. It is also assumed that phase inductors are not magnetically coupled among each other. Both mechanical and electrical components are modeled. Wherever possible, models are based on commonly available data or are simplified so that commonly only commonly available data are required to develop the model. For example, the PV array model specifically requires information readily available in datasheets of retail and commercial PV array modules. Also, for the generators, the excitation system is not modeled. Instead, a fixed voltage magnitude is utilized since, in practice detailed excitation system parameters are not readily available for retail or commercial diesel generators.

2.2 Electric Grid Builder for MATLAB/Simulink and OPAL-RT

The dynamic models developed in this chapter are modeled using MATLAB/Simulink. The model generator software is called the Electric Grid Builder (EGB) for MATLAB/Simulink and RT-LAB. The purpose of EGB is to streamline the process of a constructing dynamic hybrid ac/dc grid models in Simulink. Blocks are placed and aligned automatically without the need for the user to interface with Simulink's drag-and-drop interface. Components are defined solely by the parameters within the sheets of the Excel File. EGB reads parameters from an Excel file and automatically generates a HACDC grid model starting from a blank Simulink model file. For a more in depth discussion of EGB, the reader is referred to Appendix A.

2.3 Generator Rotor and Stator Dynamics

The stator and rotor dynamics of the generator are modeled as in Fig. 2.1 [31]. The equations involved in the three-phase system are converted to a more convenient rotor reference frame via Park's transform

$$\mathbf{K} = \frac{2}{3} \begin{bmatrix} \cos(\theta_e) & \cos(\theta_e - \frac{2}{3}\pi) & \cos(\theta_e + \frac{2}{3}\pi) \\ \sin(\theta_e) & \sin(\theta_e - \frac{2}{3}\pi) & \sin(\theta_e + \frac{2}{3}\pi) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \quad (2.1)$$

where θ_e is the rotor electrical angle. Let $F_{abc} = [F_a, F_b, F_c]^T$ and $F_{dq0} = [F_d, F_q, F_0]^T$, then

$$F_{dq0} = \mathbf{K}F_{abc}. \quad (2.2)$$

The stator dynamics are modeled as in [31]. The stator voltage equations are

$$\dot{\lambda}_d = v_{ds} + i_{ds}R_s + \lambda_{qs}\omega_r \quad (2.3)$$

$$\dot{\lambda}_q = v_{qs} + i_{qs}R_s - \lambda_{ds}\omega_r. \quad (2.4)$$

λ_{ds} and λ_{qs} are the d-axis and q-axis stator flux linkages respectively. v_{ds} and v_{qs} are the stator d-axis and q-axis voltages. i_{ds} and i_{qs} are the stator d-axis and q-axis currents. R_s is the stator resistance. ω_r is the rotor mechanical angular frequency. The rotor electrical dynamics are lumped into an ideal sinusoid with varying frequency such that

$$v_{as} = v_{rms,ref,LL} \sqrt{\frac{2}{3}} \cos \theta_e \quad (2.5)$$

$$v_{bs} = v_{rms,ref,LL} \sqrt{\frac{2}{3}} \cos \left(\theta_e - \frac{2\pi}{3} \right) \quad (2.6)$$

$$v_{cs} = v_{rms,ref,LL} \sqrt{\frac{2}{3}} \cos \left(\theta_e + \frac{2\pi}{3} \right), \quad (2.7)$$

where v_{as}, v_{bs} , and v_{cs} are the a, b, and c phase line-to-neutral voltages; $v_{rms,ref,LL}$ is the reference RMS line-to-line stator voltage; and θ_e is the rotor electrical angle. The electrical and mechanical frequency are related by

$$\omega_e = \frac{p}{2}\omega_r, \quad (2.8)$$

where p is the number of poles. The electrical voltage angle is

$$\dot{\theta}_e = \omega_e. \quad (2.9)$$

The electrical frequency is defined by the swing equation

$$J\dot{\omega}_r = (T_m - T_e - B_m|\omega_r|), \quad (2.10)$$

where the power loss due to bearing friction is approximated by the term $B_m \omega_r^2$, J is the overall inertia, and T_m is the mechanical input torque from the prime mover. The electrical torque is

$$T_e = \frac{3}{2} \frac{P}{2} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}). \quad (2.11)$$

Let

$$v_{abc} = [v_{as}, v_{bs}, v_{cs}]^T. \quad (2.12)$$

Applying Park's transform,

$$v_{dq0} = \mathbf{K} v_{abc}. \quad (2.13)$$

The generator output may be either directly coupled to a 3-phase ac bus or coupled to a dc bus through a 3-phase rectifier.

Figure 2.1: Equivalent circuit of 3-phase synchronous generator on a fixed rotor reference frame.

2.4 Diesel Engine

The prime mover for the generator is a diesel engine. A linearized diesel engine is modeled as in [32]. However, a slight modification is made. In addition to the speed controller already present, a rotor

angle control feedback loop is also added. The block diagram for the diesel engine model is shown in Fig. 2.2. ω_{ref} is the reference angular electrical frequency of the generator, ω is the measured angular frequency of the generator, δ_{ref} is the reference voltage angle at the generator terminals, and δ is the measured voltage angle at the terminals of the generator. K is the actuator gain; K_δ is the rotor angle control gain; and T_1, T_2, T_4, T_5 , and T_6 are time constants to be tuned to the specific generator being modeled. T_{min} and T_{max} are the minimum and maximum per unit torque output limits respectively. T_m is the mechanical output torque of the diesel engine. T_d is the engine dead time [32].

Figure 2.2: Diesel Engine Governor Block Diagram.

2.5 Power Electronics Interfaces

In this section, models for the power electronics interfaces (PEIs) utilized in this dissertation are discussed. These PEIs are controlled using Hamiltonian surface shaping and power flow control which is detailed in Chapter 4.

2.5.1 3-Phase Inverter

A three phase inverter is shown in Fig. 2.3. Ideal, complementary switching is assumed. $q_x \in \{0, 1\}$ is the phase x switch state and $q'_x = 1 - q_x$. The sinusoidal duty cycles of switches q_a , q_b , and q_c are D_a , D_b , and D_c respectively. v_{dc} is the dc voltage. In the dissertation, the forward direction of the inverter is considered to be from left to right in the figure. The average model for the inverter is

$$v_{dc}D_a = L \frac{di_a}{dt} + Ri_a + v_{an} + v_{ng} \quad (2.14)$$

$$v_{dc}D_b = L \frac{di_b}{dt} + Ri_b + v_{bn} + v_{ng} \quad (2.15)$$

$$v_{dc}D_c = L \frac{di_c}{dt} + Ri_c + v_{cn} + v_{ng}. \quad (2.16)$$

Assuming balanced operation, (2.14)-(2.16) may be summed to yield

$$(D_a + D_b + D_c)v_{dc} = 3v_{ng} \quad (2.17)$$

so that

$$\left(\frac{2}{3}D_a - \frac{1}{3}D_b - \frac{1}{3}D_c\right)v_{dc} = L\frac{di_a}{dt} + Ri_a + v_{an} \quad (2.18)$$

$$\left(-\frac{1}{3}D_a + \frac{2}{3}D_b - \frac{1}{3}D_c\right)v_{dc} = L\frac{di_b}{dt} + Ri_b + v_{bn} \quad (2.19)$$

$$\left(-\frac{1}{3}D_a - \frac{1}{3}D_b + \frac{2}{3}D_c\right)v_{dc} = L\frac{di_c}{dt} + Ri_c + v_{cn} \quad (2.20)$$

which simplifies to

$$D_a v_{dc} = L\frac{di_a}{dt} + Ri_a + v_{an} \quad (2.21)$$

$$D_b v_{dc} = L\frac{di_b}{dt} + Ri_b + v_{bn} \quad (2.22)$$

$$D_c v_{dc} = L\frac{di_c}{dt} + Ri_c + v_{cn}. \quad (2.23)$$

In balanced 3-phase operation, the average neutral current is zero. Therefore, for the average model, the neutral line had no effect. The two capacitors are placed in series so that the total capacitance is $C = \frac{C_1 C_2}{C_1 + C_2}$. This also hold for the output capacitors of rectifier in the next section.

Figure 2.3: 3-phase inverter with input capacitors.

2.5.2 3-Phase Rectifier

The 3-phase rectifier is shown in Fig. 2.4. The forward direction of the rectifier is assumed to be left to right in the figure. Using analysis similar to that for the inverter, the state average switch mode

equations for the rectifier currents are

$$v_{an} = L \frac{di_a}{dt} + Ri_a + \left(\frac{2}{3}D_a + \frac{1}{3}D_b + \frac{1}{3}D_c \right) v_{dc} \quad (2.24)$$

$$v_{bn} = L \frac{di_b}{dt} + Ri_b + \left(\frac{1}{3}D_a + \frac{2}{3}D_b + \frac{1}{3}D_c \right) v_{dc} \quad (2.25)$$

$$v_{cn} = L \frac{di_c}{dt} + Ri_c + \left(\frac{1}{3}D_a + \frac{1}{3}D_b + \frac{2}{3}D_c \right) v_{dc} \quad (2.26)$$

which reduces to

$$v_{an} = L \frac{di_a}{dt} + Ri_a + D_a v_{dc} \quad (2.27)$$

$$v_{bn} = L \frac{di_b}{dt} + Ri_b + D_b v_{dc} \quad (2.28)$$

$$v_{cn} = L \frac{di_c}{dt} + Ri_c + D_c v_{dc}. \quad (2.29)$$

In dq coordinates, this becomes

$$v_d = Ri_d + L \frac{di_d}{dt} - \omega Li_q + D_d v_d \quad (2.30)$$

$$v_q = Ri_q + L \frac{di_q}{dt} + \omega Li_d + D_q v_q. \quad (2.31)$$

Figure 2.4: 3-phase rectifier with output capacitors.

2.5.3 Buck converter

A buck converter is shown Fig. 2.5. The forward direction is left to right in the figure. $q \in \{0, 1\}$ is the switch state. R and L are the resistance and inductance across the inductor respectively. Assuming ideal and complementary switching and replacing the switch state q with the duty cycle $D \in [0, 1]$, the state equation for the average inductor current is

$$Dv_1 = L \frac{di}{dt} + Ri + v_2. \quad (2.32)$$

Figure 2.5: Buck converter.

2.5.4 Boost converter

A boost converter is shown Fig. 2.6. The forward directions is left to right in the figure. $q \in \{0, 1\}$ is the switch state. R and L are the resistance and inductance across the inductor respectively. Assuming ideal and complementary switching and replacing the switch state q with the duty cycle $D \in [0, 1]$, the state equation for the average inductor current is

$$v_1 = L \frac{di}{dt} + Ri + Dv_2. \quad (2.33)$$

Figure 2.6: Boost Converter.

2.6 PV array Model

Figure 2.7: PV array with output capacitor.

A datasheet model for the PV array is utilized as described in [33]. An output capacitor is connected to the PV array as shown in Fig. 2.7. In this dissertation, the output of each PV array is also connected to the output of a buck converter in order to control the power flow out of the PV array.

Figure 2.8: Five-parameter model of PV cell.

A photovoltaic (PV) cell model is shown Fig. 2.8. As [33] discusses, the standard model used for PV array modeling is the five-parameter model defined by

$$I = I_{ph} - I_0 \left(e^{\frac{q(V + IR_s)}{nkT_c}} - 1 \right) - \frac{V + IR_s}{R_{sh}}. \quad (2.34)$$

$I_{ph}(A)$ is the light generated current; $I_0(A)$ is the dark saturation current due to recombination; $q(C)$ is the charge of a single electron; $R_s(\Omega)$ and $R_{sh}(\Omega)$ are the series and shunt resistances respectively; n is the ideality factor; $k(J/K)$ is the Boltzmann constant; and $T_c(K)$ is the cell temperature. As [33] further discusses, the parameters required for this model R_s , R_{sh} , n , I_0 , and I_{ph} are not values typically published in datasheet information. Additionally, all of these values are dependent upon environmental conditions such as ambient temperature and solar irradiance. Therefore, they can only be conditionally determined for some fixed conditions.

A method which directly utilizes only datasheet parameters is proposed by Vergura [33]. The

output current I of the PV cell is

$$I = \left(1 + \frac{R_s}{R_{sh}}\right)^{-1} \left(G_{pu}(I_{sc}^0 + \alpha_{I_{sc}}\Delta T) - \beta e^{\gamma(V + \alpha_{V_{oc}}\Delta T - V_{oc}^0)} - \frac{V}{R_{sh}} \right). \quad (2.35)$$

where $\Delta T = T_a - 25$, $G_{pu} = G/1000$ is the relative solar radiance, I_{sc}^0 is the short circuit current at standard test conditions (STC), $\alpha_{I_{sc}}$ is the temperature coefficient for the short circuit current, $\alpha_{V_{oc}}$ is the temperature coefficient for the open circuit voltage, V is the output voltage of the cell, and V_{oc}^0 is the open circuit voltage at STC. The shunt resistance is

$$R_{sh} = \frac{V_{mpp}^0 - \alpha_{V_{oc}}\Delta T}{G_{pu} \frac{I_{sc}^0 - I_{mpp}^0}{2}} = \frac{V_{sh}}{I_{sh}}, \quad (2.36)$$

where V_{mpp}^0 is the maximum power point voltage at STC, I_{mpp}^0 is the maximum power point current at STC, V_{sh} is the voltage across the shunt resistance R_{sh} , and I_{sh} is the current through the shunt resistance R_{sh} . The series resistance is

$$R_s = \frac{\frac{V_{oc}^0 - V_{mpp}^0}{4}}{G_{pu}(I_{mpp}^0 + \alpha_{I_{sc}}\Delta T)} = \frac{V_s}{I_s}, \quad (2.37)$$

where V_s and I_s are the voltage drop across and current through the series resistance R_s respectively. The voltage at the terminals of the PV cell is

$$V = V_{sh} - V_s. \quad (2.38)$$

The shunt voltage and series voltage are $V_{sh} = (V_{oc}^0 - \alpha_{V_{oc}}\Delta T)$ and $V_s = \left(\frac{V_{oc}^0 - V_{mpp}^0}{4}\right)$ respectively. As discussed in [33], evaluating β and γ necessitates that some constraints be imposed. These constraints are imposed at three important operating conditions. The conditions considered are the short circuit voltage, $V = 0$; open circuit (no load) voltage $V = V_{oc}$ and maximum power point voltage, $V = V_{mpp}$. This can be stated as

$$I = I_{sc} \text{ for } V = 0 \quad (2.39)$$

$$I = 0 \text{ for } V = V_{oc} \quad (2.40)$$

$$\frac{dP}{dV} = 0 \text{ for } V = V_{mpp}. \quad (2.41)$$

Solving (2.39)-(2.41) yields

$$I_{sc} = p [G_{pu} (I_{sc}^0 + \alpha_{I_{sc}} \Delta T)] \quad (2.42)$$

$$\beta = G_{pu} (I_{sc}^0 + \alpha_{I_{sc}} \Delta T) - \frac{V_{oc}^0 - \alpha_{V_{oc}} \Delta T}{R_{sh}} \quad (2.43)$$

$$\gamma = \frac{1}{V_{mpp}^0 - V_{oc}^0} \cdot \ln \left(\frac{G_{pu} (p I_{sc}^0 - I_{mpp}^0) - (1-p) \alpha_{I_{sc}} \Delta T - \frac{p(V_{mpp}^0 - \alpha_{V_{oc}} \Delta T)}{R_{sh}}}{p \left(G_{pu} (I_{sc}^0 + \alpha_{I_{sc}} \Delta T) - \frac{V_{oc}^0 - \alpha_{V_{oc}} \Delta T}{R_{sh}} \right)} \right) \quad (2.44)$$

where

$$p = \frac{R_{sh}}{R_s + R_{sh}}. \quad (2.45)$$

The result of these calculations is that the parameters β and γ now depend on data commonly available via manufacturer datasheets [33]. The cell temperature $T_c(^{\circ}C)$ is related to the ambient temperature $T_a(^{\circ}C)$ by

$$T_c = T_a + \frac{G}{800} (NOCT - 20), \quad (2.46)$$

where G is the solar irradiance and $NOCT(^{\circ}C)$ is the nominal operating condition temperature. The cell voltage V_{cell} and current I_{cell} are related to the module voltage V_{module} and current I_{module} by

$$V_{cell} = \frac{V_{module}}{N_s} \quad (2.47)$$

$$I_{cell} = \frac{I_{module}}{N_p}, \quad (2.48)$$

where N_s and N_p are the number of cells in series and the number of parallel groups of cells respectively.

2.7 Battery Model

The battery storage is modeled as described in [34, 35]. Three points on the battery's discharge curve are used for the model. These points are $(V_{full}, 0)$, (V_{exp}, Q_{exp}) , and (V_{nom}, Q_{nom}) . As an example, the discharge curve of a Nickel-Metal Hydride battery module is shown in Fig. 2.9. Q_d is the discharge state of the battery and V_{batt} is the voltage at the terminals of the battery. $(V_{full}, 0)$ is the point at which the battery is fully charged. (V_{exp}, Q_{exp}) is the point at which the exponential zone ends. (V_{nom}, Q_{nom}) is the point at which the nominal zone of the operation ends. These points are marked from left to right respectively on Fig. 2.9.

Figure 2.9: Calculated Nickel-Metal hydride battery discharge curve.

The battery voltage is

$$V_{batt} = V_0 - K \frac{Q}{Q - Q_d} + A e^{-B Q_d}, \quad (2.49)$$

where

$$A = V_{full} - V_{exp} \quad (2.50)$$

is the voltage dip in the exponential zone, Q is the battery amp-hour (Ah) capacity,

$$B = \frac{3}{Q_{exp}} \quad (2.51)$$

is the charge at the end of the exponential zone,

$$K = \frac{V_{full} - V_{nom} + A(e^{-B Q_{nom}} - 1)(Q - Q_{nom})}{Q_{nom}} \quad (2.52)$$

is the polarization voltage,

$$Q_d = \int i_{batt} dt \quad (2.53)$$

is the discharge state of the battery, and

$$V_0 = V_{full} + K + R_{batt} i_{rated} - A \quad (2.54)$$

is the voltage constant of the battery. i_{rated} is the rated discharge current of the battery module.

For all of the battery equations described, time is in h and charge is in Ah . The series resistance of

the battery is

$$R_{batt} = V_{nom} \frac{1 - \eta}{0.2Q_{nom}}, \quad (2.55)$$

where η is the battery efficiency ranging between 0 and 1 [34, 35]. The battery characteristics vary depending upon the battery chemistry. Four common types of batteries are considered; lead-acid, Lithium-Ion(Li-Ion), Nickel-Cadmium(Ni-Cd), and Nickel-Metal-Hydride(Ni-MH). The parameters for each type are summarized in Table 2.1. The required parameters are the nominal voltage $V_{nom}(V)$ and the rated capacity $Q(Ah)$. The numerical values in Table 2.1 are taken from the Typhoon HIL software manual [34] with corrections to parameter errors present within manual corrected using values utilized within the Typhoon HIL software.

	Lead-Acid	Li-Ion	Ni-Cd	Ni-MH
V_{full}	$1.08V_{nom}$	$1.16V_{nom}$	$1.15V_{nom}$	$1.17V_{nom}$
i_{rated}	$0.05Q$	$0.2Q$	$0.2Q$	$0.2Q$
Q_{nom}	$0.5Q$	$0.935Q$	$0.905Q$	$0.762Q$
V_{exp}	$1.025V_{nom}$	$1.03V_{nom}$	$1.03V_{nom}$	$1.05V_{nom}$
Q_{exp}	$0.009Q$	$0.85Q$	$0.4Q$	$0.2Q$

Table 2.1: Battery characteristics for common material used to construct batteries.

2.8 Conclusion

The mathematical models introduced in this chapter are implemented using EGB and are used for the dynamic models of Chapters 6 and 7. The goal, again, is that the models are easy to implement based on information that is readily available. The linearized diesel engine model is, however, in stark contrast to this concept. As earlier stated, the parameters must be tuned based upon the specific diesel engine to be modeled. None of the T_i time constants are parameters that are readily available. They also do not represent readily available physical parameters such as the bore, stroke, or power rating of a diesel engine. Therefore, in future work, this simple linear model will be replaced with a more detailed nonlinear diesel engine model which specifically takes into account more representative physical quantities as parameters. In this chapter, PEI models were introduced without addressing the issue of control. Next, in Chapter 3, a nonlinear method for control called Hamiltonian surface shaping and power flow control is introduced of control of spinning machines and inverters. In Chapter 3, this method is expanded upon and is used to develop controls for all of the PEIs earlier discussed in this chapter. The controls developed are used to manage power flow

among ac and dc components to form an efficient microgrid or network of microgrids.

Chapter 3

Hamiltonian Methods of Modeling and Control of Ac Microgrids with Spinning Machines and Inverters

3.1 Introduction

Microgrids are useful tools in allowing streamlined power flow control and utilization of renewable energy as well as distributed generators [6]. They allow for optimal operation with potential results such as emissions reduction, reduced power interruptions, and optimal implementation of combined heat and power systems [7]. Ac microgrids can be very complex collections of energy sources, loads and storage. The complexity is made more pronounced with solid-state power electronic devices, such as inverters on the same network as traditional spinning synchronous generators. To synthesize advanced control structure, a simplified model is needed. Many electro-mechanical dynamic modeling approaches can be taken for an ac microgrid based on traditional electrical power systems analysis [36].

One of the main challenges for microgrid design and control is that generation capacity is very close to load demand. In addition, with the stochastic nature of most renewable energy sources there is a need for energy storage [37, 38, 39]. Energy storage mitigates both long-term and short-term transients. For example, a long term transient is the generation variation over hours and days of

Figure 3.1: Two phasor voltages across a reactance.

a wind turbine or photo-voltaic array due to weather patterns. Short-term transients include step changes in load or faults in the system where the response is on the order of seconds or fractions of a second. Therefore, a proper energy storage strategy will include devices that can respond at the proper bandwidth.

A centralized control structure allows for more optimization options and implementations, yet requires more communication channels and potential single points of failure [40]. It has been shown in [37, 41, 42] that energy storage requirements and control for dc microgrids can be optimized through a nonlinear Hamiltonian surface shaping and power flow control (HSSPFC) approach, which uses the principle of conservation of system energy as a core modeling and control technique [43]. However, the HSSPFC method has not previously been applied to ac microgrids. Typically, in a grid or microgrid, it is desired to drive the system to some reference mode of operation. This dissertation structures a model of the ac microgrid such that the HSSPFC can be applied and shown to be effective.

In this chapter an ac phasor based approach is taken for the circuit models where electrical modes of the system are approximated as complex algebraic quantities or otherwise defined as a dynamic phasors [44]. In [44], this is modeled by a truncated Taylor polynomial; however, in this dissertation the approximation is made simply by neglecting electromagnetic transients. For example, the ac voltages shown in Fig. 3.1 have rms voltage magnitudes V_1 and V_2 with relative phase angles of δ_1 and δ_2 . The two sources are connected by an inductance L . It is assumed that both sources are operating at a common angular frequency ω . From circuit analysis it can be found that the real and reactive power transfer from source 1 to source 2 is

$$P_1 = \frac{V_1 V_2 \sin(\delta_1 - \delta_2)}{\omega L} \quad (3.1)$$

$$Q_1 = \frac{V_1 V_2 \cos(\delta_1 - \delta_2) - V_2^2}{\omega L}. \quad (3.2)$$

A useful and common assumption is to define one of the angles within the circuit as a reference at 0° such that all angles are measured relative to the reference. For example, the source voltage 1 angle in Fig. 3.1 could be set as $\delta_1 = 0$ and the second angle would be defined relative to that reference

angle of zero.

3.2 Single Bus Microgrid

3.2.1 Multi-Machine Dynamic Models

For the purpose of computational efficiency, generator models are often linearized and linear control methods are applied [45]. However, for large perturbations the accuracy of the model can be greatly reduced. As an alternative, in this dissertation, a nonlinear control scheme is proposed utilizing HSSPFC. Consider the general single bus ac model shown in Fig. 3.2. Each electrical machine is modeled as a phasor voltage behind a reactance where the electrical power from the machine l to the bus is

$$P_l = \frac{E_l V_b \sin(\delta_l - \delta_b)}{\omega_l L_l}. \quad (3.3)$$

Then the torque of electrical origin upon the shaft of the machine is

$$T_{e,l} = \frac{P_l}{\omega} = \frac{E_l V_b \sin(\delta_l - \delta_b)}{\omega_l^2 L_l}. \quad (3.4)$$

From Newton's laws of motion, the sum of torques on a rotating body is equal to the time rate of change in the angular momentum. Therefore, the dynamic model for each spinning machine $l = 1, \dots, n$ is

$$J_l \frac{d\omega_l}{dt} = T_{m,l} - \frac{E_l V_b \sin(\delta_l - \delta_b)}{\omega_l^2 L_l} - D_l(\omega_l - \omega_1) - B_l \omega_l \quad (3.5)$$

$$\frac{d\delta_l}{dt} = \omega_l - \omega_1 \quad (3.6)$$

where J_l is the machine rotor inertia, D_l is the damping coefficient, B_l is the friction coefficient, ω_l is the rotor angular frequency and δ_l is the relative rotor angle. In this modeling approach machine 1 serves as the reference with $\omega_1 = \omega_l$ and $\delta_l = 0$. It should also be noted that the damping term D_l is included as a simplified model of the machines damper windings [36] and serves to synchronize the rotors relative to machine 1.

For simplicity, the prime mover, the machine exciter and electrical winding dynamics are not included but are treated as model inputs. However, further model development could be added to capture these effects [36]. Therefore, the control inputs to this machine model are the shaft torque

$T_{m,l}$ and the internal voltage magnitude E_l .

3.2.2 Inverter-Machine Dynamic Models

To include inverter based sources, such as PV and energy storage, a similar modeling approach is taken as the machine model. However, an inverter lacks a mechanical inertia, but does include a dc bus dynamics. The model for inverter $k = n + 1, \dots, n + m$ is based upon a average mode power balance approach as shown in Fig. 3.2. The power to the ac bus from the inverter is

$$P_k = \frac{E_k V_b \sin(\delta_k - \delta_b)}{\omega_1 L_k}. \quad (3.7)$$

Assuming a lossless dc to ac conversion process, the dc power must equal the ac power P_k from 3.7 and a dependent dc current source can be define as

$$i_{dc} = \frac{P_k}{v_{dc,k}} = \left(\frac{1}{v_{dc,k}} \right) \frac{E_k V_b \sin(\delta_{c,k} - \delta_b)}{\omega_k L_k} \quad (3.8)$$

then the dc capacitor in Fig. 3.2 can be modeled as

$$C_{dc,k} \frac{dv_{c,k}}{dt} = \frac{v_{dc,k} + u_{dc,k} - v_{c,k}}{R_{dc,k}} - \frac{E_k V_b \sin(\delta_{c,k} - \delta_b)}{v_{c,k} \omega_1 L_k} \quad (3.9)$$

where the $v_{dc,k}$ represents the dc energy source and $u_{dc,k}$ is the dc energy storage device. The inputs to this model are the internal voltage magnitude E_k , the voltage angle δ_k and the dc storage device voltage $u_{dc,k}$.

The complete circuit model for a multi-machine and inverter microgrid on a single bus is shown in Fig. 3.2. Included in this model is a bus energy storage device modeled as an injected ac phasor current u_b . The ac storage device u_b is modeled as an ideal ac phasor current injection such that the bus voltage and angle are maintained at their reference values. A hardware specific model is not utilized here because the purpose is to determine the required rating of a generic ideal current source.

Figure 3.2: Multi-Machine, one bus ac microgrid with dc to ac inverter.

Figure 3.3: 2 bus circuit.

3.3 Single Bus Ac Microgrid Controls

The control development for the ac single bus microgrid model described in section 3.2 should have three objectives. The first objective is to maintain the system ac synchronism through convergence of all machine frequencies. The second objective is to maintain a system reference frequency. The final objective is to enable a defined performance optimization through annunciation of system reference set-points. The following is a controls synthesis for the single bus ac microgrid shown Fig. 3.2 that enables these objectives. Feed-back control terms will be used to maintain the system frequency and synchronism, while feed-forward control terms will be used to enforce other system reference values, such as bus voltages and angles as well as optimizing the performance of the overall microgrid.

3.3.1 Spinning Machines

The rotor electrical angles of the spinning machines in error coordinates are

$$\tilde{\delta}_l = \delta_l - \delta_{l_r} \quad (3.10)$$

$$\dot{\tilde{\delta}}_l = \dot{\delta}_l = \omega_l - \omega_1 \quad (3.11)$$

$$\ddot{\tilde{\delta}}_l = \ddot{\delta}_l = \dot{\omega}_l - \dot{\omega}_1 \quad (3.12)$$

where δ_{l_r} is a constant reference angle. For machine one, $l = 1$, $\delta_{1_r} = 0$ and $\omega_{1_r} = \bar{\omega}$ is a constant reference frequency. Then the rotor error angle coordinates for machine one are

$$\tilde{\delta}_1 = \delta_1 \quad (3.13)$$

$$\dot{\tilde{\delta}}_1 = \dot{\delta}_1 = \omega_1 - \bar{\omega} = \tilde{\omega}_1 \quad (3.14)$$

$$\ddot{\tilde{\delta}}_1 = \ddot{\delta}_1 = \dot{\omega}_1 = \dot{\tilde{\omega}}_1. \quad (3.15)$$

The total torque command for each machines $l = 1, \dots, n$ is a sum of the feed-forward and feed-back control terms such that the shaft torque on the electrical machine will be

$$T_{m,l} = T_{m,l_r} + \tilde{T}_{m,l}. \quad (3.16)$$

The rotor dynamic models from (3.6) are used to derive the feed forward control of the shaft torque. Since machine one $l = 1$, is the reference angle

$$J_1 \dot{\omega}_{1_r} = 0, \quad (3.17)$$

then feed-forward torque command for machine one is

$$T_{m,1_r} = \frac{E_1 V_b \sin(0 - \delta_b)}{\bar{\omega}^2 L_1} + B_1 \bar{\omega}. \quad (3.18)$$

The feedback control torque command for machine one to maintain the reference frequency is selected to be a PI control

$$\tilde{T}_{m,1} = -K_{p,1}\dot{\delta}_1 - K_{i,1}\delta_1 \quad (3.19)$$

$$= -K_{p,1}\tilde{\omega}_1 - K_{i,1} \int \tilde{\omega}_1 dt. \quad (3.20)$$

The Hamiltonian for machine one control is

$$H_1 = \frac{1}{2}J_1\tilde{\omega}_1^2 + \frac{1}{2}K_{i,1} \left(\int \tilde{\omega}_1 dt \right)^2 \quad (3.21)$$

which is positive definite for all $J_1, K_{i,1} > 0$. Assuming that given $E_1, L_1, \bar{\omega}, \omega_1 > 0$, the time derivative of the Hamiltonian is

$$\dot{H}_1 = \tilde{\omega}_1 \left(J_1\dot{\tilde{\omega}}_1 + K_{i,1} \int \tilde{\omega}_1 dt \right) = -\tilde{\omega}_1^2 \left[[B_1 + K_{p,1}] + \frac{E_1 V_b \sin[\delta_b]}{L_1} \left[\frac{\bar{\omega} + \omega_1}{\omega_1^2 \bar{\omega}^2} \right] \right] < 0 \quad (3.22)$$

for

$$\sin(0 - \delta_b) < 0. \quad (3.23)$$

However, if

$$\sin(0 - \delta_b) > 0 \quad (3.24)$$

then, the proportional gain needs to be selected such that

$$(B_1 + K_{p,1}) > \frac{E_1 V_b \sin(0 - \delta_b)}{L_1} \left(\frac{\bar{\omega} + \omega_1}{\omega_1^2 \bar{\omega}^2} \right). \quad (3.25)$$

The feed-forward torque commands for machines $l = 2, \dots, n$ are

$$T_{m,l,r} = J_l \dot{\omega}_1 + \frac{E_l V_b \sin(\delta_{l,r} - \delta_b)}{\bar{\omega}^2 L_l} + B_l \bar{\omega}. \quad (3.26)$$

The feedback PID control torque commands for machines $l = 2, \dots, n$ are to match machine one's frequency and to maintain a reference angle relative to machine one's rotor position and is

$$\tilde{T}_{m,l,r} = -K_{p,l}\ddot{\delta}_l - K_{i,l} \int \ddot{\delta}_l dt - K_{d,l} (\omega_l - \omega_1). \quad (3.27)$$

The Hamiltonian for this control is

$$H_l = \frac{1}{2} J_l \dot{\delta}_l^2 + \frac{1}{2} K_{p,l} \ddot{\delta}_l^2 > 0 \quad \forall J_l, K_{p,l} > 0. \quad (3.28)$$

The time derivative of the Hamiltonian is

$$\begin{aligned} \dot{H}_l &= \dot{\delta}_l \left(K_{p,l} \ddot{\delta}_l + J_l \ddot{\delta}_l \right) \\ &= \dot{\delta}_l \left(K_{p,l} \ddot{\delta}_l + J_l (\dot{\omega}_l - \dot{\omega}_1) \right) \\ &= -\dot{\delta}_l K_{i,l} \int \ddot{\delta}_l dt - (B_l + D_l + K_{d,l}) \dot{\delta}_l^2 \\ &\quad - \dot{\delta}_l \frac{E_l V_b}{L_l \omega_l^2 \omega_1^2} (\omega_1^2 \sin(\delta_l - \delta_b) - \omega_l^2 \sin(\delta_{l,r} - \delta_b)) \\ &< 0 \quad \forall B_l, D_l, E_l, L_l, K_{d,l}, K_{i,l}, \omega_1, \omega_l > 0 \end{aligned} \quad (3.29)$$

for

$$\sin(\delta_l - \delta_b), \sin(\delta_{l,r} - \delta_b) > 0. \quad (3.30)$$

Specifically, for

$$\sin(\delta_l - \delta_b) = \sin(\delta_{l,r} - \delta_b) = 1 \quad (3.31)$$

then

$$(\omega_1^2 - \omega_l^2) = (\omega_1 + \omega_l)(\omega_1 - \omega_l) = -(\omega_1 + \omega_l) \dot{\delta}_l \quad (3.32)$$

then

$$\dot{\delta}_l^2 \left(\frac{E_l V_b}{L_l \omega_l^2 \omega_1^2} (\omega_1 + \omega_l) \right) > 0 \quad (3.33)$$

so the gains need to be selected such that

$$\dot{\tilde{\delta}}_l^2 \left((B_l + D_l + K_{d,l}) - \frac{E_l V_b}{L_l \omega_l^2 \omega_1^2} (\omega_1 + \omega_l) \right) > -\dot{\tilde{\delta}}_l K_{i,l} \int \tilde{\delta}_l dt.$$

In the case that

$$\sin(\delta_l - \delta_b), \sin(\delta_{l,r} - \delta_b) < 0 \quad (3.34)$$

or more specifically

$$\sin(\delta_l - \delta_b) = \sin(\delta_{l,r} - \delta_b) = -1 \quad (3.35)$$

then

$$(\omega_1^2 - \omega_l^2) = (\omega_1 + \omega_l) \dot{\tilde{\delta}}_k \quad (3.36)$$

and

$$-\dot{\tilde{\delta}}_l^2 \left(\frac{E_l V_b}{L_l \omega_l^2 \omega_1^2} (\omega_1 + \omega_l) \right) < 0 \quad (3.37)$$

demonstrates a stabilizing dissipater. The gains need to be selected such that

$$\dot{\tilde{\delta}}_l^2 \left((B_l + D_l + K_{d,l}) + \frac{E_l V_b}{L_l \omega_l^2 \omega_1^2} (\omega_1 + \omega_l) \right) > -\dot{\tilde{\delta}}_l K_{i,l} \int \tilde{\delta}_l dt. \quad (3.38)$$

For small deviations in the frequency $\omega_l \simeq \omega_1$ and $\tilde{\delta}_l = \delta_l - \delta_{l,r} \ll 1$, the frequency and angles are related by

$$\begin{aligned} & (\omega_1^2 \sin(\delta_l - \delta_b) - \omega_l^2 \sin(\delta_{l,r} - \delta_b)) \\ & \simeq \omega_1^2 (\sin(\delta_l - \delta_b) - \sin(\delta_{l,r} - \delta_b)) \\ & = \omega_1^2 (\cos(\delta_b - \delta_{l,r}) (\delta_l - \delta_{l,r})). \end{aligned} \quad (3.39)$$

Since

$$\sin(\delta_l - \delta_{l,r}) \simeq (\delta_l - \delta_{l,r})$$

$$\cos(\delta_l - \delta_{l,r}) \simeq 1$$

$$\omega_1 \simeq \bar{\omega},$$

it holds that

$$\begin{aligned} (\omega_1^2 \sin(\delta_l - \delta_b) - \omega_l^2 \sin(\delta_{l,r} - \delta_b)) &\simeq \\ \omega_1^2 \tilde{\delta}_l \cos(\delta_b - \delta_{l,r}). \end{aligned} \quad (3.40)$$

Combining (3.40) and (3.29), the Hamiltonian becomes

$$\dot{H}_l - \dot{\tilde{\delta}}_l K_{i,l} \int \tilde{\delta}_l dt - (B_l + D_l + K_{d,l}) \dot{\tilde{\delta}}_l^2 - \left(\frac{E_l V_b}{\bar{\omega}_l^2 L_l} \cos(\delta_b - \delta_{l,r}) \right) \tilde{\delta}_l \dot{\tilde{\delta}}_l. \quad (3.41)$$

The last term on the right hand side of (3.41) can be derived from a potential function which is a conservative term that can be added to the Hamiltonian function to enhance the static stability

$$V_{sm_l} = \frac{1}{2} \left(\frac{E_l V_b}{\bar{\omega}_l^2 L_l} \cos(\delta_b - \delta_{l,r}) \right) \tilde{\delta}_l^2. \quad (3.42)$$

Then the gains should be chosen such that

$$\dot{\tilde{\delta}}_l^2 (B_l + D_l + K_{d,l}) > -\dot{\tilde{\delta}}_l K_{i,l} \int \tilde{\delta}_l dt. \quad (3.43)$$

In addition, if

$$\sin(\delta_l - \delta_b) > 0 \quad (3.44)$$

$$\sin(\delta_{l,r} - \delta_b) < 0 \quad (3.45)$$

or more specifically

$$\sin(\delta_l - \delta_b) = 1 \quad (3.46)$$

$$\sin(\delta_{l,r} - \delta_b) = -1 \quad (3.47)$$

then the gains should be chosen such that

$$\dot{\tilde{\delta}}_l^2 (B_l + D_l + K_{d,l}) > -\dot{\tilde{\delta}}_l \left(K_{i,l} \int \tilde{\delta}_l dt + \frac{E_l V_b}{L_l \omega_l^2 \omega_1^2} (\omega_l^2 + \omega_1^2) \right) \quad (3.48)$$

which may require a "robust term"

$$\left(\tilde{T}_{m,l} \right)_{RT} = -K_{RT} \text{sign} \left(\dot{\tilde{\delta}}_l \right) \quad (3.49)$$

then

$$K_{RT} |\dot{\tilde{\delta}}_l| > -\dot{\tilde{\delta}}_l \left(\frac{E_l V_b}{L_l \omega_l^2 \omega_1^2} (\omega_l^2 + \omega_1^2) \right). \quad (3.50)$$

Also, $\text{sign}(\dot{\tilde{\delta}}_l)$ can be replaced with $\tanh(\beta \dot{\tilde{\delta}}_l)$ to smooth the control input.

Further model development could include prime mover models, such as a turbine or engine. For such prime mover model the input would be a throttle position and the produced torque would be applied to the electrical machine.

3.3.2 Inverter

In the inverter model, the dc storage device is used to maintain the dc capacitor bus voltage. The dc capacitor voltage reference v_{c,k_r} will be a constant,

$$\tilde{v}_{c,k} = v_{c,k} - v_{c,k_r} \quad (3.51)$$

Then the storage voltage command for all inverters $k = n + 1, \dots, n + m$ is

$$u_{dc,k} = u_{dc,k_r} + \tilde{u}_{dc,k} \quad (3.52)$$

The feedback control for the inverter dc bus storage is

$$\frac{\tilde{u}_{dc,k}}{R_{dc,k}} = -K_{P,k}\tilde{v}_{c,k} - K_{I,k} \int \tilde{v}_{c,k} dt. \quad (3.53)$$

The feed forward dc storage device voltage from (3.9) is then found from

$$u_{dc,k_r} = (v_{c,k_r} - v_{dc,k}) + R_{dc,k} \frac{E_k V_b \sin(\delta_{c,k} - \delta_b)}{\omega_1 L_k v_{c,k_r}}. \quad (3.54)$$

The Hamiltonian for the inverter is then

$$H_{dc,k} = \frac{1}{2} C_{dc,k} \tilde{v}_{c,k}^2 + \frac{1}{2} K_{I,k} \left(\int \tilde{v}_{c,k} dt \right)^2. \quad (3.55)$$

which is positive definite for all $C_{dc,k}, K_{I,k} > 0$. The time derivative of the Hamiltonian is given in (3.56),

$$\begin{aligned} \dot{H}_{dc,k} &= \tilde{v}_{c,k} \left(C_{dc,k} \dot{\tilde{v}}_{c,k} + K_{I,k} \int \tilde{v}_{c,k} dt \right) \\ &= \tilde{v}_{c,k}^2 \left(\frac{E_k V_b \sin(\delta_{c,k} - \delta_b)}{\omega_1 L_{c,k} v_{c,k_r} v_{c,k}} - \left(\frac{1}{R_{dc,k}} + K_{P,k} \right) \right) < 0 \\ &\quad \forall E_k, \omega_1, L_{c,k}, v_{c,k_r}, v_{c,k} > 0 \end{aligned} \quad (3.56)$$

and if

$$\sin(\delta_{c,k} - \delta_b) < 0 \quad (3.57)$$

then,

$$- \left(\frac{E_k V_b |\sin(\delta_{c,k} - \delta_b)|}{\omega_1 L_{c,k} v_{c,k_r} v_{c,k}} + \frac{1}{R_{dc,k}} + K_{P,k} \right) < 0 \quad (3.58)$$

is stable. However, if

$$\sin(\delta_{c,k} - \delta_b) > 0 \quad (3.59)$$

then the gains need to be chosen such that

$$\left(\frac{1}{R_{dc,k}} + K_{P,k}\right) > \left(\frac{E_k V_b |\sin(\delta_{c,k} - \delta_b)|}{\omega_1 L_{c,k} v_{c,k} v_{c,k}}\right). \quad (3.60)$$

3.3.3 Ac Bus

The bus storage device maintains the bus voltage and angle to the reference value. There are a couple of approaches to the controls for the ac bus storage device. The first is to determine the current residuals from the sources to maintain the reference bus voltage. Then the injected current from the storage device is then an algebraic relationship

$$u_b = \sum_{k=1}^{m+n} (i_k) - \frac{V_{b_r}(\cos(\delta_{b_r}) + j \sin(\delta_{b_r}))}{R_b + jX_b} \quad (3.61)$$

where i_k is the current injected from the ac sources

$$i_k = \frac{E_k(\cos(\delta_k) + j \sin(\delta_k)) - V_b(\cos(\delta_b) + j \sin(\delta_b))}{j\omega_k L_k}. \quad (3.62)$$

A second, and simpler approach is to model the storage device as a ac phasor voltage source instead of a injected current source. With this approach, the bus voltage is specified as the reference $V_{b_r} \angle \delta_{b_r}$. Then, the current (3.61) is a secondary, or post-process calculation. It is this second approach that will be used in the following examples.

3.4 Multi-Bus Ac Microgrid Model

Any power distribution system, ac or dc, can be described algebraically through the bus nodal admittance matrix [36]. Consider the following n bus system shown in Fig. 3.4, where

$$n = n_{gen} + n_{load}. \quad (3.63)$$

The number of load buses (buses not directly connected to a generator) is n_{load} . The number of generator buses is n_{gen} . S_k is the apparent power of the load at bus k . If there is no load at a bus, then $S_k=0$. Buses 1 through n_{gen} are generator buses. These may or may not have loads attached. Buses $n_{gen}+1$ through $n_{gen}+n_{load}$ are load buses. These buses may or may not have loads

Figure 3.4: Generalized black box network structure.

attached and are not directly connected to a generator. In the proposed model, electromagnetic transients are neglected. The system is analyzed in the phasor domain. For the model, it is assumed that frequency deviations are sufficiently small so that the steady state frequency assumption can accurately represent the system. Define the value of the per unit admittance between buses i and j as Y_{ij} . Also, define

$$Y_{ii} = \sum_{j=1, j \neq i}^{n_{gen} + n_{load}} Y_{ij}. \quad (3.64)$$

The bus admittance matrix $\mathbf{Y}_{bus}$ is then

$$[\mathbf{Y}_{bus}]_{ij} = \begin{cases} -Y_{ij}, & \text{if } i \neq j \\ Y_i + Y_{load,i} + \sum_{k=1, k \neq i}^n Y_{ik} & \text{if } i = j \end{cases}$$

where, Y_i is the shunt admittance due to passive elements (line charging, capacitor bank, shunt reactor, etc.) at bus i . $Y_{load,i}$ is the admittance of the load at bus i . The load at each bus is modeled as a varying shunt admittance. The load admittance at bus i is then

$$Y_{load,i} = \frac{S_{load,i}^*}{|V_i|^2}. \quad (3.65)$$

Assume that the bus indices are ordered such that all of the generator buses are $i = 1, \dots, n_{gen}$ and the load buses are $i = n_{gen} + 1, \dots, n_{gen} + n_{load}$. Partition the bus matrix into $\mathbf{Y}_{11}$, $\mathbf{Y}_{12}$, $\mathbf{Y}_{21}$, and $\mathbf{Y}_{22}$, where

$$\mathbf{Y}_{11} = \{Y_{ij}\}_{i=1, \dots, n_{gen}, j=1, \dots, n_{gen}} \quad (3.66)$$

$$\mathbf{Y}_{12} = \{Y_{ij}\}_{i=1, \dots, n_{gen}, j=n_{gen}+1, \dots, n_{gen}+n_{load}} \quad (3.67)$$

$$\mathbf{Y}_{21} = \{Y_{ij}\}_{i=n_{gen}+1,\dots,n_{gen}+n_{load},j=1,\dots,n_{gen}} \quad (3.68)$$

$$\mathbf{Y}_{22} = \{Y_{ij}\}_{i=n_{gen}+1,\dots,n_{gen}+n_{load},j=n_{gen}+1,\dots,n_{gen}+n_{load}} \quad (3.69)$$

Define the column vectors of generator bus and load bus voltages respectively

$$\mathbf{v}_G = \{v_i\}_{i=1,\dots,n_{gen}} \quad (3.70)$$

$$\mathbf{v}_L = \{v_i\}_{i=n_{gen}+1,\dots,n_{gen}+n_{load}} \quad (3.71)$$

Define the vectors of the generator bus and load bus generator armature currents respectively as

$$\mathbf{i}_G = \{i_i\}_{i=1,\dots,n_{gen}} \quad (3.72)$$

$$\mathbf{i}_L = \{i_i\}_{i=n_{gen}+1,\dots,n_{gen}+n_{load}} = \mathbf{0}_{n_{load} \times 1} \quad (3.73)$$

The armature currents injected into the load buses are zero since there are no generators at these buses. Using these equations, we can relate voltages and armature currents into the buses as

$$\mathbf{Y}_{11}\mathbf{v}_G + \mathbf{Y}_{12}\mathbf{v}_L = \mathbf{i}_G + \mathbf{u}_G \quad (3.74)$$

$$\mathbf{Y}_{21}\mathbf{v}_G + \mathbf{Y}_{22}\mathbf{v}_L = \mathbf{u}_L \quad (3.75)$$

where, $\mathbf{u}_G$ and $\mathbf{u}_L$ are vectors of current injections from storage into the generator buses and load buses respectively. The elements of $\mathbf{u}_G$ and $\mathbf{u}_L$ are defined similar to those of $\mathbf{v}_G$ and $\mathbf{v}_L$ respectively.

Define the vector of internal voltages as

$$[\mathbf{e}_s]_i = \{e_i\}_{i=1,\dots,n_{gen}} \quad (3.76)$$

The machine internal voltage angles are

$$[\boldsymbol{\delta}_s]_i = \text{angle}([\mathbf{e}_s]_i) = \{\delta_i\}_{i=1,\dots,n_{gen}} \quad (3.77)$$

where,

$$\frac{d\delta_i}{dt} = \omega_{1_r} - \omega_i \quad (3.78)$$

The magnitude of each machine internal voltage is modeled as a voltage droop controller with supplementary PI control so that

$$|e_l| = -R_{q,l} (Q_{gen,l} - Q_{gen,l,ref}) + |e_{l,ref}| + k_{pQ,l} (Q_{gen,l,ref} - Q_{gen,l}) + k_{iQ,l} \int (Q_{gen,l,ref} - Q_{gen,l}) dt \quad (3.79)$$

where, $Q_{gen,l}$ is the reactive power output of machine l , $Q_{gen,l,ref}$ is the reference reactive power output of machine l , $R_{q,l}$ is the voltage droop characteristic of machine l , $e_{l,ref}$ is the reference integral voltage for machine l , $k_{pQ,l}$ is the proportional gain for the supplementary voltage control, and $k_{iQ,l}$ is the integral gain for the supplementary voltage control. Define the diagonal matrix

$$[\mathbf{Y}_{int}]_l = \text{diag} \left\{ \left[\frac{1}{j\omega_l L_l} \right] \right\}_{l=1, \dots, n_{gen}} \quad (3.80)$$

The simplified phasor domain electrical model for the system can be written as

$$\begin{bmatrix} \mathbf{v}_G \\ \mathbf{v}_L \\ \mathbf{i}_G \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & -\mathbf{I}_{n_{gen} \times n_{gen}} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \mathbf{0}_{n_{gen} \times n_{load}} \\ \mathbf{I}_{n_{gen} \times n_{gen}} & \mathbf{0}_{n_{gen} \times n_{load}} & \mathbf{Y}_{int}^{-1} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{u}_G \\ \mathbf{u}_L \\ \mathbf{e}_s \end{bmatrix} \quad (3.81)$$

Let

$$\mathbf{u}_b = \begin{bmatrix} \mathbf{u}_G \\ \mathbf{u}_L \end{bmatrix} \quad (3.82)$$

where, $\mathbf{u}_b$ the a feedforward control for the system. Its calculation is shown in the next section (see (3.83)). Similar to the dc inverter, the current injection from storage is not limited. This, again is for the purpose of being able to determine the required storage capabilities.

3.4.1 Multi-Bus Microgrid Controls

The synchronization and system frequency are controlled using a feedback control. A feed-forward control is used to enforce the remaining system values such as bus voltages and angles. Assume that the reference generator bus voltages $\mathbf{v}_{G,ref}$ and load bus voltages $\mathbf{v}_{L,ref}$ are found using some power

flow/optimization algorithm. $\mathbf{u}_G$, $\mathbf{u}_L$, and $\mathbf{e}_{s,\text{ref}}$ can be calculated using

$$\begin{bmatrix} \mathbf{u}_G \\ \mathbf{u}_L \\ \mathbf{e}_{s,\text{ref}} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & -\mathbf{I}_{n_{\text{gen}} \times n_{\text{gen}}} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \mathbf{0}_{n_{\text{gen}} \times n_{\text{load}}} \\ \mathbf{I}_{n_{\text{gen}} \times n_{\text{gen}}} & \mathbf{0}_{n_{\text{gen}} \times n_{\text{load}}} & \mathbf{Y}_{\text{int}}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{G,\text{ref}} \\ \mathbf{v}_{L,\text{ref}} \\ \mathbf{i}_{G,\text{ref}} \end{bmatrix} \quad (3.83)$$

The angles $\boldsymbol{\delta}_{s,\text{ref}} = \text{angle}(\mathbf{e}_{s,\text{ref}})$ are targets that must be tracked using some type of trajectory, since $\boldsymbol{\delta}_s$ is a vector of continuous states. Let

$$\mathbf{x}_l = (\tilde{\omega}_l, \dot{\tilde{\omega}}_l, \tilde{\delta}_l, \dot{\tilde{\delta}}_l) \quad (3.84)$$

To insure stability in the Lyapunov sense, it must hold that (positive definiteness)

$$H_l(\mathbf{x}_l, t) \begin{cases} = \mathbf{0}, & \text{if } \mathbf{x}_l = \mathbf{0} \\ > \mathbf{0}, & \text{if } \mathbf{x}_l \neq \mathbf{0} \end{cases} \quad (3.85)$$

It must also hold that (negative semi-definiteness)

$$\dot{H}_l(\mathbf{x}_l, t) \leq \mathbf{0} \quad (3.86)$$

(3.85) is satisfied as described in equations (3.21) and (3.28). From (3.86) it must hold that the rates in (3.22) and (3.29) must be negative semi-definite. In (3.22) this can be enforced by choosing a sufficiently large $K_{p,1}$ such that over the region of operation, the rate remains negative semi-definite. Consider the definiteness of (3.29). $B_l \geq 0$ and $D_l \geq 0$; therefore, if $K_{d,l} \geq 0$, then the term $-(B_l + D_l + K_{d,l})\dot{\delta}^2$ satisfies (3.86). If $\text{sign}(K_{i,1}) = \text{sign}(\dot{\tilde{\delta}}_l \int \tilde{\delta}_l dt)$, then the term $-\dot{\tilde{\delta}}_l K_{i,l} \int \tilde{\delta}_l dt$ satisfies (3.86). For the remaining term in (3.29), there are no degrees of freedom, therefore the definiteness of the term cannot be directly controlled. For this term, let us first make a few practical considerations. The first consideration is that the angular speed ω_l is bounded. Another assumption we will make is that the generators will operate above some minimum nonzero operating speed $\omega_{\min}$. The exact values of speed $\omega_{\min}$ and speed $\omega_{\max}$ may not be known, but their existence allows us to make an important leap in the discussion of stability. Here, instead of direct analysis of the

derivative of the Hamiltonian, we can instead analyze its bounds. Define

$$\dot{H}_l^a = -\dot{\delta}_l K_{i,l} \int \tilde{\delta} dt - (B_l + D_l + K_{d,l}) \dot{\delta}_l^2 + \dot{\delta}_l \frac{|e_l| |v_{bk}|}{L_l \omega_l^2 \omega_1^2} (\omega_l^2 + \omega_1^2) \quad (3.87)$$

It can readily be seen that $\dot{H}_l \leq \dot{H}_l^a$. Also, define

$$\dot{H}_l^b = -\dot{\delta}_l K_{i,l} \int \tilde{\delta} dt - (B_l + D_l + K_{d,l}) \dot{\delta}_l^2 + \dot{\delta}_l \frac{E_{max} V_{max}}{L_l \omega_l^2 \omega_1^2} (\omega_l^2 + \omega_1^2) \quad (3.88)$$

where, E_{max} and V_{max} are the maximum internal voltage and maximum bus voltage for the entire system respectively. Then, $\dot{H}_l^a \leq \dot{H}_l^b$. Lastly, define

$$\dot{H}_l^c = -\dot{\delta}_l K_{i,l} \int \tilde{\delta} dt - (B_l + D_l + K_{d,l}) \dot{\delta}_l^2 + \dot{\delta}_l \frac{E_{max} V_{max}}{L_l \omega_{min}^4} (2\omega_{max}^2) \quad (3.89)$$

It can be seen that, $\dot{H}_l^b \leq \dot{H}_l^c$. If $\text{sign}(K_{i,l}) = \text{sign}(\dot{\delta}_l \int \tilde{\delta} dt)$ as earlier described and the integral error $\int \tilde{\delta}_l dt$ is nonzero, then it is possible to choose $K_{d,l}$ and $K_{i,l}$ sufficiently large that $\dot{H}_l^c \leq 0$. The requirement that $\int \tilde{\delta}_l dt$ is nonzero can be removed if we choose $K_{i,l} = 0$. In which case, it is sufficient to choose a sufficiently large $K_{d,l}$. We then have

$$\dot{H}_l \leq \dot{H}_l^a \leq \dot{H}_l^b \leq \dot{H}_l^c \leq 0. \quad (3.90)$$

Thus, we have shown that (3.29) satisfies (3.86). Therefore, the system is stable in the Lyapunov sense over the defined region of operation establishing asymptotic stability within the prescribed region of operation.

3.4.2 Multi-Bus Simulation Example

The proposed model is applied to a 20 bus system shown in Fig. 3.5. The generators are marked with a 'G' and the inverted dc sources are marked with an 'I'. The system consists of 5 generators, 2 inverted dc powered sources, and 3 loads. The voltage references for the 4 generators and the 3 inverters are updated every 5 minutes with loss minimization as the objective for the optimal power flow calculation. The loads are held constant for 480 seconds. Then, the load at bus 101 is stepped from no load to 3 per unit (30 kVA) at a lagging power factor of 0.9. The full simulation time is 960 seconds. The gains for the machine 1 control are set to $K_{p,1} = 1000$ and $K_{i,1} = 5$. The gains

Figure 3.5: Initial 20 bus, 5 generator, 2 inverter network.

for machines $l = 2, \dots, n_{gen}$ are $K_{p,l} = 1, K_{i,l} = 0$ and $K_{d,l} = 1000$. The system is modeled and simulated in MATLAB/Simulink.

The initial voltages are all set to 1.0 per unit at an angle of 0 at time $t = 0$ s. The Newton OPF earlier described is applied in Simulink using the MATLAB Function block. It is desired that the system be able to track a reference voltage. The optimizer utilizes metered load power (real and reactive) to determine the optimal voltage levels for the buses. All load buses have some storage available while the generator buses and inverted dc source buses do not have storage available. It is desired to determine the necessary real storage power required for the system to operate over the given load profile. Therefore, the real power and reactive power from storage (current injection) are minimized.

When the simulation starts, the optimizer makes its first run. The optimizer runs every 300 s. At 360 s, the load at bus 101 is stepped. The frequencies start at 0.98 pu and accelerate to 1.0 pu. In Fig. 3.6 there is no noticeable frequency deviation at the time of the load step. However, when the new optimal power flow is calculated at the 600 second mark, the frequencies deviate to allow the voltage angles to be adjusted as desired as seen in Fig. 3.6. The reason for the flat frequency at the time of the load step is that the real and reactive power injection from storage at bus 101 spike in order to compensate for the rapid change in load. This can be seen in the real power shown in Fig. 3.8. The heightened levels of the real and reactive power injection are maintained until the next optimal power flow is calculated at 600 seconds. It can also be seen in Fig. 3.8 that the current

injection is minimized as desired. Current is only injected from storage when transient frequency swings occur or when there is a mismatch between the present loading condition and the loading condition at the time of the last optimal power flow calculation.

Figure 3.6: Generator and inverter electrical frequencies.

Figure 3.7: Generator real power outputs.

Figure 3.8: Real power injection from storage.

The voltage magnitudes and angles are adjusted when the new optimal power flow is calculated. Any adjustment in voltage angles at generator buses requires a temporary deviation in frequency. Rounding up to the nearest half unit power, a charge/discharge power of 0.5 per unit at each load bus is sufficiently large to provide the necessary power to the system during transient events. For bus 101, rounding up to the nearest half unit power, a storage rate (power) of 3 per unit is sufficient. The worst case occurs when the load is stepped at the next infinitesimal time after the optimal power flow is calculated. In this case, the required standby energy at bus 101 must be 810 per unit. In other words, the storage must be capable of delivering 2.7 per unit active power for at least 5 minutes.

The generator output powers are shown in Fig. 3.7. The bus voltages are maintained within 0.07 per unit during transient events and with negligible error during steady state operation. The bus voltage magnitudes are shown in Fig. 3.9. The optimal power flow limits the target voltage magnitudes to a maximum of 1.05 and a minimum of 0.95 per unit at the generator and inverter buses and limits to voltages to a range of 0.8 to 1.2 at the load buses. As can be seen in Fig. 3.9., the voltages remain within these bounds at all times.

Figure 3.9: Bus voltage magnitude.

3.5 Conclusions

This chapter described a simplified spinning machine and inverter model for ac microgrids. The modeling is based on an ac phasor approach and uses complex admittances and power balances to derive the dynamic models. It was found that the modeled systems were capable of maintaining frequency stability and voltage stability while tracking an optimal voltage and responding to a step load change. While optimizations were not addressed directly in this dissertation, the proposed modeling approach offers tools optimization of system operation. In this chapter, HSSPFC was applied to the inverter using power balance to determine the necessary power input from storage to be passed through the inverter. This approach is expanded in Chapter 4 to directly determine the duty cycles of various power electronics interfaces.

Chapter 4

Hamiltonian Surface Shaping and Power Flow Control for Power Electronics Interfaces

4.1 Introduction

It has been shown in [46, 47, 48] that energy storage requirements and control for DC microgrids can be optimized through a nonlinear Hamiltonian surface shaping and power flow control (HSSPFC) approach which uses the principle of conservation of system energy as a core modeling and control technique [49]. Typically, in a grid or microgrid, it is desired to drive the system to some reference mode of operation. In Chapter 3, a time-varying phasor approximation is utilized for forming the Hamiltonian control and power balance is utilized in lieu of direct analysis of the power electronics interfaces (PEIs) and generators. For the generator, the simplified time varying phasor approximation is found to accurately model the dynamic system. This allowed for a simplified but stable control over a wide range of operating condition for the generator [50]. In this dissertation, a direct computation is utilized for power electronics interfaces in contrast to the power balance method utilized in Chapter 3 in order to allow for direct duty cycle computation. In this chapter, HSSPFC is applied to PEIs in order to determine the average duty cycle needed to achieve a stable operating condition.

4.2 HSSPFC Applied to the Voltage Source Rectifier with Unknown Load

A rectifier with generic switches is shown in Fig. 4.1. v_{an} , v_{bn} , and v_{cn} are the phase to neutral voltages. R and L are the resistance and inductance of the inductor respectively. Magnetic coupling among inductor phases is neglected. q_a , q_b , and q_c are the ideal switch states and D_a , D_b , and D_c are the respective sinusoidal duty cycles. Also, $q'_a = 1 - q_a$, $q'_b = 1 - q_b$, and $q'_c = 1 - q_c$. The Hamiltonian is defined as

$$H = \mathcal{T} + \mathcal{V}, \quad (4.1)$$

where $\mathcal{T}$ is the kinetic energy and $\mathcal{V}$ is the potential energy. This Hamiltonian defines the energy surface of the system. This surface may be reshaped by developing a control law for the system. The goal is to drive the system to operate along some stable limit cycle where the energy generation and dissipation are balanced [49]. For the VSR under consideration, the Hamiltonian analysis is limited to the control region marked in Fig 4.1. Within this control region, potential energy is present in the form of energy storage in the phase inductors, but there is no kinetic energy present. The Hamiltonian on a per-phase basis is then

$$H_a = \frac{1}{2}Li_a^2 \quad (4.2)$$

$$H_b = \frac{1}{2}Li_b^2 \quad (4.3)$$

$$H_c = \frac{1}{2}Li_c^2, \quad (4.4)$$

where the total Hamiltonian is the sum

$$H = H_a + H_b + H_c. \quad (4.5)$$

Figure 4.1: 3-phase rectifier with output capacitors.

The voltage equations are

$$L \frac{di_a}{dt} = v_{an} - Ri_a + D_a v_{dc} \quad (4.6)$$

$$L \frac{di_b}{dt} = v_{bn} - Ri_b + D_b v_{dc} \quad (4.7)$$

$$L \frac{di_c}{dt} = v_{cn} - Ri_c + D_c v_{dc}. \quad (4.8)$$

As discussed in [50], the Hamiltonian is stable in the Lyapunov sense if

$$H(\mathbf{x}) > 0 \quad (4.9)$$

$$H(\mathbf{0}) = 0 \quad (4.10)$$

$$\dot{H}(\mathbf{x}) \leq 0. \quad (4.11)$$

It can readily be seen that the Hamiltonian immediately satisfies (4.9) and (4.10). That, is the

Hamiltonian is positive definite. The derivative of the per-phase Hamiltonian is

$$\dot{H}_a = L \frac{di_a}{dt} i_a \quad (4.12)$$

$$\dot{H}_b = L \frac{di_b}{dt} i_b \quad (4.13)$$

$$\dot{H}_c = L \frac{di_c}{dt} i_c. \quad (4.14)$$

Here, the goal will be to enforce $\dot{H} = 0$. Since the currents i_a , i_b , and i_c are not identically zero, this requires

$$L \frac{di_a}{dt} = v_a n - R i_a + D_a v_{dc} = 0 \quad (4.15)$$

$$L \frac{di_b}{dt} = v_b n - R i_b + D_b v_{dc} = 0 \quad (4.16)$$

$$L \frac{di_c}{dt} = v_c n - R i_c + D_c v_{dc} = 0. \quad (4.17)$$

Applying Parks transform, the right hand side in dq coordinates becomes

$$v_d - R i_d - D_d v_{dc} = 0 \quad (4.18)$$

$$v_q - R i_q - D_q v_{dc} = 0. \quad (4.19)$$

Let $H_{d,gen}$ and $H_{q,gen}$ be the d- and q-axis energy generation and $H_{d,diss}$ and $H_{q,diss}$ be the d- and q-axis energy dissipation. The energy generation rate is

$$\dot{H}_{d,gen} = v_d i_d - D_d v_{dc} i_d \quad (4.20)$$

$$\dot{H}_{q,gen} = v_q i_q - D_q v_{dc} i_q \quad (4.21)$$

and the energy dissipation rate is

$$\dot{H}_{d,diss} = R i_d^2 \quad (4.22)$$

$$\dot{H}_{q,diss} = R i_q^2. \quad (4.23)$$

Consider some balanced three phase sinusoidal function where $f_a = f_{peak} \cos \theta$, $f_b = f_{peak} \cos (\theta - \frac{2\pi}{3})$, and $f_c = f_{peak} \cos (\theta + \frac{2\pi}{3})$. Assume that Park transform yields the dq coordinates f_d and f_q . The

phase angle is then

$$\phi = \cos^{-1} \left(\frac{f_q}{\sqrt{f_d^2 + f_q^2}} \right) - \frac{\pi}{2}. \quad (4.24)$$

For unity power factor, it must then hold that

$$\frac{v_q}{\sqrt{v_d^2 + v_q^2}} = \frac{i_q}{\sqrt{i_d^2 + i_q^2}}. \quad (4.25)$$

This yields

$$i_{q,ref} = \pm \frac{i_d v_q}{v_d}. \quad (4.26)$$

The '+' solution is chosen arbitrarily. Define the lumped variables

$$u_d = \frac{v_d - Ri_d}{v_{dc}} \quad (4.27)$$

$$u_q = \frac{v_q - Ri_q}{v_{dc}} \quad (4.28)$$

with references

$$u_{d,ref} = \frac{v_d - Ri_d}{v_{dc,ref}} \quad (4.29)$$

$$u_{q,ref} = \frac{v_q - Ri_{q,ref}}{v_{dc}}. \quad (4.30)$$

The control law chosen for the duty cycle is

$$D_d = \left[k_p (u_{d,ref} - u_d) + k_i \int (u_{d,ref} - u_d) dt \right] \quad (4.31)$$

$$D_q = \left[k_p (u_{q,ref} - u_q) + k_i \int (u_{q,ref} - u_q) dt \right]. \quad (4.32)$$

Substituting into (4.18) and (4.19) yields

$$(u_d v_{dc} + Ri_d) - Ri_d - v_{dc} \left(k_p (u_{d,ref} - u_d) + k_i \int (u_{d,ref} - u_d) dt \right) = 0 \implies \quad (4.33)$$

$$u_d - \left(k_p (u_{d,ref} - u_d) + k_i \int (u_{d,ref} - u_d) dt \right) = 0 \quad (4.34)$$

and

$$(u_q v_{dc} + Ri_q) - Ri_q - v_{dc} \left(k_p(u_{q,ref} - u_q) + k_i \int (u_{q,ref} - u_q) dt \right) = 0 \implies \quad (4.35)$$

$$u_q - \left(k_p(u_{q,ref} - u_q) + k_i \int (u_{q,ref} - u_q) dt \right) = 0. \quad (4.36)$$

Converting to Laplace domain, this becomes

$$u_d(s) = \frac{k_p s + k_i}{s + k_i + k_p s} u_{d,ref}(s) \quad (4.37)$$

$$u_q(s) = \frac{k_p s + k_i}{s + k_i + k_p s} u_{q,ref}(s). \quad (4.38)$$

The limiting behavior is then

$$\lim_{s \rightarrow 0} u_d(s) = \frac{k_p(0) + k_i}{0 + k_i + k_p 0} u_{d,ref}(0) = u_{d,ref}(0) \quad (4.39)$$

$$\lim_{s \rightarrow 0} u_q(s) = \frac{k_p(0) + k_i}{0 + k_i + k_p 0} u_{q,ref}(0) = u_{q,ref}(0). \quad (4.40)$$

Therefore,

$$\lim_{t \rightarrow \infty} u_d(t) = u_{d,ref}(\infty) \quad (4.41)$$

$$\lim_{t \rightarrow \infty} u_q(t) = u_{q,ref}(\infty). \quad (4.42)$$

The exact behavior of $u_{d,ref}$ and $u_{q,ref}$ is unknown. However it can be assumed that these references do not have poles given that the dc voltage does not go to zero. The pole for both (4.37) and (4.38) is then

$$s = -\frac{k_i}{1 + k_p}. \quad (4.43)$$

Therefore, the system is stable when k_i and $1 + k_p$ have the same sign.

4.2.1 Load Step Tracking

Consider again the Fig. 4.1. Assume that a constant power dc load is connected to the dc output of the rectifier (connection to right of capacitors). Starting at no load, the load is stepped by 10 kW every 10 seconds until the load reaches 50 kW. The prime mover is a diesel engine as modeled

in section 2.4. The maximum output of the diesel engine is 55 kW. The overall bearing friction of the 4-pole generator is $B_m = 0.1 \text{ Nm s/rad}$. Here poles refers to the number of magnetic poles not to be confused with the mathematical poles introduced earlier in the Laplace domain analysis. The proportional and integral gains for the Hamiltonian controller are $k_p = 1$ and $k_i = 100$. The dc voltage is shown in Fig. 4.2. The voltage is accurately tracked to its target of 750 V throughout the simulation with short lived transient deviations following load steps. The generator frequency is shown in Fig. 4.3. The frequency is maintained at 60 Hz with minor transient deviations following load steps. However, once the generator becomes overloaded, the frequency begins to droop to its new operating point of about 56.5 Hz. Throughout this process, however, the dc voltage output is maintained. The diesel engine mechanical power and generator active power are shown in Fig. 4.4. It can be seen that after each load step, the generator quickly reaches a new steady state. Also, it can be seen in Fig. 4.5 the reactive power maintains its target value of 0 kVAR so that the desired power factor $pf = 1$ is maintained except of during short lived transients after load steps.

Figure 4.2: Dc output voltage of VSR.

Figure 4.3: Generator frequency.

Figure 4.4: Generator active power.

Figure 4.5: Generator reactive power.

4.2.2 Voltage Reference Step Tracking: HSSPFC v. Feedback Linearization

Now consider the case depicted in Fig. 4.6 where the neutral line is no longer present. Here, the ideal complementary swithing will be applied. The HSSPFC method is compared to the feedback linearization result of [51]. The proportional and integral gains for the Hamiltonian control are $k_p = 2$ and $k_i = 100$. The Hamiltonian control earlier introduced is not altered. However, with the neutral line removed the rectifier model itself is altered. The voltage equations are now

$$v_{an} = L \frac{di_a}{dt} + Ri_a + q_a v_{dc} + v_{gn} \quad (4.44)$$

$$v_{bn} = L \frac{di_b}{dt} + Ri_b + q_b v_{dc} + v_{gn} \quad (4.45)$$

$$v_{cn} = L \frac{di_c}{dt} + Ri_c + q_c v_{dc} + v_{gn}, \quad (4.46)$$

where v_{gn} is the voltage drop from ground to neutral. Without the neutral line present, the phase currents must sum to zero. Also, the input voltages are balanced. Summing (4.44)-(4.46) yields

$$0 = (q_a + q_b + q_c)v_{dc} + 3v_{gn}. \quad (4.47)$$

Solving for v_{gn} and substituting back into (4.44)-(4.46) yields

$$v_{an} = L \frac{di_a}{dt} + Ri_a + \left(\frac{2}{3}q_a - \frac{1}{3}q_b - \frac{1}{3}q_c \right) v_{dc} \quad (4.48)$$

$$v_{bn} = L \frac{di_b}{dt} + Ri_b + \left(-\frac{1}{3}q_a + \frac{2}{3}q_b - \frac{1}{3}q_c \right) v_{dc} \quad (4.49)$$

$$v_{cn} = L \frac{di_c}{dt} + Ri_c + \left(-\frac{1}{3}q_a - \frac{1}{3}q_b + \frac{2}{3}q_c \right) v_{dc} \quad (4.50)$$

Figure 4.6: 3-phase rectifier without line connection.

The load resistance R_L is held at $300\ \Omega$. The 3-phase input voltage is 80 V peak line-to-neutral. The switching frequency is $f_{sw} = 10\text{ kHz}$ and the voltage frequency is 50 Hz . The target dc voltage is step from 300 V to 350 V at $t = 0.1\text{ s}$, then back down to 300 V at $t = 0.9\text{ s}$. It can in be seen from comparing Fig. 4.7 to Fig. 4. of [51] that there is less overshoot using the HSSPFC method that the feedback linearization approach. Also, the gains were chosen somewhat arbitrarily. With further analysis it may be possible to ever further improve performance. Not only does the HSSPFC method outperform feedback linearization, but it does so without requiring any load information. The feedback linearization method of [51] requires load information. It can also be seen in Fig. 4.8 that the voltage and current are in phase so that the unity power goal is enforced.

Figure 4.7: Dc voltage for HSSPFC application to example of [51].

Figure 4.8: Zoomed ac voltage and current for HSSPFC application to example of [51].

4.2.3 Voltage Reference Step Tracking: HSSPFC v. Direct Power Control Based on Fuzzy Sliding Mode

The method proposed in [52] is direct power control based on fuzzy sliding mode. For the remainder of this section this method will be referred to as the direct power control method. The details of this method are not discussed here. However, it is important to note that this method requires a reference load power as part of the control mechanism. That is this control method requires load

information to function properly. Consider again the rectifier of Fig. 4.6. The electrical frequency is $f = 50 \text{ Hz}$, the peak line to neutral voltage is 100 V , $R = 0.3 \Omega$, $L = 2 \text{ mH}$, $C = 4700 \mu\text{F}$, the switching frequency is $f_{sw} = 9.5 \text{ kHz}$, and the load resistance is 20Ω . The dc voltage reference is stepped from 300 V to 330 V at $t = 0.195 \text{ s}$. The proportional and integral gains for the Hamiltonian control are $k_p = 7$ and $k_i = 225$.

Figure 4.9: Zoomed ac voltage for HSSPFC application to example of [52].

Figure 4.10: Power factor for HSSPFC application to example of [52].

It can be seen in Fig. 6 of [52] that it takes about 10 ms to reach the new steady state after the load step using the direct power control method. Also, using this method there is a voltage dip of about 10 V during the transient which occurs after the reference is stepped. For the proposed HSSPFC method, it can be seen in Fig. 4.9 that the voltage dip is negligible. In addition, there is no overshoot as is also the case for the direct power control method. For the HSSPFC method, the transients

last about 20 seconds which is twice as long as for the direct power control method. However, the power flow method requires the load information to be fully known whereas the HSSPFC method requires no load information. Therefore, if there were a loss of communication in an interconnected grid, the direct power control method would effectively become unusable. In addition, the gains for the HSSPFC method have not been optimized, so there may be potential for further improving performance. In addition to accurately and quickly tracking the voltage reference, it can be seen in Fig. 4.10 that the HSSPFC method also maintains near unity power factor throughout the simulation, even during transients. During the transient, the power factor dips to about 97.5% which is still a relatively high power factor.

4.3 HSSPFC Applied to the Current Source Rectifier (CSR)

Consider again the rectifier of Fig. 4.1. Now the goal is to drive the inductor current to some reference value. Let $i_{d,ref}$ and $i_{q,ref}$ be the d- and q-axis reference currents. These currents are sent from some centralized controller or optimizer. Define the variables

$$u_d = v_d - Ri_d \quad (4.51)$$

$$u_q = v_q - Ri_q \quad (4.52)$$

with references

$$u_{d,ref} = v_d - Ri_{d,ref} \quad (4.53)$$

$$u_{q,ref} = v_q - Ri_{q,ref} \quad (4.54)$$

The control law chosen for the duty cycle is

$$D_d = \frac{1}{v_{dc}} \left[k_p (u_{d,ref} - u_d) + k_i \int (u_{d,ref} - u_d) dt \right] \quad (4.55)$$

$$D_q = \frac{1}{v_{dc}} \left[k_p (u_{q,ref} - u_q) + k_i \int (u_{q,ref} - u_q) dt \right]. \quad (4.56)$$

Substitution yields

$$(u_d + Ri_d) - Ri_d - v_{dc} \left(\frac{1}{v_{dc}} \left(k_p(u_{d,ref} - u_d) + k_i \int (u_{d,ref} - u_d) dt \right) \right) = 0 \implies \quad (4.57)$$

$$u_d - k_p(u_{d,ref} - u_d) - k_i \int (u_{d,ref} - u_d) dt = 0 \quad (4.58)$$

and

$$(u_q + Ri_q) - Ri_q - v_{dc} \left(\frac{1}{v_{dc}} \left(k_p(u_{q,ref} - u_q) + k_i \int (u_{q,ref} - u_q) dt \right) \right) = 0 \implies \quad (4.59)$$

$$u_q - k_p(u_{q,ref} - u_q) - k_i \int (u_{q,ref} - u_q) dt = 0 \quad (4.60)$$

Note that (4.56) and (4.58) are identical to (4.34) and (4.36) respectively. Therefore, the analysis and resultant convergence results are identical.

4.4 HSPFC Applied to the Voltage Source Inverter

Consider the rectifier shown in Fig. 4.11. The control region is marked similar to that of the rectifier. Here the goal is to drive the sinusoidal output voltage to the desired value. The ac output terminals are to the far right. Following similar reasoning to that for the rectifier, it can be seen that the Hamiltonian is positive definite and that driving the derivative of the Hamiltonian to zero requires that

$$D_d v_{dc} - Ri_d - v_d = 0 \quad (4.61)$$

$$D_q v_{dc} - Ri_q - v_q = 0. \quad (4.62)$$

Define the variables

$$u_d = Ri_d + v_d \quad (4.63)$$

$$u_q = Ri_q + v_q \quad (4.64)$$

and with respective references

$$u_{d,ref} = Ri_d + v_{d,ref} \quad (4.65)$$

$$u_{q,ref} = Ri_q + v_{q,ref}. \quad (4.66)$$

The control law is chosen as

$$D_d = \frac{1}{v_{dc}} \left[k_p(v_{d,ref} - v_{d,ref}) + k_i \int (v_{d,ref} - v_{d,ref}) dt \right] \quad (4.67)$$

$$D_q = \frac{1}{v_{dc}} \left[k_p(v_{q,ref} - v_{q,ref}) + k_i \int (v_{q,ref} - v_{q,ref}) dt \right]. \quad (4.68)$$

Figure 4.11: 3-phase inverter with input capacitors.

Substitution yields

$$\frac{1}{v_{dc}} \left(k_p(u_{d,ref} - u_d) + k_i \int (u_{d,ref} - u_d) dt \right) v_{dc} - Ri_d - (u_d - Ri_d) = 0 \implies \quad (4.69)$$

$$k_p(u_{d,ref} - u_d) + k_i \int (u_{d,ref} - u_d) dt - u_d = 0 \quad (4.70)$$

and

$$\frac{1}{v_{dc}} \left(k_p(u_{q,ref} - u_q) + k_i \int (u_{q,ref} - u_q) dt \right) v_{dc} - Ri_q - (u_q - Ri_q) = 0 \implies \quad (4.71)$$

$$k_p(u_{q,ref} - u_q) + k_i \int (u_{q,ref} - u_q) dt - u_q = 0 \quad (4.72)$$

It can be seen that (4.70) and (4.72) are identical to (4.34) and (4.34) respectively. Therefore the analysis and resultant convergence results are identical.

4.5 HSSPFC Applied to the Buck Converter

Consider the buck converter shown in Fig. 2.5. Following similar reasoning to that for the VSR, it can be seen that the Hamiltonian is positive definite and that driving the derivative of the Hamiltonian to zero requires that

$$Dv_1 - Ri - v_2 = 0. \quad (4.73)$$

Define the variable

$$u = Ri + v_2 \quad (4.74)$$

with reference

$$u_{ref} = Ri + v_{2,ref}. \quad (4.75)$$

The control law is chosen as

$$D = \frac{1}{v_1} \left[k_p(u_{ref} - u) + k_i \int (u_{ref} - u) dt \right]. \quad (4.76)$$

Substitution yields

$$\frac{1}{v_{dc}} \left(k_p(u_{ref} - u) + k_i \int (u_{ref} - u) dt \right) v_{dc} - Ri - (u - Ri) = 0 \implies \quad (4.77)$$

$$k_p(u_{ref} - u) + k_i \int (u_{ref} - u) dt - u = 0. \quad (4.78)$$

It can be seen that (4.78) is identical to (4.34) in structure. Therefore the analysis and resultant convergence results are identical.

4.6 HSSPFC Applied to the Boost Converter

Consider the boost converter shown in Fig. 2.6. Following similar reasoning to that for the buck converter, it can be seen that the Hamiltonian is positive definite and that driving the derivative of the Hamiltonian to zero requires that

$$v_1 - Ri - Dv_2 = 0. \quad (4.79)$$

Define the variable

$$u = \frac{v_1 - Ri}{v_{dc}} \quad (4.80)$$

with reference

$$u_{ref} = \frac{v_1 - Ri}{v_{2,ref}}. \quad (4.81)$$

The control law is chosen as

$$D = k_p(u_{ref} - u) + k_i \int (u_{ref} - u)dt. \quad (4.82)$$

Substitution yields

$$(uv_2 + Ri) - Ri - \left(k_p(u_{ref} - u) + k_i \int (u_{ref} - u)dt \right) v_2 = 0 \implies \quad (4.83)$$

$$u - k_p(u_{ref} - u) - k_i \int (u_{ref} - u)dt = 0. \quad (4.84)$$

It can be seen that (4.84) is identical to (4.78) in structure. Therefore the analysis and resultant convergence results are identical.

4.7 Conclusion

For all converters discussed in this chapter, the convergence criteria were identical. Therefore, the proportional and integral gains k_p and k_i may be chosen as system-wide values without the need for separate stability analysis for each component. In Chapter 6, the Hamiltonian based controls for the CSR, VSI, boost, and buck converters introduced in this chapter are applied to a centralized network of interconnected microgrids. For the VSR, the HSSPFC was shown to have

fast response to large and rapid load steps. For the switching models of VSR without a neutral line, the HSSPFC method was compared to 2 methods from literature which were feedback linearization and direct power control. In both methods, load information is directly utilized in the development and implementation of the controller. For the feedback linearization method, the HSSPFC method has similar transient duration. However, the HSSPFC had negligible overshoot whereas overshoot for the feedback linearization method was substantial. When compared to direct power control, the HSSPFC method has slightly slower voltage tracking. However, the transient voltage dip during the transient even was negligible for the HSSPFC method, whereas for the direct power control method the dip was substantial. The most important takaway in these comparisons is that both the feedback linearization method and the direct power flow method effective had an unfair advantage in that they both relied on load information. Were this information lost to either, they would both be rendered useless. The HSSPFC method was able to outperform the methods for certain criteria while at the same time using no load information. The HSSPFC method proposed in this chapter for VSR control is more versatile in that in the event of a loss of communication with the grid, it can continue operate without any loss of performance. For cases discussed, the proposed HSSPFC method was able to accurately track the desired dc output voltage at while at the same time maintaining unity power factor. Because the proposed HSSPFC method allows for the VSR to be controlled without any load information being utilized, the HSSPFC method is later applied to the decentralized control in Chapter 7 where interconnection information is unknown to the local controller.

Chapter 5

Constrained Optimization Methods

5.1 Introduction

In this chapter, popular methods for centralized steady-state power flow control are discussed. Optimization methods fall under two classifications: deterministic and stochastic. For both cases, an initial guess for a solution is required. Typical deterministic methods are derivative based. These methods seek a local optimum. Solutions of derivative based methods are directly dependent upon the initial guess. This single solution is perturbed until a final solution (local optimum) is found. Stochastic methods allow for parallel searches to be carried out to increase the likelihood of arriving at a global optimum regardless of the quality of the initial guess. Although stochastic methods improve the likelihood of arriving at a global optimum, these methods require repeated trials, which increases the solution time. From a pure optimization standpoint, stochastic methods are ideal. However, from a controls standpoint, faster deterministic methods are more ideal. Two standard stochastic methods and two standard deterministic methods are discussed in this chapter. The genetic algorithm (GA) and particle swarm optimization (PSO) are stochastic global optimization algorithms; whereas Newton's method and the interior point method are deterministic local optimization algorithms. These methods are compared as it relates to controls application and overall optimality of solution.

5.2 Genetic Algorithm (GA)

The genetic algorithm is inherently a method for unconstrained optimization. However, as discussed in [53], modifications may be made to this method in order for it to be applied to the constrained problem. The genetic algorithm (GA) consists of 3 major processes: reproduction (selection), crossover, and mutation. The GA is a derivative-free method. Unlike derivative-based methods, which seek to iteratively improve a single solution, the GA during its implementation is dealing with a population of potential solutions at any given time. A fitness value is assigned based upon the objective function value is assigned to each member of the aforementioned population. A higher fitness value means that the member will survive longer. Therefore, better solutions are assigned higher fitness values. A random initial population is selected to start the process. Then successive populations are created by reproduction, crossover, and mutation algorithms to yield an improved population of solutions which approach the global optimal solution of the problem as generations progress. Characteristics that make good candidates for an optimal solution are dominant so that they have a better chance of being inherited by future generations. Poor characteristics are recessive, but the probability that they get passed on to future generations is still nonzero as is the case with true genetics. Stabilization occurs when all members of the population are identical. Once the population has stabilized, the algorithm is said to have converged [53].

Figure 5.1: Basic Genetic Algorithm.

A flowchart depicting the complete genetic algorithm is shown in Fig. 5.1. Gen is the generation number and MaxGen is the maximum number of generations. The process starts with random initial population. The remaining processes in the flow chart are detailed in the subsections that follow.

5.2.1 Encoding and Decoding

As discussed in [54], the values of the control variables must first be encoded to a binary string. The control variables proposed in [54] for optimal power flow in ac grids are active power generation $P_{Gi}, \forall i \in \{1, \dots, N\}$, bus voltage $V_i, \forall i \in \{1, \dots, N\}$, transformer tap settings $t_i, \forall i \in \{1, \dots, N_t\}$, and bus shunt admittances $b_{SHi}, \forall i \in \{1, \dots, N\}$, where N is the number of buses and N_t is the number of tap-changing transformers. Power generation and voltage are continuous variables whereas, tap position and shunt admittance are discrete variables. A chromosome will take the form shown in Fig. 5.2. In order for the continuous variables to be coded as binary variables, they must be discretized into a finite number of data points.

Figure 5.2: Example chromosome for 4-bus system.

For control variable u_i , if u_i is a continuous variable on the interval $[u_i^{min}, u_i^{max}]$, then the binary representation of the variable k is

$$k = \hat{k} - \text{mod}(\hat{k}, 1), \quad (5.1)$$

where

$$\hat{k} = \frac{(u_i - u_i^{min})(2^{N_{u_i}} - 1)}{u_i^{max} - u_i^{min}} \quad (5.2)$$

and N_{u_i} is the number of bits used to store the variable u_i . If u_i is a discrete variable such that $u_i \in \{u_i^1, \dots, u_i^M\}$, then the binary representation of the variable k is

$$k = \left\lceil \frac{(m - 1.5)2^{N_{u_i}}}{M} \right\rceil,$$

where

$$N_{u_i} = \lceil \log_2 M \rceil,$$

and $m \in \{1, \dots, M\}$ is the m^{th} value in the ordered solution space of u_i . Once a final solution has been found, the binary string must be converted back into the original form of the variable. If u_i is

continuous on the interval $[u_i^{min}, u_i^{max}]$, the binary representation k is decoded as

$$u_i = u_i^{min} + (u_i^{max} - u_i^{min}) \frac{k}{2^{N_{u_i}} - 1}. \quad (5.3)$$

If u_i is discrete taking on the M values $u_i^1, \dots, u_i^M$, then the variable u_i is decoded as

$$u_i = u_i^m, \quad (5.4)$$

where

$$m = \text{integer} \left[\frac{M}{2^{N_{u_i}}} k + 1.5 \right]. \quad (5.5)$$

The minimum necessary number of bits should be utilized so that $N_{u_i} = \lceil \log_2(M) \rceil$. It should be noted that k is defined such that $k = 0$ represents the binary $[0, \dots, 0]$ and $k = 2^{N_{u_i}} - 1$ represents the binary string $[1, \dots, 1]$, where the strings are of length N_{u_i} [54].

5.2.2 Fitness Function (FF)

As discussed in [54], the goal of the GA is to minimize some cost function given some functional constraints. To accomplish this, a fitness function (FF) is assigned taking both into account. Note that only the inequality constraints are directly handled using the FF. Assume that the general optimization problem is of the form

$$\min f(\mathbf{x}, \mathbf{u}), \text{ subject to} \quad (5.6)$$

$$\mathbf{g}(\mathbf{x}, \mathbf{u}) = \mathbf{0} \quad (5.7)$$

$$\mathbf{h}(\mathbf{x}, \mathbf{u}) \leq \mathbf{0} \quad (5.8)$$

$$\mathbf{u} \in U. \quad (5.9)$$

The FF for the GA is then

$$FF = \frac{A}{\sum_{i=1}^{N_G} F_i(P_{Gi}) + \sum_{i=1}^{N_C} w_i Pen_j + \sigma \sum_{i=1}^{2N} |g_i(\mathbf{x}, \mathbf{u})|}, \quad (5.10)$$

where

$$Pen_i = |h_i(\mathbf{x}, \mathbf{u})|H(h_i(\mathbf{x}, \mathbf{u})). \quad (5.11)$$

N is the total number of buses, N_G is the number of generating units, N_C is the number functional inequality constraints, $H(\cdot)$ is the Heaviside step function, $F_i(P_{Gi})$ is the fuel cost function for unit i , w_i is the weighting factor for violation of inequality constraint $h_i(\mathbf{x}, \mathbf{u})$, σ (sufficiently larger than w_i) is the weighting factor for violating equality (load flow/power balance) constraints, and Pen_i is the penalty function for the i th inequality constraint. In the transformer model, shunt admittances are neglected. For clarity, the same transformer will be modeled in 2 different manners, step-down and step-up. The step-down transformer is modeled as shown in Fig. 5.3 and the step-up transformer is modeled as in Fig. 5.4. For the step-down transformer, the active and reactive power flow equations for bus i are respectively

$$P_{ij} = \frac{V_i V_j}{t_{ij}} Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i - \tau_{ij}) \quad (5.12)$$

and

$$Q_{ij} = -\frac{V_i V_j}{t_{ij}} Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i - \tau_{ij}), \quad (5.13)$$

where t_{ij} and τ_{ij} are the turns ratio and phase shift of the tap changing transformer respectively; Y_{ij} and θ_{ij} are the magnitude and angle of the bus admittance matrix respectively; V_i and V_j are the bus i and j voltage magnitudes respectively; and δ_i and δ_j are the bus i and j voltage angles respectively. The step-down (forward) direction of the transformer is assumed from left to right in Fig. 5.3. For the step-down transformer, the active and reactive power flow equations for bus i are respectively

$$P_{ij} = \frac{V_i V_j}{t_{ji}} Y_{ij} \cos(\theta_{ij} + \delta_j + \tau_{ji} - \delta_i) \quad (5.14)$$

and

$$Q_{ij} = -\frac{V_i V_j}{t_{ji}} Y_{ij} \sin(\theta_{ij} + \delta_j + \tau_{ji} - \delta_i). \quad (5.15)$$

The step-up (forward) direction of the transformer is assumed from left to right in Fig. 5.3.

Figure 5.3: Step-down tap-changing transformer with core admittance neglected. Voltages and admittances are per-unit.

Figure 5.4: Step-up transformer with core admittance neglected. Voltages and admittances are per-unit.

As detailed in [54], given some candidate for a solution, the FF is evaluate by the following steps:

- Step 1: Decode the binary chromosomes to determine decimal value of the control parameter $\mathbf{u}$ using (5.3)-(5.5).
- Step 2: Solve the power flow to calculate the state vector x . As discussed in [54], the state variables $\mathbf{x}$ are calculated by substituting the control variables $\mathbf{u}$ into the load flow (or power balance) equations.
- Step 3: Evaluate the penalty functions (5.11) to asses boundary constraint violations.
- Step 4: Use (5.10) to compute the FF.

For the load flow calculation in Step 2, [54] recommends using fast decoupled load flow (FDLF) for fast convergence.

5.2.3 Selection

The two selection techniques are discussed in [55] are the tournament selection and the random selection technique. In the tournament selection technique, 3 chromosomes are randomly selected

from the population. The one with the largest FF replicates itself twice in the mating pool and the one with the 2nd largest FF replicates itself once in the mating pool. This process is repeated until the population of the mating pool reaches the magnitude of the original population. In the random selection technique, 2 integers are randomly generated. These integers represent the binary coded chromosomes as shown in Fig. 5.2. The one with the higher FF is sent to the mating pool. This process is repeated until the population of the mating pool reaches the initial population [55].

5.2.4 Crossover

As discussed in [54], crossover is the main process by which structure recombination occurs. In this process, 2 parents are randomly selected from the gene pool and combined to produce a new chromosome which inherits segments of information stored in the parent chromosomes. Segments of the parent chromosomes are exchanged with relatively high probability. For example, the crossover probability of 0.6-0.9 is suggested in [54]. For an in depth review of many classical crossover methods, the reader is referred to [56]. The method chosen in [54] is uniform crossover. In uniform crossover the exchange happens to individual bits instead of strings. The two parents exchange bits with probability p . So that the expected number of bits to be exchanged is pL , where L is the chromosome length [56]. As [54] discusses, no new information is introduced during the crossover process, old information from a pair of parents is simply passed on to the next generation.

5.2.5 Mutation

The mutation process is where new information is introduced to the population. During this process, each bit in a chromosome will flip with some small probability. [54] recommends a mutation probability in the range of range of 0.0001-0.001. The the reason that this probability is low is to minimize the possibility that good genes are destroyed. That is dominant traits (better solutions) should be discarded with low, but nonzero, probability. If the probability is too high, the evolution process degrades to a random search [57].

5.3 Particle Swarm Optimization

Particle swarm optimization (PSO) is another stochastic method. Similarly to GA, PSO improves the likelihood of arriving at a global optimal solution by searching the entire solution space. Particle

swarm optimization, as detailed in [58], is summarized in Fig. 5.5.

Figure 5.5: Particle swarm optimization flowchart.

As discussed in [58], PSO was developed based upon observations seen in individuals within a flock of animals searching for food. Members are able to independently search while at the same time sharing information with other members of the flock. Each individual within the group represents a potential solution to the problem. The position, velocity, individual past experience, and past experience of neighbors are taken into account to approach an optimal solution [58].

Let $\mathbf{x}_i^k$ be the position vector of individual i at iteration k and $\mathbf{v}_i^k$ be the velocity vector of individual i at iteration k . Also let $\alpha_i \in [0, 1]$ and $\beta_i \in [0, 1]$ come from a uniform random distribution. Also, let $\mathbf{x}_i^{lbest}$ be the best solution for individual i and $\mathbf{x}^{gbest}$ be the best global solution for the swarm. The iterates for position and velocity vectors of individual i are given by

$$\mathbf{x}_i^{(x+1)} = \mathbf{x}_i^{(k)} + \mathbf{v}_i^{(k+1)} \quad (5.16)$$

and

$$\mathbf{v}_i^{(x+1)} = \mathbf{v}_i^{(k)} + \alpha_i \left(\mathbf{x}_i^{lbest} - \mathbf{x}_i^{(k)} \right) + \beta_i \left(\mathbf{x}_i^{gbest} - \mathbf{x}_i^{(k)} \right) \quad (5.17)$$

respectively [58]. As mentioned in [59], PSO was originally intended to solve for the continuous unconstrained problem. In [59], discrete variables are solved as continuous, then rounded to the nearest discrete value. The optimal per flow problem is formulated as (5.6)-(5.8). Penalty terms are used to account for constraints. As [58] discusses, the penalty term $\Omega(\mathbf{x})$ is

$$\Omega(\mathbf{x}) = \rho \left(\sum_{k=1}^{n_e} g_i^2 + \sum_{k=1}^{n_i} \max(0, h_i(\mathbf{x}))^2 \right), \quad (5.18)$$

where n_e and n_i are the number of equality and inequality constraints respectively and ρ is the penalty factor. The penalty function is then

$$P(\mathbf{x}) = f(\mathbf{x}) + \Omega(\mathbf{x}). \quad (5.19)$$

The constrained problem (5.6)-(5.8) has been converted to the unconstrained problem (5.19) so that the goal is now the minimize the unconstrained function $P(\mathbf{x})$ [58]. PSO is similar to GA in that it allows for parallel searches of the entire solution space improving the likelihood of moving towards a globally optimal solution. However, a major difference in GA and PSO is that PSO does not require the separate solution of a power flow at each iteration allowing for faster iterations.

5.4 Optimal Power Flow by Newton Approach

Newton's method was the first nonlinear optimization method to be successfully applied to solving the ACOPF problem in 1968 [11]. The Newton approach to optimal power flow is a deterministic method. First and second order derivatives are utilized directly to find local optimal solutions. The quality of the solution is directly dependent on the starting point. Here, a single solution is iterated to approach a local optimum. As discussed in [60, 26], the Lagrangian of (5.6)-(5.7) may be written as

$$\mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \sum_{i=1}^{n_e} \lambda_i g_i(\mathbf{x}), \quad (5.20)$$

where λ_i is the Lagrange multiplier associated with equality constraint g_i . The Newton Approach is specifically designated to solve optimization problems with equality constraints. Where inequality constraints are present as is the case for any OPF problem, [60, 26] suggest that a penalty function be utilized similar to the case of (5.18). The penalty term $\Psi_i(\mathbf{x})$ corresponding to inequality constraint

$h_i(\mathbf{x})$ may be defined as the quadratic

$$\Psi_i(\mathbf{x}) = \rho_i \max(0, h_i(\mathbf{x}))^2, \quad (5.21)$$

where $\rho_i > 0$ is a penalty factor. The quantity $\max(0, h_i(\mathbf{x}))$ is squared to ensure that the first order derivatives remain continuously differentiable. The terms (5.21) are added to the objective function $f(\mathbf{x})$ so that the Lagrangian for (5.6)-(5.8) may be written as

$$\mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \sum_{i=1}^{n_i} \Psi_i(\mathbf{x}) + \sum_{i=1}^{n_e} \lambda_i g_i(\mathbf{x}), \quad (5.22)$$

where n_e and n_i are the number of equality and inequality constraints respectively. Notice that in this approach there is no distinction between control variables and state variables. The variable $\mathbf{x}$ encompass both control and state variables since all variables are able to be solved simultaneously. Let $\mathbf{z} = [\mathbf{x}^T \lambda^T]^T$. The Karush-Kuhn-Tucker (KKT) conditions for (5.22) are

$$\nabla_x \mathcal{L}(\mathbf{z}) = \mathbf{0} \quad (5.23)$$

$$\nabla_\lambda \mathcal{L}(\mathbf{z}) = \mathbf{0} \implies \mathbf{g} = \mathbf{0}. \quad (5.24)$$

The linearization of (5.23)-(5.24) is given by

$$\begin{bmatrix} \nabla_{xx}^2 \mathcal{L}(\mathbf{z}^k) & \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \\ \frac{\partial \mathbf{g}^T}{\partial \mathbf{x}} & 0 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x}^k \\ \Delta \lambda^k \end{bmatrix} = - \begin{bmatrix} \nabla_x \mathcal{L}(\mathbf{z}^k) \\ \mathbf{g}(\mathbf{x}^k) \end{bmatrix}, \quad (5.25)$$

where $\frac{\partial \mathbf{g}^T}{\partial \mathbf{x}} = \left[\frac{\partial g_1}{\partial x_1}, \dots, \frac{\partial g_{n_e}}{\partial x_{n_x}} \right]$, n_x is the number of variables, $\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta \mathbf{x}$, $\lambda^{k+1} = \lambda^k + \Delta \lambda$, and

$$\nabla_{xx}^2 \mathcal{L}(\mathbf{z}^k) = \nabla_{xx}^2 f(\mathbf{x}^k) + \nabla_{xx}^2 \Psi(\mathbf{x}^k) + \sum_{i=1}^{n_e} \lambda_i \left[\frac{\partial^2 g_i}{\partial x^2} \right] \quad (5.26)$$

is the Hessian. Note that the reformulation of the inequality constraints using penalty terms results in these constraints being soft constraints. That is, there is no tolerance specified for violation of these constraints. It is possible that, depending how the penalty factors ρ_i are chosen, the violation of the boundary constraints may exceed the error tolerance of the optimizer. Care must be chosen in how ρ_i are chosen. If values are too small, inequality constraints may be excessively violated; and

if ρ_i is too large, ill-conditioning may occur during the solution process. The optimal power flow has converged if:

- All active and reactive power mismatches are within some specified tolerance.
- The gradient of the Lagrangian is near zero within some specified tolerance.

[60, 26].

5.5 Interior-Point Logarithmic Barrier Method

Let n_e , denote the number of equality constraints, n_i denote the number of inequality constraints, and n_x denote the number of variables. In order to solve (6.26), the problem is rewritten using the slack variable $\mathbf{s} \geq \mathbf{0} \in \mathbb{R}^{n_i}$ [61]. (6.26) is rewritten as the system of equations

$$\text{Minimize } f(\mathbf{x}), \text{ subject to } \mathbf{c_E}(\mathbf{x}) = \mathbf{0}, \mathbf{c_I}(\mathbf{x}) - \mathbf{s} = \mathbf{0} \quad (5.27)$$

The Lagrangian of (5.27) is

$$\mathcal{L} = f(\mathbf{x}) - \mathbf{y}^T \mathbf{c_E}(\mathbf{x}) - \mathbf{z}^T (\mathbf{c_I}(\mathbf{x}) - \mathbf{s}), \quad (5.28)$$

where $\mathbf{y} \in \mathbb{R}^{n_e}$ and $\mathbf{z} \in \mathbb{R}^{n_i}$ are Lagrange multipliers for the equality and inequality constraints respectively.

It was earlier stated that $\mathbf{s} > \mathbf{0}$; however, there is nothing enforcing this. As is, it is possible for the elements of $\mathbf{s}$ to become negative during the solution process. As discussed in [61], in order to force $\mathbf{s}$ to remain positive, a logarithmic barrier is utilized such that $f(\mathbf{x})$ is replaced with $f(\mathbf{x}) - \mu \sum_{i=1}^{n_i} \ln(s_i)$. The Lagrangian of the augmented system is then

$$\mathcal{L} = f(\mathbf{x}) - \mathbf{y}^T \mathbf{c_E}(\mathbf{x}) - \mathbf{z}^T (\mathbf{c_I}(\mathbf{x}) - \mathbf{s}) - \mu \sum_{i=1}^{n_i} \ln(s_i). \quad (5.29)$$

Newton's method is used to find the search directions $\mathbf{p_x}$, $\mathbf{p_s}$, $\mathbf{p_y}$, and $\mathbf{p_z}$ at each iteration for $\mathbf{x}$, $\mathbf{s}$, $\mathbf{y}$, and $\mathbf{z}$ respectively. The fraction to boundary rule (5.34-5.35) is utilized to prevent the solution from approaching the boundary too quickly. The new iterate is then

$$\mathbf{x}^+ = \mathbf{x} + \alpha_s^{max} \mathbf{p_x} \quad (5.30)$$

$$\mathbf{s}^+ = \mathbf{s} + \alpha_s^{max} \mathbf{p}_s \quad (5.31)$$

$$\mathbf{y}^+ = \mathbf{y} + \alpha_z^{max} \mathbf{p}_y \quad (5.32)$$

$$\mathbf{z}^+ = \mathbf{z} + \alpha_z^{max} \mathbf{p}_z, \quad (5.33)$$

where

$$\alpha_s^{max} = \max\{\alpha \in (0, 1] : \mathbf{s} + \alpha \mathbf{p}_s \geq (1 - \tau) \mathbf{s}\} \quad (5.34)$$

$$\alpha_z^{max} = \max\{\alpha \in (0, 1] : \mathbf{z} + \alpha \mathbf{p}_z \geq (1 - \tau) \mathbf{z}\}. \quad (5.35)$$

The two linear optimization problems, (5.34) and (5.35), with $\tau \in (0, 1)$ form what is called the fraction to boundary rule [61]. Here the parameter τ shall be referred to as the bound rate. A typical value is $\tau = 0.995$. (5.34) and (5.35) are two independent linear optimization problems in a single variable α . For this problem, a closed form may be derived. Note that the elements of $\mathbf{p}_s$ and $\mathbf{p}_z$ may take on any positive, negative, or zero value. Consider (5.34). The subscript s is dropped for convenience so that

$$\alpha^{max} = \max\{\alpha \in (0, 1] : \mathbf{s} + \alpha \mathbf{p} \geq (1 - \tau) \mathbf{s}\}. \quad (5.36)$$

Let $\mathcal{N}$ and $\mathcal{P}$ be the set of indices for the negative and positive elements of $\mathbf{p}$ respectively. The constraints of (5.36) may be rewritten in a more convenient form as

$$\max \left\{ \max_{k \in \mathcal{P}} \left\{ \frac{-\tau s_k}{p_k} \right\}, 0 \right\} < \alpha \leq \min \left\{ \min_{l \in \mathcal{N}} \left\{ \frac{-\tau s_l}{p_l} \right\}, 1 \right\}. \quad (5.37)$$

From (5.37), it can be seen that if $\mathbf{p}$ has negative components, the maximum value of α is

$$\alpha^{max} = \min \left\{ \min_{l \in \mathcal{N}} \left\{ \frac{-\tau s_l}{p_l} \right\}, 1 \right\}. \quad (5.38)$$

If $\mathbf{p}$ has no negative elements, then the solution is $\alpha^{max} = 1$. Similar reasoning may be applied to (5.35), so that

$$\alpha_s^{max} = \min \left\{ \min_{l \in \mathcal{N}} \left\{ \frac{-\tau s_l}{p_l} \right\}, 1 \right\} \quad (5.39)$$

and

$$\alpha_z^{max} = \min \left\{ \min_{l \in \mathcal{N}} \left\{ \frac{-\tau z_l}{p_l} \right\}, 1 \right\}. \quad (5.40)$$

Let $\mathbf{A}_E(\mathbf{x})$ and $\mathbf{A}_I(\mathbf{x})$ denote the Jacobians of the equality and inequality constraints respectively.

As discussed in [61], the basic interior point method may be summarized as follows:

Let

$$E = \max \{ \|\nabla f(\mathbf{x}) - \mathbf{A}_E^T(\mathbf{x})\mathbf{y} - \mathbf{A}_I^T(\mathbf{x})\mathbf{z}\|, \|S\mathbf{z} - \mu\|, \|\mathbf{c}_E(\mathbf{x})\|, \|\mathbf{c}_I(\mathbf{x}) - \mathbf{s}\| \}. \quad (5.41)$$

Choose some initial value for $\mathbf{x}_0$ and $\mathbf{s}_0 > \mathbf{0}$ and compute the initial values for the Lagrange multipliers $\mathbf{y}_0$ and $\mathbf{z}_0$. The initial value of $\mathbf{z}$ is

$$\mathbf{z}_0 = -\mu \mathbf{S}_0^{-1} \mathbf{e}, \quad (5.42)$$

where $\mathbf{S}_0 = \text{diag}(\mathbf{s}_0)$ and $\mathbf{e} = [1, \dots, 1]^T \in \mathbb{R}^{n_i}$. n_i denotes the number of inequality constraints.

Also, choose some initial barrier parameter $\mu_0 > 0$ and $\sigma, \tau \in (0, 1)$. The initial value of $\mathbf{y}$ is

$$\mathbf{y}_0 = (\mathbf{A}_E(\mathbf{x}_0) \mathbf{A}_E^T(\mathbf{x}_0))^{-1} (\mathbf{A}_E(\mathbf{x}_0) (\nabla f(\mathbf{x}_0) - \mathbf{A}_I^T(\mathbf{x}_0)\mathbf{z})). \quad (5.43)$$

while *stopping criteria for nonlinear program (5.27) are not satisfied* **do**

while $E > \mu_k$ **do**

 1) Solve

$$\begin{bmatrix} \nabla_{xx}^2 \mathcal{L} & \mathbf{0}_{n \times n_i} & \mathbf{A}_E^T(\mathbf{x}) & \mathbf{A}_I^T(\mathbf{x}) \\ \mathbf{0}_{n_i \times n} & \mathbf{S}^{-1} \mathbf{Z} & \mathbf{0}_{n_i \times n_e} & -\mathbf{I}_{n_i \times n_i} \\ \mathbf{A}_E(\mathbf{x}) & \mathbf{0}_{n_e \times n_i} & \mathbf{0}_{n_e \times n_e} & \mathbf{0}_{n_e \times n_i} \\ \mathbf{A}_I(\mathbf{x}) & -\mathbf{I}_{n_i \times n_i} & \mathbf{0}_{n_i \times n_e} & \mathbf{0}_{n_i \times n_i} \end{bmatrix} \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_s \\ -\mathbf{p}_y \\ -\mathbf{p}_z \end{bmatrix} = - \begin{bmatrix} \nabla_x f(\mathbf{x}) - \mathbf{A}_E^T(\mathbf{x})\mathbf{y} - \mathbf{A}_I^T(\mathbf{x})\mathbf{z} \\ \mathbf{z} - \mu \mathbf{S}^{-1} \mathbf{e} \\ \mathbf{c}_E(\mathbf{x}) \\ \mathbf{c}_I(\mathbf{x}) - \mathbf{s} \end{bmatrix} \quad (5.44)$$

 to obtain the search direction $\mathbf{p} = [\mathbf{p}_x^T, \mathbf{p}_s^T, \mathbf{p}_y^T, \mathbf{p}_z^T]^T$.

 2) Compute $\alpha_s^{max}, \alpha_z^{max}$ using (5.34) and (5.35) respectively.

 3) Compute the new iterate $(\mathbf{x}_{k+1}, \mathbf{s}_{k+1}, \mathbf{y}_{k+1}, \mathbf{z}_{k+1})$ using (5.30) -(5.33).

 4) Set $\mu_{k+1} \leftarrow \mu_k$ and $k \leftarrow k + 1$.

end

 Choose $\mu_k \in (0, \sigma \mu_k)$

end

Algorithm 1: Interior point logarithmic barrier method (primal-dual interior point method) for constrained nonlinear optimization.

To ensure convergence of this algorithm, care must be taken in how the μ_k is decreased. Con-

vergence is highly dependent upon how this parameter is decreased. For further discussion of this method and convergence as it relates to the sequence $\{\mu_k\}$, the reader is referred to [61].

5.6 Conclusion

The focus of this dissertation is on control. For faster response to load changes, deterministic methods are preferred even though they can only guarantee locally optimal solutions. In addition, a solution is only accurate for as long as the load or any other parameters or variables remain constant. If the solution takes several minutes to calculate and the load is changing every few seconds, even the globally optimal solution will be inaccurate by the time it is calculated. Therefore, local optimization methods are utilized in this dissertation. These methods do, however, have some disadvantages. For example, in Newton's method, if penalty functions are utilized, attempting to enforce hard constraints (using large penalty factors) may introduce ill-conditioning into the system. Also the interior point method, requires careful reduction of the perturbation factor μ_k at each iteration. Also, in either case it is not possible to guarantee that the inequality constraints are satisfied within some set tolerance. To circumvent these issues, an alternative method is proposed in Chapter 6.

Chapter 6

A Squared Slack Interior Point Method for Hybrid Ac/Dc Grid Optimization

6.1 Introduction

A new interior point method is developed to handle centralized control in HACDC microgrids. The interior point method developed in this dissertation is referred to as the squared slack interior-point method. The reason that this method was developed was to avoid the use of penalty functions in handling inequality constraints and to develop a method requiring minimal tuning of parameters to better ensure convergence of the methods. A typical penalty function as suggested in [62, 63] is

$$\phi_1(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^{n_i} \rho_i [\max(0, h_i(\mathbf{x}))]^2, \quad (6.1)$$

where n_i is the number of inequality constraints, $h_i(\mathbf{x}) \leq 0$ is the i th inequality constraint and $\mathbf{x}$ is the array of control and state variables. $\rho_i > 0$ is the constant penalty factor for the i th inequality constraint. If $F(\mathbf{x})$ is the objective function and only inequality constraints are present, the modified objective function is

$$F_1(\mathbf{x}, \boldsymbol{\rho}) = F(\mathbf{x}) + \frac{1}{2} \sum_{i=1}^{n_i} \rho_i [\max(0, h_i(\mathbf{x}))]^2. \quad (6.2)$$

Higher values of ρ_i result in stronger enforcement of inequality constraints. However, if ρ_i are too large, ill-conditioning is potentially introduced into (6.2)[63]. Other penalty functions such as the log-barrier method utilized in solving the interior point method also exist. In each case the inequality constraints are similarly weighted in order to penalize constraint violations. In the case of the interior point method with logarithmic barriers, the penalty factors must be carefully modified each iteration to ensure convergence.

Another downside of penalty functions is that hard tolerances cannot be set for violations of boundary constraints. This dissertation introduces a method where inequality constraints are converted into equality constraints so that all inequality may be treated as hard constraints. A hard constraint is one such that when it is violated beyond a certain error tolerance, the solution is rejected. If a solution to the problem actually exists, it will be found within some specified error tolerance of the inequality constraints. If the solution does not exist, the problem is ill-defined and the method will fail to converge. This is in contrast to penalty function methods which are allowed approach solutions outside the feasible region, except where such solution are not mathematically defined, such as those requiring division by zero. Constraint violation is allowed, but discouraged for penalty function methods.

Once the inequality constraints are converted to equality constraints, the Karush-Kuhn-Tucker (KKT) conditions are drastically simplified and simple Lagrangian methods may be applied directly to the problem. This method does introduce ill-conditioning. However, applying the fraction to boundary rule described in [61] is found to alleviate the known ill-conditioning issue caused by the introduction of squared slack variables [64]. The centralized optimization introduced in this chapter which consists of converting inequality constraints to equality constraints using squared slack variables and applying the fraction to boundary rule to avoid ill-conditioning is referred to as the squared slack interior point method.

In microgrids, the generation capacity is often very close to the load demand which poses a challenge in the design and control of such a system. Another issue in microgrids is the stochastic nature of renewable energy sources [46, 65, 39]. Faster optimization results allow for more accurate response to changing load demand and varying environmental conditions. The squared slack interior point method is found to provide fast and accurate results for efficient control of power flow in HACDC microgrids as well as ac grids in general.

6.2 Grid Structure

Let $\mathbf{Y}_{\text{bus}}$ be the bus admittance matrix of an n bus HACDC grid. Assume that all loads are given in terms of active and reactive power. At dc buses there are no shunt resistors to ground. At ac buses there are no shunt resistors or inductors to neutral.

Let $\mathcal{A}$ and $\mathcal{D}$ be the set of ac and dc buses respectively. Also, let $\mathcal{ACDC}$ and $\mathcal{DCDC}$ be the set of ac/dc (or dc/ac) and dc/dc converters respectively. Define $\mathbf{v}_{ac} = [V_k]_{k \in \mathcal{A}} \in \mathbb{R}^{n_{ac}}$, $\mathbf{v}_{dc} = [V_k]_{k \in \mathcal{D}} \in \mathbb{R}^{n_{dc}}$, $\boldsymbol{\delta}_{ac} = [\delta_k]_{k \in \mathcal{A}} \in \mathbb{R}^{n_{ac}}$, $\boldsymbol{\lambda}_{ac} = [\lambda_k]_{k \in \mathcal{ACDC}} \in \mathbb{R}^{n_{acdc}}$, $\boldsymbol{\mu}_{dc} = [\mu_k]_{k \in \mathcal{DCDC}} \in \mathbb{R}^{n_{dcdc}}$, and $\boldsymbol{\gamma}_{ac} = [\gamma_k]_{k \in \mathcal{ACDC}} \in \mathbb{R}^{n_{acdc}}$ as the vectors of ac bus voltage RMS magnitudes; dc bus voltages; ac bus voltage angles; RMS duty cycle magnitudes for inverters/rectifiers; duty cycles for dc/dc converters; and angles for inverter or rectifier duty cycles. For the HACDC system, 6 basic components are considered: three-phase ac lines, dc lines, buck converters, boost converters, three-phase rectifiers, and three-phase inverters. Note that the ac RMS voltages $\{V_k\}_{k \in \mathcal{A}}$ are line-to-neutral values.

6.3 Per-Unit Values in the HACDC Grid

For the networks considered in this dissertation, no voltages transformers are present. Therefore, a single base voltage is utilized. The line-to-line base voltage is $V_{base,LL}$, and the line-to-neutral base voltage is $V_{base,LN} = \frac{V_{base,LL}}{\sqrt{3}}$. The base three-phase power for the system is $P_{base,3\Phi}$. This is also the base for the dc power so that

$$P_{base,dc} = P_{base,3\Phi}. \quad (6.3)$$

The base voltage for all dc components is

$$V_{base,dc} = V_{base,LN}. \quad (6.4)$$

The steady-state admittance of the dc line, buck converter, or boost converter between buses i and bus j be defined as

$$Y_{i,k} = \frac{1}{R_{i,k}}, \quad (6.5)$$

where $R_{i,k}$ is the line resistance. In this work, it is assumed that all ac lines are 3-phase and that admittances among line phases are equal. A one-line representation is utilized as in Fig. 6.1. Using this representation, the per-phase admittances should be multiplied by a factor of 3 so that the one

line representation of the line is treated as the equivalent of 3 parallel lines. That is, the current of the 3 phase load is treated as though it is carried by a single equivalent line. Also, let the steady-state admittance of the ac line, inverter, or rectifier between bus i and k be defined as

$$\tilde{Y}_{i,k} = \frac{3}{R_{i,k} + j\omega L_{i,k}}, \quad (6.6)$$

where $L_{i,k}$ is the line inductance and ω is the angular frequency of the ac voltage. Inductance is present in both ac and dc lines. However, inductance only affects the dc portion of the system during transient events. If there is an ideal capacitance to neutral C_i at an ac bus i , then the phase to neutral admittance due to the capacitance is

$$\tilde{Y}_i^{C_n} = j3\omega C_i. \quad (6.7)$$

The complex bus admittance matrix is then

$$\tilde{\mathbf{Y}}_{i,k}^{\text{bus}} = \begin{cases} -Y_{i,k}, & \text{if } (i,k) \text{ is a dc line and } i \neq k \\ -\tilde{Y}_{i,k}, & \text{if } (i,k) \text{ is an ac line and } i \neq k \\ \sum_{k=1, \dots, n_{dc}+n_{ac}} Y_{i,k}, & \text{if } i \text{ is a dc bus and } i = k \\ \tilde{Y}_i^{C_n} + \sum_{k=1, \dots, n_{dc}+n_{ac}} \tilde{Y}_{i,k}, & \text{if } i \text{ is an ac bus and } i = k \end{cases}. \quad (6.8)$$

The base admittance for dc lines and dc/dc converters is

$$Y_{base,dc} = \frac{P_{base,dc}}{V_{base,dc}^2} \quad (6.9)$$

and the base admittance for ac lines, inverters, rectifiers, and phase-to neutral ac capacitors is

$$Y_{base,ac} = \frac{P_{base,3\Phi}}{V_{base,LL}^2}. \quad (6.10)$$

The initial values chosen for the HACDC grid optimizer are

$$\mathbf{x}_0 = [\mathbf{v}_{min}^T, \mathbf{0}_{n_{ac} \times 1}^T, \frac{0.5}{\sqrt{2}} \mathbf{1}_{n_{acdc} \times 1}^T, \frac{1}{2} \mathbf{1}_{n_{dc} \times 1}^T, \mathbf{0}_{n_{acdc} \times 1}^T, \mathbf{0}_{(n_{ac}+n_{dc}) \times 1}^T, \mathbf{0}_{n_{ac} \times 1}^T]^T \quad (6.11)$$

$$\mathbf{s}_0 = \mathbf{1}_{n_i \times 1} / \epsilon_{tol} \quad (6.12)$$

$$\mathbf{z}_0 = \mathbf{1}_{n_i \times 1} / \epsilon_{tol} \quad (6.13)$$

$$\mathbf{B}_0 = \mathbf{I}_n / \epsilon_{tol}. \quad (6.14)$$

6.4 Power Balance

In this subsection, the power balance equation for the HACDC power grid is detailed. A solution is feasible if and only if there is both active and reactive power balance at each node. For inverters, reactive power may be injected up to some defined limit at the ac output. Therefore, although reactive power cannot be transferred from the dc source, it can be injected by the inverter itself.

Define the matrix

$$\mathbf{T}_{i,k} = \begin{cases} 0, & \text{if } i=k \\ 1, & \text{if } (i,k) \text{ is dc line} \\ 2, & \text{if } (i,k) \text{ is ac line} \\ 3, & \text{if } (i,k) \text{ is buck converter} \\ 4, & \text{if } (i,k) \text{ is boost converter} \\ 5, & \text{if } (i,k) \text{ is rectifier} \\ 6, & \text{if } (i,k) \text{ is inverter} \end{cases} \quad (6.15)$$

and the vector

$$\mathbf{t}_k = \begin{cases} 1, & \text{if bus } k \text{ is dc bus} \\ 2, & \text{if bus } k \text{ is ac bus} \end{cases}. \quad (6.16)$$

Also, since a buck converter is a reversed boost converter and a rectifier is a reversed inverter, the

following relation also holds

$$\mathbf{T}_{k,i} = \begin{cases} 0, & \text{if } \mathbf{T}_{i,k} = 0 \\ 1, & \text{if } \mathbf{T}_{i,k} = 1 \\ 2, & \text{if } \mathbf{T}_{i,k} = 2 \\ 4, & \text{if } \mathbf{T}_{i,k} = 3 \\ 3, & \text{if } \mathbf{T}_{i,k} = 4 \\ 6, & \text{if } \mathbf{T}_{i,k} = 5 \\ 5, & \text{if } \mathbf{T}_{i,k} = 6 \end{cases} \quad (6.17)$$

Also, define

$$N_{i,k}^{dc} = \text{index of buck or boost converter from bus } i \text{ to bus } k, \quad (6.18)$$

$$N_{i,k}^{ac} = \text{index of inverter or rectifier from bus } i \text{ to bus } k, \quad (6.19)$$

$$\mu_k = \text{duty cycle of } kth \text{ of dc/dc converter}, \quad (6.20)$$

and

$$\tilde{\lambda}_k = \text{rms phasor duty cycle } kth \text{ of ac/dc (or dc/ac) converter} \quad (6.21)$$

Assume that all ac components are balanced 3-phase. Then, for a HACDC grid, the apparent power

balance at bus k is

$$\begin{aligned}
\tilde{z}_k(\mathbf{x}) = 0 = & \tilde{S}_{G,k} - \tilde{S}_{L,k} \\
& - \sum_{\{\forall k|\mathbf{T}_{k,l}=1\}} V_k(V_k - V_l)Y_{k,l} \\
& - \sum_{\{\forall l|\mathbf{T}_{k,l}=2\}} \tilde{V}_k \left[(\tilde{V}_k - \tilde{V}_l) \tilde{Y}_{k,l} \right]^* \\
& - \sum_{\{\forall l|\mathbf{T}_{k,l}=3\}} \mu_{N_{k,l}^{dc}} V_k (\mu_{N_{k,l}^{dc}} V_k - V_l) Y_{k,l} \\
& - \sum_{\{\forall l|\mathbf{T}_{k,l}=4\}} V_k (V_k - \mu_{N_{k,l}^{dc}} V_l) Y_{k,l} \\
& - \sum_{\{\forall l|\mathbf{T}_{k,l}=5\}} \tilde{V}_k \left[(\tilde{V}_k - \tilde{\lambda}_{N_{k,l}^{ac}} V_l) \tilde{Y}_{k,l} \right]^* \\
& - \sum_{\{\forall l|\mathbf{T}_{k,l}=6\}} \tilde{\lambda}_{N_{k,l}^{ac}} V_k \left[(\tilde{\lambda}_{N_{k,l}^{ac}} V_k - \tilde{V}_l) \tilde{Y}_{k,l} \right]^*
\end{aligned} \tag{6.22}$$

For bus k , the active and reactive power balance are respectively

$$g_k(\mathbf{x}) = \text{Re} \{ \tilde{z}_k(\mathbf{x}) \} = 0 \tag{6.23}$$

and

$$h_k(\mathbf{x}) = \text{Im} \{ \tilde{z}_k(\mathbf{x}) \} = 0. \tag{6.24}$$

Note that $\tilde{\lambda}_{N_{k,j}^{ac}} = \lambda_{N_{k,j}^{ac}} \angle \gamma_{N_{k,j}^{ac}}$ is the phasor representation of the sinusoidal duty cycle of ac/dc or dc/ac converter $N_{k,j}^{ac}$. Since $\lambda_{N_{k,j}^{ac}}$ is RMS the value of a sinusoid of maximum peak value 1, it must hold that

$$0 \leq \lambda_{N_{k,j}^{ac}} \leq \frac{1}{\sqrt{2}}. \tag{6.25}$$

These equations can be further simplified using trigonometric identities. However, MATLAB/Simulink is capable of handling these complex values. Therefore, forms of (6.22), (6.23) and (6.24) are sufficient for code development.

6.5 Optimization Problem Form

The general optimization problem is stated as

$$\text{Minimize } f(\mathbf{x}) \text{ subject to } \mathbf{c_E}(\mathbf{x}) = \mathbf{0}, \mathbf{c_I}(\mathbf{x}) \geq \mathbf{0}, \quad (6.26)$$

where $f(\mathbf{x})$ is the scalar objective function, $\mathbf{c_E}(\mathbf{x})$ is the vector of equality constraints, and $\mathbf{c_I}(\mathbf{x})$ is the vector of inequality constraints. In a power grid, the inequality constraints are typically box constraints forming the lower and upper bounds on each variable. The optimization problem for an n bus the steady-state HACDC power grid therefore reduces to

$$\text{Minimize } f(\mathbf{x}) \text{ subject to } \mathbf{g}(\mathbf{x}) = \mathbf{0}, \mathbf{h}(\mathbf{x}) = \mathbf{0}, \mathbf{x_{min}} \leq \mathbf{x} \leq \mathbf{x_{max}}, \quad (6.27)$$

where

$$\mathbf{x} = [V_1, \dots, V_n, \delta_1, \dots, \delta_{n_{ac}}, \lambda_1, \dots, \lambda_{n_{acdc}}, \mu_1, \dots, \mu_{n_{dcdc}}, \gamma_1, \dots, \gamma_{n_{acdc}}, P_{G_1}, \dots, P_{G_n}, Q_{G_1}, \dots, Q_{G_{n_{ac}}}]^T, \quad (6.28)$$

$$\mathbf{x_{min}} = \text{lower bounds on elements of } \mathbf{x}, \quad (6.29)$$

$$\mathbf{x_{max}} = \text{upper bounds on elements of } \mathbf{x}, \quad (6.30)$$

and n_{dc} , n_{ac} , n_{acdc} , n_{dcdc} are the number of dc buses, ac buses, ac/dc (or dc/ac) converters and dc/dc converters respectively. As this relates to (6.26),

$$\mathbf{c_E}(\mathbf{x}) = \begin{bmatrix} \mathbf{g}(\mathbf{x}) \\ \mathbf{h}(\mathbf{x}) \end{bmatrix} \quad (6.31)$$

and

$$\mathbf{c_I}(\mathbf{x}) = \begin{bmatrix} \mathbf{x} - \mathbf{x_{min}} \\ \mathbf{x_{max}} - \mathbf{x} \end{bmatrix}. \quad (6.32)$$

6.6 Background: Using Squared Slack Variables to Solve Nonlinear Optimization Problems

As discussed in [64], the method of converting a general nonlinear optimization problem to one only containing equality constraints by introducing squared slack variables has been known for decades. The increase in problem dimension is one reason that this method has historically been avoided. However, with the advancement of computational technology this has become far less of a hindrance [64]. In the optimization community, the main reason that this method has been avoided is because of potential numerical instabilities introduced by the reformulation [64]. In kind, [66] illustrates issues that may arise in attempting to use squared slack variables in solving inequality constrained optimization problems. [66] uses a few simple examples to illustrate to the reader potential pitfalls of using the squared slack reformulation. The example that [66] uses is the problem

$$\min_{x,y \in \mathbb{R}} f(x) \tag{6.33}$$

$$\text{subject to} \tag{6.34}$$

$$ax - e^x \leq 0, \tag{6.35}$$

where the problem is reformulated as

$$\min_{x,y \in \mathbb{R}} f(x) \tag{6.36}$$

$$\text{subject to} \tag{6.37}$$

$$ax - e^x + y^2 = 0. \tag{6.38}$$

In [66] constrained nonlinear optimization examples solved by using with squared slack variables directly are examined. Particularly, the cases $a = 0$, $a = -1$, and $a = 2$ were discussed. An initial guess $(x_0, 0)$ and it is shown that each of these cases fails to converge or converges to a local minimum instead of the global minimum. One limitation of [66] is that it only focuses on the fact that once a solution where the slack either starts at or becomes zero is attained, the method cannot escape this condition so that the problem will fail to converge. However, for the optimal power flow, the inequality constraints are linear bounds on the control variables as in (6.27). It can readily be seen that solution $\mathbf{s} = \mathbf{0}$ is only possible if $\mathbf{x}_{\min} = \mathbf{x} = \mathbf{x}_{\max}$. In which case, either the solution is

trivial or does not exist. Therefore, the convergence issues described in [66] do not apply to the HACDC optimal power flow problem.

6.7 Proposed Method: Squared Slack Interior-Point

The proposed method is similar to the interior-point logarithmic barrier method. The key differences are that the outer loop of the algorithm is removed, the logarithmic barrier is dropped, and the slack variables $[s_i]_{i=1,\dots,n_i}$ are replaced with $[s_i^2]_{i=1,\dots,n_i}$. The Lagrangian of the modified system becomes

$$\mathcal{L} = f(\mathbf{x}) - \mathbf{y}^T \mathbf{c}_E(\mathbf{x}) - \mathbf{z}^T (\mathbf{c}_I(\mathbf{x}) - \mathbf{s}^2), \quad (6.39)$$

where $\mathbf{s}^2 = [s_1^2, s_2^2, \dots, s_{n_i}^2]^T$. Note that locally optimal solutions occur when the gradient of the Lagrangian is zero. Applying Newton's method to (6.39) yields

$$\begin{bmatrix} \mathbf{B} & \mathbf{0}_{n_i \times n_i} & -\mathbf{A}_E^T(\mathbf{x}) & -\mathbf{A}_I^T(\mathbf{x}) \\ \mathbf{0}_{n_i \times n} & 2\mathbf{Z} & \mathbf{0}_{n_i \times n_e} & 2\mathbf{S} \\ -\mathbf{A}_E(\mathbf{x}) & \mathbf{0}_{n_e \times n_i} & \mathbf{0}_{n_e \times n_e} & \mathbf{0}_{n_e \times n_i} \\ -\mathbf{A}_I(\mathbf{x}) & 2\mathbf{S} & \mathbf{0}_{n_i \times n_e} & \mathbf{0}_{n_i \times n_i} \end{bmatrix} \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_s \\ \mathbf{p}_y \\ \mathbf{p}_z \end{bmatrix} = - \begin{bmatrix} \nabla_x f(\mathbf{x}) - \mathbf{A}_E^T(\mathbf{x})\mathbf{y} - \mathbf{A}_I^T(\mathbf{x})\mathbf{z} \\ 2\mathbf{s} \otimes \mathbf{z} \\ -\mathbf{c}_E(\mathbf{x}) \\ -\mathbf{c}_I(\mathbf{x}) + \mathbf{s}^2 \end{bmatrix}, \quad (6.40)$$

where $\otimes$ denotes the element-wise Hadamard product. As discussed in [67], the Broyden-Fletcher-Goldfarb-Shanno (BFGS) Hessian approximation at iteration $k+1$ is

$$\mathbf{B}_{k+1} = \mathbf{B}_k + \frac{\mathbf{q}_k \mathbf{q}_k^T}{\mathbf{q}_k^T \mathbf{q}_k} - \frac{\mathbf{B}_k \mathbf{d}_k \mathbf{d}_k^T \mathbf{B}_k^T}{\mathbf{d}_k^T \mathbf{B}_k \mathbf{d}_k}, \quad (6.41)$$

where

$$\mathbf{d}_k = \mathbf{x}_{k+1} - \mathbf{x}_k \quad (6.42)$$

and

$$\mathbf{q}_k = \nabla f(\mathbf{x}_{k+1}) - \nabla f(\mathbf{x}_k). \quad (6.43)$$

Let ϵ_{tol} be the error tolerance of the optimization algorithm. The proposed algorithm is as follows. For the proposed algorithm, the function $E(\mathbf{x}, \mathbf{s}, \mathbf{y}, \mathbf{z})$ is defined as

$$E(\mathbf{x}, \mathbf{s}, \mathbf{y}, \mathbf{z}) = \{ \|2\mathbf{s} \otimes \mathbf{z}\|, \|\mathbf{c}_E(\mathbf{x})\|, \|\mathbf{c}_I(\mathbf{x}) - \mathbf{s}^2\| \}. \quad (6.44)$$

initialize: $\mathbf{x}_0 \in \mathbb{R}^n$, $\mathbf{s}_0 \in \mathbb{R}^{n_i}$, $\mathbf{y}_0 \equiv (5.43)$, and $\mathbf{z}_0 \in \mathbb{R}^{n_i}$ and $\mathbf{B}_0 \in \mathbb{R}^{n_i \times n_i}$; and choose some $\tau \in (0, 1)$.

while $E > \epsilon_{tol}$ **do**

- 1) Solve (6.40) to get the search direction $\mathbf{p} = [\mathbf{p}_x^T, \mathbf{p}_s^T, \mathbf{p}_y^T, \mathbf{p}_z^T]^T$.
- 2) Compute $\alpha_s^{max}, \alpha_z^{max}$ using (5.34) and (5.35) respectively.
- 3) Compute the new iterate $(\mathbf{x}_{k+1}, \mathbf{s}_{k+1}, \mathbf{y}_{k+1}, \mathbf{z}_{k+1})$ using (5.30) -(5.33).

end

Algorithm 2: Squared slack interior point method. This is a superlinear method for numerically locally optimizing the nonlinear programming problem. Inequality constraints are converted to equality constraints using squared slack variables and ill-conditioning introduced by this is offset by applying the fraction to boundary rule.

In this dissertation it is assumed that no prior data about the system to be optimized are known initially. As with any deterministic optimizer, the initial guess for the solution will affect both the rate of convergence of the optimizer and the quality of the solution (i.e the value of the objective). This is in slight contrast to standard ac optimal power flow problems such as the IEEE 30 bus for example, where a power flow solution is initially known and the goal is to improve starting from the prior known solution. Initialization parameters specific to the HACDC OPF problem will be later defined.

6.8 Feasibility and Local Convergence

In this section, feasibility and first order optimality are established for Algorithm 2. The goal is to establish that if the Algorithm 2 converges to a solution, then the solution must be both feasible and locally optimal. For simplicity, the system with only inequality constraints is considered initially. Consider the general inequality constrained nonlinear optimization problem

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && f(\mathbf{x}) \\ & \text{subject to} && c_i(\mathbf{x}) \geq 0, \quad i \in \mathcal{I}, \end{aligned}$$

where $\mathcal{I}$ is the set of inequality constraints. Define the subset of active constraints of $\mathcal{I}$ as $\mathcal{A}$. In order to establish optimality, the goal is to establish that the result of Algorithm 2 will satisfy the

KKT conditions

$$\nabla_x \mathcal{L}(\mathbf{x}^*, \mathbf{z}^*) = 0 \quad (6.45)$$

$$c_i(\mathbf{x}^*) \geq 0, \forall i \in \mathcal{I} \quad (6.46)$$

$$z_i^* \geq 0, \forall i \in \mathcal{I} \quad (6.47)$$

$$z_i^* c_i(\mathbf{x}^*) \geq 0, \forall i \in \mathcal{I}, \quad (6.48)$$

where z_i are Lagrange multipliers for the inequality constraints.

Theorem 1. *Suppose that Algorithm 2 generates an infinite sequence of iterates $\{\mathbf{x}_k\}$ and that $\epsilon_{tol} \rightarrow 0$. Further, suppose that f and $\mathbf{c}_i$ are continuously differentiable. Then, any limit point $\hat{\mathbf{x}}$ of the sequence $\{\mathbf{x}_k\}$ is feasible. Furthermore, if $\forall i \notin \mathcal{A}$ the gradients of the inequality constraints are linearly independent, then first-order optimality conditions hold at the limit point $\hat{\mathbf{x}}$.*

The qualification that the gradients of the active inequality constraints and the equality constraints are linearly independent is called the linear independence constraint qualification (LICQ) [61]. The proof for Theorem 1 is as follows.

Proof. Let $\hat{\mathbf{x}}$ be a limit point of the sequence $\{\mathbf{x}_k\}$. Also, let $\{\mathbf{x}_{k_l}\}$ be a convergent subsequence of $\{\mathbf{x}_k\}$ so that $\{\mathbf{x}_{k_l}\} \rightarrow \hat{\mathbf{x}}$. Choose the error tolerance $\epsilon_{tol} = 0$ so that convergence in an infinite number of iterations is to the exact solution. Since $\epsilon_{tol} = 0$, it must also hold that the error E converges to zero so that $(\mathbf{c}_{k,l} - \mathbf{s}_{k,l}^2) \rightarrow 0$. By continuity of $\mathbf{c}$, it holds that $\hat{\mathbf{c}} = \mathbf{c}(\hat{\mathbf{x}}) \geq 0$, $\mathbf{s}_{k_l}^2 \rightarrow \hat{\mathbf{s}}^2 = \hat{\mathbf{c}}$. The fact that $\mathbf{c}(\hat{\mathbf{x}}) \geq 0$ establishes the feasibility of $\hat{\mathbf{x}}$.

For optimality, a slight alteration of Algorithm 2 is utilized where (6.44) is replaced with

$$E(\mathbf{x}, \mathbf{s}, \mathbf{y}, \mathbf{z}) = \{ \|\nabla_x f(\mathbf{x}) - \mathbf{A}_f^T(\mathbf{x})\mathbf{z}\|, \|2\mathbf{s} \otimes \mathbf{z}\|, \|\mathbf{c}_f(\mathbf{x}) - \mathbf{s}^2\| \}. \quad (6.49)$$

Now consider the active set of inequality constraints such that LICQ holds at the limit point $\hat{\mathbf{x}}$

$$\mathcal{A} = \{i : \hat{c}_i = 0\}. \quad (6.50)$$

For $i \notin \mathcal{A}$, $\hat{c}_i > 0$ and $\hat{s}_i^2 > 0$. Also, $\hat{s}_i^2 > 0 \implies \hat{s}_i \neq 0$. Thus, from the complementary condition

$2\mathbf{s} \otimes \mathbf{z} = 0$, it must hold that $[\mathbf{z}_{k_l}]_i \rightarrow 0$. From this result and $\nabla f_{k_l} - \mathbf{A}_{\mathbf{I}_{k_l}} \mathbf{z}_{k_l} \rightarrow 0$ it must hold that

$$\nabla f_{k_l} - \sum_{i \in \mathcal{A}} [z_{k_l}]_i \nabla c_i(\mathbf{x}_{k_l}) \rightarrow 0. \quad (6.51)$$

By LICQ, the vectors $\{\nabla \hat{c}_i : i \in \mathcal{A}\}$ are linearly independent. Therefore, from (6.51) and continuity of ∇f and $c_i, \forall i \in \mathcal{E}$ the positive sequence $\{\mathbf{z}_{k_l}\}$ must converge to some value $\hat{\mathbf{z}} \geq 0$. Taking the limit of (6.51) as $k \rightarrow \infty$ yields

$$\nabla f(\hat{\mathbf{x}}) = \hat{\mathbf{z}}^T \nabla \hat{\mathbf{c}}(\hat{\mathbf{x}}). \quad (6.52)$$

This, along with the fact that $\hat{\mathbf{c}}^T \hat{\mathbf{z}} = 0$ completes the proof.

This may proof may be extended to include equality constraints by simply representing the constraint c_{E_i} as

$$c_{E_i} + \epsilon \geq 0 \quad (6.53)$$

$$\epsilon - c_{E_i} \geq 0 \quad (6.54)$$

for some small value $\epsilon > 0$. □

From a practical standpoint, it is possible that gradient Lagrangian of the system in some neighborhood of the optimal solution has very slow convergence. Therefore $\|\nabla_x f(\mathbf{x}) - \mathbf{A}_{\mathbf{E}}^T(\mathbf{x})\mathbf{y} - \mathbf{A}_{\mathbf{I}}^T(\mathbf{x})\mathbf{z}\|$ is dropped from E for the applications in this dissertation. Therefore, at best the solution is guaranteed to be feasible and in some neighborhood of the optimal solution. Further proof to determine the size of this neighborhood is planned for future work.

6.9 Numerical Results

In this section, the proposed algorithm is applied to a HACDC grid consisting 9 ac buses and 9 dc buses. There are 6 rectifiers, 3 inverters and 3 boost converters, and 3 buck converters. The dynamic system is modeled in MATLAB/Simulink. For the ac optimization process there is a total of 9 ac voltage magnitudes, 9 ac voltage angles, 9 dc voltages, 9 ac/dc converter duty cycle magnitudes, 9 ac/dc converter duty cycle angles, 6 dc/dc converter duty cycles, 18 power active power injections, and 9 reactive power injections. This is a total of 78 variables (not including the 156 corresponding slack variables). A one line diagram of the system is shown below in Fig. 6.1. All

ac lines and converters are balanced 3-phase. The sets of buses $\{1, 2, 7, 10, 11, 12\}$, $\{3, 4, 8, 13, 14, 15\}$, and $\{5, 6, 9, 16, 17, 18\}$ form 3 distinct microgrids which are interconnected via ac transmission lines through voltage source inverters.

Figure 6.1: One line of 18 bus HACDC Grid.

The objective function $f(\mathbf{x})$ for the system is the power loss

$$\begin{aligned}
 f(\mathbf{x}) = P_{loss}(\mathbf{x}) = & \sum_{k=1}^n \sum_{\{\forall l | \mathbf{T}_{k,l}=1\}} (V_k - V_j)^2 G_{k,l} + \sum_{k=1}^n \sum_{\{\forall l | \mathbf{T}_{k,l}=2\}} |\tilde{V}_k - \tilde{V}_l|^2 G_{k,l} \\
 & + \sum_{k=1}^n \sum_{\{\forall l | \mathbf{T}_{k,l}=3\}} (\mu_{N_{k,l}^{dc}} V_k - V_l)^2 G_{k,l} + \sum_{k=1}^n \sum_{\{\forall l | \mathbf{T}_{k,l}=4\}} (V_k - \mu_{N_{k,l}^{dc}} V_l)^2 G_{k,l} \\
 & + \sum_{k=1}^n \sum_{\{\forall l | \mathbf{T}_{k,l}=5\}} |\tilde{V}_k - \tilde{\lambda}_{N_{k,l}^{ac}} V_l|^2 G_{k,l} + \sum_{k=1}^n \sum_{\{\forall l | \mathbf{T}_{k,l}=6\}} |\tilde{\lambda}_{N_{k,l}^{ac}} V_k - \tilde{V}_l|^2 G_{k,l}, \quad (6.55)
 \end{aligned}$$

where $G_{k,j} = \text{Re} \left\{ \tilde{Y}_{k,j} \right\}$.

The line and power electronics interface parameters are summarized in Table 6.1, the bus parameters are given in Table 6.2, and the power constraints are given in Table 6.3. The arrows in Fig. 6.1 signify the forward direction of each converter. The battery voltage is not controllable because it is a direct function of its state of charge (SOC), therefore, the upper and lower bounds on the battery voltage are equal to the measured voltage at the terminals of the battery V_{batt} .

As for the PV arrays, the power output and voltage are both controllable. In order to maximize utilization of solar energy, the PV array voltage and output power are tracked to the maximum power point (mpp). The maximum power point voltage and maximum power output are V_{mpp} and P_{mpp} respectively for the PV arrays.

Table 6.1: HACDC line and power electronics parameters

from	to	R(Ω)	L(mH)	type	ac/dc conv. no.	dc/dc conv. no.
7	8	0.01	0.1	ac line	-	-
8	9	0.01	0.1	ac line	-	-
9	7	0.01	0.1	ac line	-	-
12	7	0.1	1	inverter	1	-
15	8	0.1	1	inverter	2	-
18	9	0.1	1	inverter	3	-
1	12	0.1	1	rectifier	4	-
2	12	0.1	1	rectifier	5	-
3	15	0.1	1	rectifier	6	-
4	15	0.1	1	rectifier	7	-
5	18	0.1	1	rectifier	8	-
6	18	0.1	1	rectifier	9	-
10	12	0.1	1	boost	-	1
11	12	0.1	1	boost	-	2
13	15	0.1	1	boost	-	3
14	15	0.1	1	boost	-	4
16	18	0.1	1	boost	-	5
17	18	0.1	1	boost	-	6

Table 6.2: HACDC bus parameters and voltage constraints

bus	$C(\mu F)$	V_{min}	V_{max}	type	source
1	-	220	220	ac	generator
2	-	220	220	ac	generator
3	-	220	220	ac	generator
4	-	220	220	ac	generator
5	-	220	220	ac	generator
6	-	220	220	ac	generator
7	200	198	242	ac	-
8	200	198	242	ac	-
9	200	198	242	ac	-
10	100	V_{mpp}	V_{mpp}	dc	PV array
11	-	V_{batt}	V_{batt}	dc	battery
12	1600	675	825	dc	-
13	100	V_{mpp}	V_{mpp}	dc	PV array
14	-	V_{batt}	V_{batt}	dc	battery
15	1600	675	825	dc	-
16	100	V_{mpp}	V_{mpp}	dc	PV array
17	-	V_{batt}	V_{batt}	dc	battery
18	1600	675	825	dc	-

Table 6.3: HACDC bus power constraints

bus	$P_{min}(kW)$	$P_{max}(kW)$	$Q_{min}(kVAr)$	$Q_{max}(kVAr)$
1	0	15	-15	15
2	0	15	-15	15
3	0	15	-15	15
4	0	15	-15	15
5	0	15	-15	15
6	0	15	-15	15
7	0	0	-30	30
8	0	0	-30	30
9	0	0	-30	30
10	P_{mpp}	P_{mpp}	0	0
11	-15	15	0	0
12	0	0	0	0
13	P_{mpp}	P_{mpp}	0	0
14	-15	15	0	0
15	0	0	0	0
16	P_{mpp}	P_{mpp}	0	0
17	-15	15	0	0
18	0	0	0	0

The optimization program was written using MATLAB. The optimizations were run on a Dell Latitude E5570 laptop with 8.00 GB RAM and an Intel Core i5-6440HQ 2.60 GHz processor. For the

dynamic simulation, $\tau = 0.95$ is used for the bound rate and the optimizer is run every 5 simulation seconds.

6.9.1 Optimizer Validation: Dynamic Simulation Results

In order to ensure accuracy of optimization results, the optimizer results are validated against dynamic time domain results. The error tolerance for the optimizer is 0.001%. Therefore, under steady-state conditions, all variables should be within 0.01% of their reference values. The base power for the system is 10 kW and the base voltage is $V_{base,dc} = V_{base,rms,LL} = 380$ V. At the chosen error tolerance $\epsilon_{tol} = 0.0001$, the voltages should accurately be within 0.038V, the power generation should be accurate within 10 W, and the voltage angles should be accurate within 0.0001 rad. The loads are stepped every 40 s over a 120 s interval and kept flat between steps. The balanced 3-phase load profile is shown in Table 6.4. All other active and reactive loads are zeros. There are no dc loads. The PV array parameters are shown in Tables 6.5-6.7. The battery parameters are given in Table 6.8. The generator parameters are given in Table 6.9. The diesel engine parameters are given in Tables 6.10-6.11. Throughout the simulation, the irradiance is held at 750 W/m² and the temperature is held at 25°C.

The simulation results are summarized in Fig. 6.2 and Fig. 6.3. All variables successfully track their steady state reference values within the desired error tolerance. From Fig. 6.2a. It can be seen that the ac voltages are all able to track their reference values with short-lived transient spikes immediately either the load changes or after a new set point has been sent from the optimizer. Similarly, it can be seen in Fig. 6.3, that the desired power output for each power source is maintained accurately through the simulation with minor deviations during transient event. It can readily be seen that the dynamic calculations are consistent with the steady state optimization result computed using the squared slack interior point method. A direct proof for the convergence of the method has not yet been developed; however, in the next section, a few trials are run and conjectures are made about overall convergence based upon these results. In future work, the goal will be to establish convergent criteria as it relates to the bound rate τ .

Table 6.4: HACDC load profile

bus	0s	40s	80s
7	15 kW	0 kW	0 kW
8	40 kW	40 kW	60 kW
9	17 kW	30 kW	30 kW

Table 6.5: PV array parameters

bus	$P_n(W)$	$V_{oc,stc}(V)$	$I_{sc,stc}(A)$	$V_{mpp}(V)$
10	330	45.6	9.39	36.8
13	330	45.6	9.39	36.8
16	330	45.6	9.39	36.8

Table 6.6: PV array parameters

bus	$I_{mpp}(A)$	$NOCT(^{\circ}C)$	$\alpha_{I_{sc}}(\%/^{\circ}C)$
10	8.84	45	0.1
13	8.84	45	0.1
16	8.84	45	0.1

Table 6.7: PV array parameters

bus	$\alpha_{V_{oc}}(\%/^{\circ}C)$	N_s	N_p	$C(\mu F)$
10	-0.37	15	1	100
13	-0.37	15	1	100
16	-0.37	15	1	100

Table 6.8: Battery parameters

bus	type	$V_{nom}(V)$	Ah	efficiency(%)	SOC(0)
11	Li-Ion	518	620	99.5	0.8
14	Li-Ion	518	620	99.5	0.8
17	Li-Ion	518	620	99.5	0.8

Table 6.9: Generator parameters

bus	poles	$J(kgm^2)$	$B_m(Nms/rad)$	$R_s(\Omega)$	K
1	4	1.662	0.1	0.087	40
2	4	1.662	0.1	0.087	40
3	4	1.662	0.1	0.087	40
4	4	1.662	0.1	0.087	40
5	4	1.662	0.1	0.087	40
6	4	1.662	0.1	0.087	40

Table 6.10: Diesel engine parameters

bus	T_1	T_2	T_3	T_4	T_5	T_6
1	0.01	0.02	0.2	0.25	0.009	0.0384
2	0.01	0.02	0.2	0.25	0.009	0.0384
3	0.01	0.02	0.2	0.25	0.009	0.0384
4	0.01	0.02	0.2	0.25	0.009	0.0384
5	0.01	0.02	0.2	0.25	0.009	0.0384
6	0.01	0.02	0.2	0.25	0.009	0.0384

Table 6.11: Diesel engine parameters

bus	$T_{min}(pu)$	$T_{max}(pu)$	$T_d(s)$	kW_{rated}	K_δ
1	0	1.1	0.024	50	0.01
2	0	1.1	0.024	50	0.01
3	0	1.1	0.024	50	0.01
4	0	1.1	0.024	50	0.01
5	0	1.1	0.024	50	0.01
6	0	1.1	0.024	50	0.01

(a) Line-to-neutral RMS voltage magnitudes. The solid lines are the actual voltages and the dashed lines are the reference voltages. The voltage magnitudes accurately track their targets throughout the simulation with short transient spikes immediately after load steps and OPF updates.

(b) Line-to-neutral RMS voltage angles. The solid lines are actual angles and the dashed lines are the reference angles. The voltage angles accurately track their targets throughout the simulation with short transient spikes immediately after load steps and OPF updates. These transients are slower than those for the magnitudes because they directly depend upon the mechanical inertia of the generator.

(c) Dc voltages. The solid lines are the actual voltages and the dashed lines are the target voltages. The power electronics interfaces are able to accurately tightly track their target values with very minor transients immediately after load step changes and OPF updates.

Figure 6.2: Bus Voltages.

Figure 6.3: Source active power outputs. The solid lines are the actual power generation and the dashed lines are the reference power generation. The power output of each source is accurately tracked to its target value throughout the simulation with short-lived transients immediately after load step changes and OPF updates.

6.9.2 Optimization Results: Optimizer Convergence Analysis

More detailed optimization results are analyzed in this subsection. In this subsection, the tolerance is set to $\epsilon_{tol} = 0.001$. Again, the system of Fig. 6.1 is examined. The loads at buses 7, 8 and 9 are $S_{L7} = 20 + j15$ kVA, $S_{L8} = 15 + j10$ kVA, and $S_{L9} = 50 + j25$ kVA respectively. Note that one goal in the development of this optimizer is that it requires minimal tuning of parameters. The only parameter to tune for the proposed optimization algorithm is the bound rate τ . In this subsection, τ is perturbed and the rate of convergence is examined along with the optimality of the objective. As was the case earlier, the goal is to minimize power losses. The results of these optimizations are summarized in Table 6.12. It can be seen from Table 6.12 that beyond $\tau = 0.5$, the power losses become relatively flat. Also, since the error tolerance for the power is 100W, all of the power losses in the table are effectively equal.

Of the values in Table 6.12, $\tau = 0.75$ yields the fastest result. Optimization results for $\tau = 0.75$ are summarized in Tables 6.13-6.17. All constraints are as earlier defined. As can be seen in Table 6.13, the ac voltages are within the desired ranges. Due to differing loads, the ac load buses (7-9) are at different voltage magnitudes and angles. The magnitudes of the generator bus voltages (1-6) are restricted to exactly 220V, which is also reflected in Tables 6.13. $V_{rms,LN}$ is the RMS line to neutral voltage magnitude, and δ is the angle in degrees. It can also be seen in Table 6.14, that all dc voltages V_{dc} are within the desired ranges. The ac/dc (or dc/ac) converter sinusoidal duty cycles are given in Tables 6.15. λ is the RMS magnitude of the sinusoidal duty cycle and γ is the phase angle. It can be seen from Tables 6.15, that the duty cycles of the ac/dc converters all satisfy the desired constraints. The optimal results for the dc/dc converter duty cycles are summarized in

Tables 6.16. These values are all within the desire range of $[0, 1]$.

Table 6.12: Optimization results

τ	$t(s)$	$P_{loss}(W)$	$iterations$
0.05	34.9075	1719.0519	1004
0.10	18.8114	1718.3958	535
0.15	17.1552	1714.1246	491
0.20	7.7910	1719.0507	217
0.25	6.3156	1720.3035	176
0.30	6.2270	1718.604	171
0.35	3.7833	1763.2165	102
0.40	3.2068	1764.2049	83
0.45	2.8386	1765.3105	75
0.50	2.7358	1764.5623	71
0.55	3.2115	1764.4936	87
0.60	3.8683	1764.6039	104
0.65	3.0635	1764.9604	82
0.70	2.8356	1764.6048	75
0.75	2.4790	1764.8435	65
0.80	2.6965	1764.9467	71
0.85	2.7550	1764.6782	73
0.90	2.8473	1764.6272	75
0.95	2.8524	1772.3075	76
0.995	4.7662	1724.7781	131

Table 6.13: Optimal ac voltages for $\tau = 0.75$.

bus	$V_{rms,LN}(V)$	$\delta(^{\circ})$
1	220	0
2	220	0
3	220	0.00084103
4	220	0.00084103
5	220	-0.0088006
6	220	-0.0088006
7	241.89	-2.077
8	242.00	-2.0619
9	241.58	-2.1904

Table 6.14: Optimal dc bus voltages for $\tau = 0.75$.

<i>bus</i>	$V_{dc}(V)$
10	518.4
11	569.2
12	750
13	518.4
14	569.2
15	750
16	518.4
17	569.2
18	750

Table 6.15: Optimal ac/dc converter phasor duty cycles for $\tau = 0.75$.

ac/dc conv. no.	λ	$\gamma(^{\circ})$
1	0.32831	1.3619
2	0.32847	1.3761
3	0.32782	1.2261
4	0.29169	-1.267
5	0.29169	-1.267
6	0.2917	-1.2677
7	0.2917	-1.2677
8	0.29168	-1.2585
9	0.29168	-1.2585

Table 6.16: Optimal dc/dc converter duty cycles for $\tau = 0.75$.

<i>bus</i>	$V_{dc}(V)$
1	0.6903
2	0.75693
3	0.6903
4	0.75693
5	0.69032
6	0.75695

Table 6.17: Optimal power generation for $\tau = 0.75$.

bus	$P_{gen}(W)$	$Q_{gen} (VAr)$	bus type
1	8469.6	0	ac load
2	8469.6	0	ac load
3	8478.3	0	ac load
4	8478.3	0	ac generator
5	8364.8	0	ac generator
6	8364.8	0	ac generator
7	0.017096	3464	ac generator
8	0.017096	3440.3	ac generator
9	0.017096	3473.8	ac generator
10	3503	-	PV array
11	8565.9	-	battery
12	0.017696	-	dc load
13	3503	-	PV array
14	8570.8	-	battery
15	0.017725	-	dc load
16	3503	-	PV array
17	8492.1	-	battery
18	0.017754	-	dc load

Also, it can be seen in Tables 6.17, that all power generation is within the defined limits of the sources. Note that at the load buses (7,8,9,12,15,18), a negligible amount of power appears to be generated at this bus although there is no active power generation present at these buses. This this does not represent actual power generation, but computational error in the optimizer. these values are well below the error tolerance of 100W (0.001 pu). For each inverter, reactive power may be generated or by absorbed up to 30 kVAr. Since delivering (or absorbing) reactive power to the reactive loads and capacitors from the generators would incur additional line losses, the reactive power should be generated by the inverters if available and not ac generators, unless the reactive power injection from the capacitors and inverters is insufficient. Also, since the ac generators are coupled to the load through a dc link, it is not possible in this case to transfer reactive power from the generators to the loads, this is why the reactive power generation of each generator is zero.

6.9.3 Additional note on convergence

The optimization code written for this dissertation was written in a functioning, but not optimal manner. Also, analysis of how τ should be chosen was only developed via repeated trials. Optimality of the solutions is also affected by loading. Therefore, improvements to the rate of convergence can

be made if the code is written in a more optimal manner and if a dependency can be established among τ , the rate of convergence, the loading conditions, and resultant objective function value.

6.10 Conclusions

The squared slack interior-point method was developed and applied to the HACDC optimal power flow problem. Convergence of the method was consistent over a broad range of choices of the parameter τ . All constraints are treated as hard constraints in this method. Therefore, all solutions found were within the desired limits. Were there not a solution within these constraints, the solver would fail by design, since no exists. Thus all solutions found using this method satisfied all of the required inequality constraints. Feasibility was established for Algorithm 2. However, a concrete proof for first order optimality and not been completed. This proof will be addressed in future work. Also, the rate of convergence has not been concretely established. Simple tuning of bound rate τ can determine the rate of convergence and optimality of the solution. This too will be further analyzed in future work.

This chapter has been focused on centralized control of HACDC grids. However, the squared slack interior point method can be applied to any constrained nonlinear optimization problem. This method is also applied to the decentralized control of Chapter 7. In Chapter 7, the squared slack interior point method is applied locally to each player in a continuous constrained game in order to allow for each local optimizer to quickly arrive at a solution.

Chapter 7

A Game Theoretic Approach to OPF in Microgrids without a Communication Infrastructure

7.1 Introduction

Ideally, in a power grid or microgrid, power sources would collaborate to achieve a best operating point taking all components and their operational constraints into consideration. This approach requires complete knowledge of the system which requires a communication infrastructure to be present. That is, all admittances and loads must be known in order for a valid optimization to be performed. This was the underlying assumption of the methods detailed in Chapters 5-6. However, if communication is lost or not is available from the start, an alternative is required. In this chapter, constrained non-cooperative game theory along with solution perturbation is proposed as a method for controlling power flow in systems where communication is not available.

7.2 Non-Cooperative Games

Consider a case where N players are competing for resources. If a centralized communications and controls network is not available, loads will have to compete for resources out of necessity. However,

this may result in undesired operation of energy sources if additional measures are not taken. As an example, consider the simple case of a 3-bus dc system as shown in Fig. 7.1. The initial load is at bus 3 is 10 kW. Source 1 is rated at 15 kW and source 2 is rated at 5 kW. Also, voltages at buses 1 and 2 are $V_1 = 652.45 \text{ V}$ and $V_2 = 651.05 \text{ V}$ respectively.

Figure 7.1: Simple 3 bus dc example.

Performing sensitivity analysis, it can be seen that for any increase in load, the source at bus 2 will exceed its power output limit. For example, if the load is increased to 13 kW, the resultant power generation at buses 1 and 2 respectively will be 5530.11 and 7510.47 W respectively. At this operating point, the source at bus 1 is at 36.87% capacity and the source at bus 2 is at 150.21% capacity. Sustained operation at this level can result in instability or damage to the power supply at bus 2. If the system is well designed, temporarily operating at these levels will not cause damage to equipment. However, sustained operation at these levels must be avoided to ensure that equipment is not damaged or prematurely aged. Ideally, a centralized optimizer would send updated setpoints to the local voltage controllers at buses 1 and 2. However, if a centralized control is not available, an alternative local optimization is needed. Constrained non-cooperative continuous game theory is considered in this chapter.

7.2.1 Unconstrained Non-Cooperative Games

As described in [68, 69], consider an N-player game with the set of players $p = \{p_1, \dots, p_N\}$. Let A_i define the action set of player i . The action set is the set of all possible actions of player i . The action profile for the game is defined as the Cartesian product of all player action sets

$$A = \prod_{i=1}^n A_i. \quad (7.1)$$

The product space for player i is defined as

$$A_{-i} = \prod_{j \neq i} A_j. \quad (7.2)$$

The product space for player i is the set of all player actions other than i . Each player has a utility function $J_i : A_i \times A_{-i} \rightarrow \mathbb{R}$. In terms of optimization, the utility function J_i is the local objective for player i . Let $a_i \in A_i$ be the action of player i and $a_{-i} \in A_{-i}$ be the actions of all players other than i . If the goal is to minimize $J_i(a_i, a_{-i})$, then the best response for player i is

$$BR_i(a_{-i}) = \arg \min_{a_i \in A_i} J_i(a_i, a_{-i}). \quad (7.3)$$

That is, the best response of player i occurs when player i 's utility function cannot be further decreased by taking unilateral action. A Nash Equilibrium occurs when

$$a_i^* = BR_i(a_{-i}), \forall i \in \{1, \dots, N\}. \quad (7.4)$$

That is, a Nash equilibrium occurs when each player is operating at its best response. Any unilateral move by a player will result in a lower utility for the player [68, 69]. For the continuous unconstrained game, the Nash Equilibrium (NE) occurs when

$$\frac{\partial J(a_i, a_{-i})}{\partial a_i} = 0, \forall i \in \{1, \dots, N\} \quad (7.5)$$

and

$$\frac{\partial^2 J(a_i, a_{-i})}{\partial a_i^2} \geq 0, \forall i \in \{1, \dots, N\} \quad (7.6)$$

[70].

7.2.2 Constrained Non-Cooperative Games

In any power grid or microgrid, whether ac, dc, or HACDC the optimal power flow formulation is a constrained problem in nature. Inequality constraints are typically box constraints. That is, for each variable, there is a static lower and upper bound. Assume that there are N players. Also let $\mathcal{E}$ be the set of equality constraints and $\mathcal{I}$ be the set of inequality constraints. For some variable z_i with lower and lower upper bounds $z_i^{\min}$ and $z_i^{\max}$ respectively, these constraints will take some

variation of the form

$$z_i^{min} \leq z_i \leq z_i^{max}, \forall i \in \mathcal{I}. \quad (7.7)$$

This constraint alone can be handled by utilizing penalty functions in the single variable z_i without much difficulty. These constraints are decoupled and do not pose much of a hindrance in finding the NE for the problem. However, the equality constraints will take the form

$$g_k(z_1, \dots, z_N) = 0, \forall k \in \mathcal{E} \quad (7.8)$$

Unconstrained game theory methods can no longer be applied directly to this problem. A method for achieving NE taking all constraints into account is proposed in [71] for ac grids and microgrids. For an n bus ac system, define the power balance equations

$$P_i = \sum_{j=1}^n V_i V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (7.9)$$

$$Q_i = \sum_{j=1}^n V_i V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)]. \quad (7.10)$$

P_i , Q_i , V_i , and θ_i are the net active power injection, net reactive power injection, voltage magnitude, and voltage angle at bus i respectively. G_{ij} and B_{ij} are the real and imaginary parts of the complex bus admittance matrix $\mathbf{Y} \in \mathbb{C}^{n \times n}$. Also define $P_i = P_{Gi} - P_{Li}$ and $Q_i = Q_{Gi} - Q_{Li}$, where the subscripts G and L denote generation and load quantities respectively. Bus r is chosen as a reference where V_r is known and $\theta_r = 0$. For more rapid computation, [71] utilizes a dc approximation of (7.9) and (7.10). For this approximation, it is assumed that transmission line resistances are small relative to their reactances. Assuming small differences among voltage angles, the approximations $\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$ and $\cos(\theta_i - \theta_j) \approx 1$ are applied. It is also assumed that $V_i = 1, \forall i \in \{1, \dots, n\}$. Since $\mathbf{G} \approx \mathbf{0}$, no variables remain in (7.10) and it is therefore removed from consideration. The power flow equations are approximated by the linear equations

$$P_i = \sum_{j=1}^n B_{ij}(\theta_i - \theta_j), \forall i \in \{1, \dots, n\}, \forall i \neq r \quad (7.11)$$

$$\theta_r = 0 \quad (7.12)$$

This may be written as

$$\hat{\mathbf{P}} = -\hat{\mathbf{B}}\boldsymbol{\theta}, \quad (7.13)$$

where $\hat{\mathbf{P}} = [P_1, \dots, P_{r-1}, 0, P_{r+1}, \dots, P_n]^T$, $\boldsymbol{\theta} = [\theta_1, \dots, \theta_r, \dots, \theta_n]^T$, and

$$\hat{\mathbf{B}} = \begin{bmatrix} B_{1,1} & \dots & B_{1,r-1} & 0 & B_{1,r+1} & \dots & B_{1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{r-1,1} & \dots & B_{r-1,r-1} & 0 & B_{r-1,r+1} & \dots & B_{r-1,n} \\ 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ B_{r+1,1} & \dots & B_{r+1,r-1} & 0 & B_{r+1,r+1} & \dots & B_{r+1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{n,1} & \dots & B_{n,r-1} & 0 & B_{n,r+1} & \dots & B_{n,n} \end{bmatrix}. \quad (7.14)$$

In [71], the players are the buses at which renewable resources are present. Define the set of buses with renewable energy sources as $\mathcal{N}_d$. [71] defines a renewable generation game so that for player $i \in \mathcal{N}_d$, the goal is to optimize

$$\min_{P_{Gi}, \theta_i} U_i(P_{Gi}, \theta_i), \text{ subject to} \quad (7.15)$$

$$\hat{\mathbf{P}} = -\hat{\mathbf{B}}\boldsymbol{\theta}, \quad (7.16)$$

$$P_{min,i} \leq P_{Gi} \leq P_{max,i}, \quad (7.17)$$

where $P_{min,i}$ and $P_{max,i}$ are the lower and upper bounds on active power generation at bus i respectively. Unlike for the unconstrained continuous game, a NE is a solution pair $(\mathbf{P}_{\mathbf{G}}^*, \boldsymbol{\theta}_{\mathbf{G}}^*)$, where $\mathbf{P}_{\mathbf{G}}^* = [P_{Gi}^*]_{i \in \mathbb{N}_d}$ and $\boldsymbol{\theta}_{\mathbf{G}}^* = [\theta_i^*]_{i \in \mathbb{N}_d}$ [71]. (7.15)-(7.17) may be solve using the Lagrangian methods detailed in [72]. However, there is a caveat that has not yet been addressed. Notice that the formulation of (7.16) requires knowledge of all loads. For such information to be common knowledge throughout the system, there must be a communication infrastructure present.

7.3 Constrained Game Theoretic Optimal Power Flow without Communication in Ac Grids

Recall the general form of the constrained optimization problem

$$\text{Minimize } f(\mathbf{x}), \text{ subject to } \mathbf{c_E}(\mathbf{x}) = \mathbf{0}, \mathbf{c_I}(\mathbf{x}) \geq \mathbf{0}, \quad (7.18)$$

where $\mathbf{c_E}(\mathbf{x}) \in \mathbb{R}^{n_e}$, $\mathbf{c_I}(\mathbf{x}) \in \mathbb{R}^{n_i}$, n_e is the number of equality constraints, and n_i is the number of inequality constraints. If a bus has a power supply present, it is referred to as a control bus. Let $CTRL$ be the set of control buses. Let $\mathbf{x}_i = [V_i, \delta_i, P_{Gi}, Q_{Gi}]$. Each control bus has the local objective function $J_i(\mathbf{x}_i, \mathbf{x}_{-i}), \forall i \in CTRL$. Also, let $\mathbf{c_{Ei}}(\mathbf{x}_i)$ be the subset of elements of $\mathbf{c_E}(\mathbf{x})$ specifically corresponding to variables associated with bus i only. Let $\mathbf{c_{Ii}}(\mathbf{x}_i)$ be the subset of elements of $\mathbf{c_I}(\mathbf{x})$ corresponding exclusively to variables associated with bus i . At bus (player) i , the constrained game is then

$$\min_{\mathbf{x}_i} J_i(\mathbf{x}_i, \mathbf{x}_{-i}), \text{ subject to } \mathbf{c_{Ei}}(\mathbf{x}_i) = \mathbf{0}, \mathbf{c_{Ii}}(\mathbf{x}_i) \geq \mathbf{0}, \quad (7.19)$$

The remaining values needed to compute (7.19), that is, elements of $\mathbf{x}_{-i}$ corresponding to adjacent nodes are held constant at their measured (or estimated) values throughout the optimization process so that partial derivatives are only in elements of the variable $\mathbf{x}_i$. Non-adjacent variables are not needed and have no affect in the calculation since current flow is always zero for these absent connections.

The SSIP method introduced in section 6.7 is used to solve (7.19). Similar to the method described in [70], a turn-based approach is utilized. Players takes turns optimizing their locally constrained objective. This is done for some number of cycles n_{cyc} . That is, each player gets to optimize its own objective exactly n_{cyc} times. Once each player settles at its own best local operating point another concern may become evident. Since the players actions are not decoupled, each player's choice may have an adverse on other players true operating condition. More importantly, some players may be exceeding their rated power output.

Assume that each player is able to temporarily maintain overloading or under-loading of power. Once the Nash Equilibrium is found, each player then needs to perturb its solution to drive itself back into a sustainable operating point.

7.3.1 Perturb and Observe Method for Ac Power Flow Control

In this subsection, a closed-loop method for driving the power generation of sources back within their power constraints is introduced. This method is a variation of the perturb and observe method for maximum power point tracking in PV array control. Adjustments of fixed magnitude are applied to the voltages and angles at generation sources and then power outputs are measured afterward. This process is repeated until all sources are within their desired power constraints.

Perturb and Observe Method for MPPT of a PV Array

As [73, 74] discusses, the perturb and observe (P&O) method is the most frequently utilized method for maximum power point tracking in a PV array. The goal of P&O is to find the voltage V_{mpp} and current I_{mpp} where the power of the PV array is maximized given some set of environmental conditions. In the P&O method, first the power is measured. Then the duty cycle of the controller is adjusted slightly to determine if there is an improvement in the power output. If there is an improvement, then next perturbation is in the same direction. Otherwise, the a perturbation is done in the opposite direction. Let d_{old} be the original duty cycle and d_{new} be the updated duty cycle. Also, let ϕ represent some constant value by which the duty cycle is to be perturbed, P represents the most recent power measurement and P_{old} represent the prior measured power output (before perturbation). The P&O method is summarized in Fig. 2.3.

Figure 7.2: Flowchart of conventional P&O algorithm for mppt of a PV array.

$P(k)$, $V_{PV}(k)$, and I_{PV} are the power, voltage, and current out of the PV array at iteration k respectively. ΔP , ΔV_{PV} , ΔI_{PV} denote changes in the power, voltage, and current from the prior iteration $k-1$ respectively. The algorithm is said to have found a steady-state operating point when $|\Delta P|$ becomes sufficiently small. Suppose that the voltage is adjusted by perturbing the duty cycle of the boost converter by some fixed value $\pm\phi$. If ϕ is large P&O will have faster convergence, but the steady-state oscillation will be larger [73, 74].

Proposed Perturb and Observe Method for Power Generation Constraint Enforcement

Once a NE has been attained, players violating their active power constraints must adjust power output to back within required constraints. In order to achieve this, a modified P&O method is proposed where the voltage is adjusted until the power moves back into the feasible region. Note

that since there is no centralized control present, temporarily violating power constraints may be unavoidable. Therefore, it is required that all generating units are able to handle temporary overload until they can be driven back into the desired operating range. The algorithm is summarized in Fig. 7.3.

Figure 7.3: Flowchart of modified P&O algorithm for localized power constraint management.

P_G is the measured active power outputs at the bus, Q_G is the measured reactive power outputs at the bus, P_{min} is the minimum active power output at the bus, Q_{min} is the minimum reactive power output at the bus, P_{max} is the maximum active power output at the bus, Q_{max} is the maximum reactive power output at the bus, ΔP_G is the change in active power output induced by the last alteration of the angle δ , ΔQ_G is the change in reactive power output induced by the last alteration of the voltage V , ϕ is the value by which the angle δ is to be perturbed at the current iteration, and ψ is the value by which the angle V is to be perturbed at the current iteration. The magnitudes of ϕ and ψ remains constant, but their signs can change as detailed in Fig. 7.3. The algorithm of Fig. 7.3 will be referred to as the perturb and observe squeeze algorithm (POSA).

7.3.2 Constrained NE with Supplementary POSA Feedback

The algorithm for localized optimization is summarized in this section. The optimization is a two-stage process. In stage 1, local NE are calculated for each bus. Each bus takes a turn optimizing with time allowed for steady-state to be reached after each optimization. In stage 2, POSA is performed at each bus, one at a time, cyclically. If a generator is already within its power limits, it is skipped in the current round. The process repeats until all generators active power outputs are within their desired limits.

initialization;

STAGE 1: Locally Constrained Nash Equilibrium

for $i = 1, \dots, n$ **do**

if *bus i is generator bus* **then**

- 1) Calculate the NE of (7.19) for bus i ;
- 2) Set V_i and δ_i based on NE result for bus i ;
- 3) For dynamic simulation or application to hardware allow time delay for system to reach steady-state before NE calculation at next source;

else

 Perform no action;

end

end

STAGE 2: Perturb and Observe

while $\min(\mathbf{P}_G - \mathbf{P}_{\min} < 0)$ *OR* $\min(\mathbf{P}_G - \mathbf{P}_{\max} > 0)$ *OR* $\min(\mathbf{Q}_G - \mathbf{Q}_{\min} < 0)$ *OR*

$\min(\mathbf{Q}_G - \mathbf{Q}_{\max} > 0)$ **do**

- 1) Start with generator at bus j .
- 2) Check that $P_{\min,j} \leq P_{Gj} \leq P_{\max,j}$ *OR* $Q_{\min,j} \leq Q_{Gj} \leq Q_{\max,j}$. If true skip to step 5. If false, go to step 3.
- 3) Perform POSA for bus j
- 4) Update V_j and δ_j at bus j .
- 5) Repeat $j := j + 1$ until the next bus with a power supply present is reached.
- n is the number of buses. If $j := n + 1$ is reached, reset j to $j = 1$.

end

Algorithm 3: Constrained Nash Equilibrium with POSA power constraint enforcement. In stage 1, a constrained NE is performed individually at each bus in a turn-based fashion. In stage 2, any violated power constraints are reinforced using POSA in a turn based fashion until all sources are within operating limits.

7.3.3 Numerical Results

As earlier discussed, sharing information to achieve an optimal solution taking all variables into account is the ideal method for optimizing a power grid (or microgrid). The centralized control will be the base case for the results that follow. A turn-based approach is utilized. The optimization if

performed periodically by each player. For the constrained Nash problem at bus i

$$J_i(\mathbf{x}_i, \mathbf{x}_{-i}) = (a_i^p P_{Gi}^2 + b_i^p P_{Gi} + c_i^p) + (a_i^q Q_{Gi}^2 + b_i^q Q_{Gi} + c_i^q), \quad (7.20)$$

$$P_{Gi} - P_{Li} - \sum_{j=1}^n Y_{ij} V_i V_j \cos(\theta_{ij} + \delta_j - \delta_i) = 0 \quad (7.21)$$

$$Q_{Gi} - Q_{Li} + \sum_{j=1}^n Y_{ij} V_i V_j \sin(\theta_{ij} + \delta_j - \delta_i) = 0 \quad (7.22)$$

The steady-state system response is modeled algebraically. For the cases in this dissertation, $a_i^p = 1$, $b_i^p = 0$, $c_i^p = 0$, $a_i^q = 0$, $b_i^q = 0$, $c_i^q = 0$. The IEEE 14-bus and IEEE 30-bus systems are studied [75]. Transformer tap settings are neglected in this analysis. P_L is the active load power, Q_L is the reactive load power, B_s is the shunt admittance, V_{min} is the minimum voltage, V_{max} is the maximum voltage, P_{min} is the minimum active power output, P_{max} is the maximum active power output, Q_{min} is the minimum reactive power output, and Q_{max} is the maximum reactive power output.

For the line parameters, R is the branch resistance, X is the branch reactance, and B_{chg} is the full line charging admittance. In Algorithm 3, perturbation magnitudes of $\phi = 0.001$ and $\psi = 0.001$ are chosen respectively for the voltage magnitudes and angles.

IEEE 14-bus test case

bus	P_L	Q_L	B_s	V_{min}	V_{max}	P_{min}	P_{max}	Q_{min}	Q_{max}
1	0	0	0	0.94	1.06	0	3.324	0	0.10
2	0.217	0.127	0	0.94	1.06	0	1.40	-0.40	0.50
3	0.942	0.19	0	0.94	1.06	0	1.00	0	0.40
4	0.478	0	0	0.94	1.06	—	—	—	—
5	0.76	0.015	0	0.94	1.06	—	—	—	—
6	0.112	0.075	0	0.94	1.06	0	1.00	-0.06	0.24
7	0	0	0	0.94	1.06	—	—	—	—
8	0	0	0	0.94	1.06	0	1.00	-0.06	0.24
9	0.295	0.166	0.19	0.94	1.06	—	—	—	—
10	0.09	0.058	0	0.94	1.06	—	—	—	—
11	0.035	0.018	0	0.94	1.06	—	—	—	—
12	0.061	0.016	0	0.94	1.06	—	—	—	—
13	0.135	0.058	0	0.94	1.06	—	—	—	—
14	0.149	0.05	0	0.94	1.06	—	—	—	—

Table 7.1: Bus parameters of IEEE 14-bus test system. All values are per-unit.

from	to	R	X	B_{chg}
1	2	0.1938	0.19797	0.0528
1	5	0.05403	0.17632	0.0492
2	3	0.4699	0.17388	0.0438
2	4	0.05811	0.17103	0.034
2	5	0.05695	0.04211	0.0346
3	4	0.06701	0.20912	0.0128
4	5	0.01335	0.55618	0
4	7	0	0.25202	0
4	9	0	0.1989	0
5	6	0	0.25581	0
6	11	0.09498	0.13027	0
6	12	0.12291	0.17615	0
6	13	0.06615	0.13027	0
7	8	0	0.17615	0
7	9	0	0.11001	0
9	10	0.03181	0.0845	0
4 9	14	0.12711	0.27038	0
10	11	0.08205	0.19207	0
12	13	0.22092	0.19988	0
13	14	0.17093	0.34802	0

Table 7.2: Line parameters of IEEE 14-bus test system. All values are per-unit.

The bus parameters for the IEEE 14-bus test system are summarized in Table 7.1 and the line parameters are summarized in Table 7.2. All values are per-unit quantities. Initially a centralized optimization is performed using the SSIP method with $\tau = 0.95$ and a tolerance of 0.001. The objective of the centralized optimization is to minimize transmission losses. After this operating point is attained, the communications system is lost and all loads subsequently increase by 50%. In response to the loss of communication, Algorithm 3 is implemented. The results for the centralized optimization prior to the loss of the communication network and the 50% load increase are summarized in Figs. 7.4, 7.5, 7.6, and 7.7. The voltage levels and power generation computed by the centralized optimizer are all within the desired ranges.

Figure 7.4: Bus voltage magnitudes for IEEE 14-bus after centralized optimization using proposed earlier squared slack interior-point method.

Figure 7.5: Bus voltages angles for IEEE 14-bus after centralized optimization using proposed earlier squared slack interior-point method. No reference angle is chosen so that there is no angle that is identically zero.

Figure 7.6: Active power generation for IEEE 14-bus after centralized optimization using proposed squared slack interior-point method.

Figure 7.7: Reactive power generation for IEEE 14-bus after centralized optimization using proposed squared slack interior-point method.

The voltages and power generation after the 50% load step are summarized in Figs. 7.8, 7.9, 7.10, and 7.11. Recall that at this operating state, no additional optimization has been carried due to a loss of communication prior to the load step. At this state all buses voltages remain within the desired range. The the voltage angles of the ac sources remain unchanged as did the voltage magnitudes of the sources. Slight changes in the load voltage angles have occurred, but are of little concern since the angles are unbounded. Also, the active power generation remains within the desired range except for that of generator 3 and the reactive power of the generator at bus 6 it outside of the desired range. In order to drive the outputs of generators 3 and 6 back within the desire range, Algorithm 3 is implemented.

It is assumed that the communication system is lost before the centralized controller has a chance to update the operating point. As can be seen in Figs. 7.12 and Fig. 7.13, all of the active and power constraints are satisfied once the final operating point is attained. Only the upper bounds are shown in the figure, but the lower bounds are also satisfied for each generator. During the transition process, several generators violate their power constraints to varying degrees. In system design, this temporary overloading should be taken into account if there is a risk of losing the communication network or if none is present from the start. Also, the progression of the voltage magnitudes and angles are are shown in Figs. 7.14 and 7.15 respectively. Once the NE is attained for all sources, the source voltages magnitudes are systematically perturbed to drive the power generation back within the constraints. As was seen in discussed prior, this process was successful. However, the new operating point the several of the bus voltages magnitudes are violating the original constraints. Therefore, if there are voltage sensitive loads, additional voltage regulation may be required close to the load to ensure voltage quality. For example ac/dc/ac converters may be required to maintain the desired voltage levels at the distribution end.

Figure 7.8: Bus voltage magnitudes for IEEE 14-bus after load is stepped by 50% and new steady-state has been reached.

Figure 7.9: Bus voltage angles for IEEE 14-bus after load has been stepped by 50%.

Figure 7.10: Active power generation for IEEE 14-bus after the loads have been all increased by 50%.

Figure 7.11: Reactive power generation for IEEE 14-bus after the loads have been all increased by 50%.

Figure 7.12: Progression of active power delivery in IEEE 14-bus test case.

Figure 7.13: Progression of reactive power delivery in IEEE 14-bus test case.

Figure 7.14: Progression of voltage angles IEEE 14-bus test case.

Figure 7.15: Progression of voltage angles IEEE 14-bus test case.

Once the new steady-state solution is attained, the overall losses are 0.1582 pu or 3.9138%. If communications were maintained after the 50% load step, the centralized squared slack interior-point

solver with the loss minimization objective would arrive at a solution where the losses are 0.1692 pu or 4.1736%. There appears to be an improvement in the system when the communication system is removed. However, this result is somewhat deceptive. For the centralized solution all parameters are properly constrained. However, after the NE is attained and the perturbation process is complete, the voltages are no longer within the desired ranges. This is effectively optimizing the original problem with relaxed voltage constraints which would most certainly improve the optimality of the solution.

IEEE 30-bus test case

The bus parameters for the IEEE 30-bus test system are summarized in Table 7.3 and the line parameters are summarized in Table 7.4. All values are per-unit quantities. Initially a centralized optimization is performed using the squared slack interior point method with $\tau = 0.95$ and a tolerance of 0.001. The objective of the centralized optimization is to minimize transmission losses. After this operating point is attained, the communications system is lost and all loads subsequently increase by 50%. In response to the loss of communication, Algorithm 3 is implemented.

bus	P_L	Q_L	B_s	V_{min}	V_{max}	P_{min}	P_{max}	Q_{min}	Q_{max}
1	0	0	0	0.95	1.05	0.50	2.00	-0.20	1.50
2	0.217	0.127	0	0.95	1.05	0.20	0.80	-0.20	0.60
3	0.024	0.012	0	0.95	1.05	—	—	—	—
4	0.076	0.016	0	0.95	1.05	—	—	—	—
5	0	0	0.19	0.95	1.05	0.15	0.50	-0.15	0.625
6	0	0	0	0.95	1.05	—	—	—	—
7	0.228	0.109	0	0.95	1.05	—	—	—	—
8	0.30	0	0	0.95	1.05	0.10	0.35	-0.15	0.487
9	0	0	0	0.95	1.05	—	—	—	—
10	0.058	0.02	0	0.95	1.05	—	—	—	—
11	0	0	0	0.95	1.05	0.10	0.30	-0.10	0.40
12	0.112	0.075	0	0.95	1.05	—	—	—	—
13	0	0	0	0.95	1.05	0.12	0.40	-0.15	0.447
14	0.062	0.016	0	0.95	1.05	—	—	—	—
15	0.082	0.025	0	0.95	1.05	—	—	—	—
16	0.035	0.018	0	0.95	1.05	—	—	—	—
17	0.09	0.058	0	0.95	1.05	—	—	—	—
18	0.032	0.009	0	0.95	1.05	—	—	—	—
19	0.095	0.034	0	0.95	1.05	—	—	—	—
20	0.022	0.007	0	0.95	1.05	—	—	—	—
21	0.175	0.112	0	0.95	1.05	—	—	—	—
22	0	0	0	0.95	1.05	—	—	—	—
23	0.032	0.016	0	0.95	1.05	—	—	—	—
24	0.087	0.067	0.04	0.95	1.05	—	—	—	—
25	0	0	0	0.95	1.05	—	—	—	—
26	0.035	0.023	0	0.95	1.05	—	—	—	—
27	0	0	0	0.95	1.05	—	—	—	—
28	0	0	0	0.95	1.05	—	—	—	—
29	0.024	0.009	0	0.95	1.05	—	—	—	—
30	0.106	0.019	0	0.95	1.05	—	—	—	—

Table 7.3: Bus parameters of IEEE 30-bus test system. All values are per-unit.

from	to	R	X	B_{chg}
1	2	0.0192	0.0575	0.0528
1	3	0.0452	0.1652	0.0408
2	4	0.057	0.1737	0.0368
3	4	0.0132	0.0379	0.0084
2	5	0.0472	0.1983	0.0418
3	6	0.0581	0.1763	0.0374
4	6	0.0119	0.0414	0.009
5	7	0.046	0.116	0.0204
6	7	0.0267	0.082	0.017
6	8	0.012	0.042	0.009
6	9	0	0.208	0
6	10	0	0.556	0
9	11	0	0.208	0
9	10	0	0.11	0
4	12	0	0.256	0
12	13	0	0.14	0
12	14	0.1231	0.2559	0
12	15	0.0662	0.1304	0
12	16	0.0945	0.1987	0
14	15	0.221	0.1997	0
16	17	0.0524	0.1923	0
15	18	0.1073	0.2185	0
18	19	0.0639	0.1292	0
19	20	0.034	0.068	0
10	20	0.0936	0.209	0
10	17	0.0324	0.0845	0
10	21	0.0348	0.0749	0
10	22	0.0727	0.1499	0
21	22	0.0116	0.0236	0
15	23	0.1	0.202	0
22	24	0.115	0.179	0
23	24	0.132	0.27	0
24	25	0.1885	0.3292	0
25	26	0.2544	0.38	0
25	27	0.1093	0.2087	0
28	27	0	0.396	0
27	29	0.2198	0.4153	0
27	30	0.3202	0.6027	0
29	30	0.2399	0.4533	0
8	28	0.0636	0.2	0.0428
6	28	0.0169	0.0599	0.013

Table 7.4: Line parameters of IEEE 30-bus test system. All values are per-unit.

The results for the centralized optimization prior to the loss of the communication network and the 50% load increase are summarized in Fig. 7.16, 7.17, 7.18, and 7.19. It can be seen in Fig. 7.16 that all voltage magnitudes are within the desired range of 0.95-1.05. The voltage angles

are summarized in Fig. 7.17. The angles were effectively unconstrained since they were limited to $\pm 180^\circ$. The reactive power generation for the centralized optimization is shown in Fig. 7.19. It can be seen that all reactive power generation constraints are also satisfied. The active power generation for the centralized optimization is depicted in Fig. 7.18. All active power generation is within desired limits.

Figure 7.16: Bus voltage magnitudes for IEEE 30-bus after centralized optimization using proposed earlier squared slack interior point method.

Figure 7.17: Bus voltages angles for IEEE 30-bus after centralized optimization using proposed earlier squared slack interior-point method. No reference angle is chosen so that there is no angle that is identically zero.

Figure 7.18: Active power generation for IEEE 30-bus after centralized optimization using proposed earlier squared slack interior point method.

Figure 7.19: Reactive power generation for IEEE 30-bus after centralized optimization using proposed earlier squared slack interior point method.

Figure 7.20: Bus voltage magnitudes for IEEE 30-bus after load is stepped by 50% and new steady-state has been reached.

Figure 7.21: Bus voltage angles for IEEE 30-bus after load has been stepped by 50%.

Figure 7.22: Active power generation for IEEE 30-bus after the loads have been all increased by 50%.

Figure 7.23: Reactive power generation for IEEE 30-bus after the loads have been all increased by 50%.

The voltages magnitudes after the load step are summarized in Fig. 7.20. At this state all buses voltages several voltage are already outside of the desired range of 0.95-1.05 pu with bus 30 being the worst offender at 0.89755 pu. The voltages angles after the load step are summarized in Fig. 7.21. Slight changes in the load voltage angles have occurred, but are of little concern since the angles are unbounded. The steady-state active power generation after the 50% load step has occurred are summarized in Fig. 7.22. Buses 8 and 13 are violating their active power generation constraints. Also, The steady-state reactive power generation after the 50% load step has occurred is summarized in Fig. 7.23. All generators are satisfying their reactive power generation constraints. The loss of communication triggers Algorithm 3 to be implemented. It is assumed that the communication system is lost before the centralized controller has a chance to update the operating point. As can be seen in Fig. 7.24, all of the active power constraints are satisfied once the final operating point is attained. Only the upper bounds are shown in the figure, but the lower bounds are also satisfied for each generator. During the transition process, several generators violate their active generation constraints to varying degrees. Also, it can be seen in Fig. 7.25 that all of the reactive power constraints are satisfied once the final operating point is attained. During the transition process, only the generator at bus 8 violates its reactive generation constraint. For the reactive power generation, the boundary violations are relatively minor.

Figure 7.24: Progression of active power delivery in IEEE 30-bus test case.

Figure 7.25: Progression of reactive power delivery in IEEE 30-bus test case.

The progression of the voltage magnitudes is shown in Fig. 7.26. Once the NE is attained for all sources, the source voltage magnitudes are systematically perturbed to drive the reactive power back within the constraints. As was seen in earlier in Fig. 7.25, this process was successful. However, the new operating points the several of the bus voltage magnitudes are violating the original constraints. Therefore, if there are voltage sensitive loads, additional voltage regulation may be required close to the load to ensure voltage quality as earlier discussed for the IEEE 14-bus case. The progression of the voltage angles is shown in Fig. 7.27. Once the NE is attained for all sources, the source voltage angles are systematically perturbed (in parallel with the magnitudes) to drive the active power back within the constraints. As was seen in earlier in Fig. 7.24, this process was successful. The angles all shift together with their spread remaining relatively uniform. Once the new steady-state solution is attained, the overall losses are 0.0835 pu or 2.8565%. If communications were maintained after the 50% load step, a solution of the 150% load problem could not be found within the voltage limits of 0.95-1.05. However when the voltage constraints are relaxed to the range of 0.90-1.10 the centralized squared slack interior-point solver with the loss minimization objective would arrive at a solution where the losses are 0.0700 pu or 2.4070%. The centralized solution with similarly relaxed constraints improves over the NE solution as should be expected.

Figure 7.26: Progression of voltage angles IEEE 30-bus test case.

Figure 7.27: Progression of voltage angles IEEE 30-bus test case.

7.4 Constrained Game Theoretic Optimal Power Flow in Hybrid Ac/Dc Microgrids

The example of Fig. 6.1 is again considered in this section. The load profile is shown in Table 7.5. Each inverter is bidirectional and rated at 30kW. Buses 12, 15, and 18 are controlled using the voltages source rectifiers controlled as described in section 4.2. Note again that there is no communication infrastructure. Therefore, other than scheduled charge/discharge cycles, the batteries have no current references available. The battery is in charging mode. It is assumed that battery controller is able to estimate the state of the PV array by assessing environmental conditions. Therefore the input power of the battery is set to the output power of the PV array.

Table 7.5: HACDC load profile

bus	0s	30s	60s
7	0 kW	50 kW	25 kW
8	0 kW	25 kW	50 kW
9	75 kW	0 kW	0 kW

The overall objective is to minimize power losses in the system. This is not, in general, possible in an exact sense without communication. This is because for each optimizer power balance is only possible to calculate power balance at the local bus. That is, even if the full structure of the system is known to each player, the loads are only known locally. For inverter $N_{x,y}^{ac}$, connecting dc bus x to

3-phase ac bus y , the constrained game theory problem is then

$$\begin{aligned}
& \text{Minimize } J_{N_{x,y}^{ac}} = 10 \sum_{i=1}^n P_{Gi}^2, \text{ subject to} \\
& - \sum_{\{\forall j | \mathbf{T}_{x,j}=1\}} V_x(V_x - V_j)Y_{x,j} - \sum_{\{\forall j | \mathbf{T}_{x,j}=2\}} \tilde{V}_x \left[(\tilde{V}_x - \tilde{V}_j) \tilde{Y}_{x,j} \right]^* \\
& - \sum_{\{\forall j | \mathbf{T}_{x,j}=3\}} \mu_{N_{x,j}^{dc}} V_x (\mu_{N_{x,j}^{dc}} V_x - V_j) Y_{x,j} - \sum_{\{\forall j | \mathbf{T}_{x,j}=4\}} V_x (V_x - \mu_{N_{x,j}^{dc}} V_j) Y_{x,j} \\
& - \sum_{\{\forall j | \mathbf{T}_{x,j}=5\}} \tilde{V}_x \left[\left(\tilde{V}_x - \tilde{\lambda}_{N_{x,j}^{ac}} V_j \right) \tilde{Y}_{x,j} \right]^* - \sum_{\{\forall j | \mathbf{T}_{x,j}=6\}} \tilde{\lambda}_{N_{x,j}^{ac}} V_x \left[\left(\tilde{\lambda}_{N_{x,j}^{ac}} V_x - \tilde{V}_j \right) \tilde{Y}_{x,j} \right]^* \\
& - \sum_{\{\forall j | \mathbf{T}_{y,j}=1\}} V_y(V_y - V_j)Y_{y,j} - \sum_{\{\forall j | \mathbf{T}_{y,j}=2\}} \tilde{V}_y \left[(\tilde{V}_y - \tilde{V}_j) \tilde{Y}_{y,j} \right]^* \\
& - \sum_{\{\forall j | \mathbf{T}_{y,j}=3\}} \mu_{N_{y,j}^{dc}} V_y (\mu_{N_{y,j}^{dc}} V_y - V_j) Y_{y,j} - \sum_{\{\forall j | \mathbf{T}_{y,j}=4\}} V_y (V_y - \mu_{N_{y,j}^{dc}} V_j) Y_{y,j} \\
& - \sum_{\{\forall j | \mathbf{T}_{y,j}=5\}} \tilde{V}_y \left[\left(\tilde{V}_y - \tilde{\lambda}_{N_{y,j}^{ac}} V_j \right) \tilde{Y}_{y,j} \right]^* - \sum_{\{\forall j | \mathbf{T}_{y,j}=6\}} \tilde{\lambda}_{N_{y,j}^{ac}} V_y \left[\left(\tilde{\lambda}_{N_{y,j}^{ac}} V_y - \tilde{V}_j \right) \tilde{Y}_{y,j} \right]^* = 0, \\
& V_{j,min} \leq V_j \leq V_{j,max}, \forall j \in \{1, \dots, n\}, \\
& \delta_{j,min} \leq \delta_j \leq \delta_{j,max}, \forall j \in \{1, \dots, n_{ac}\}, \\
& \lambda_{j,min} \leq \lambda_j \leq \lambda_{j,max}, \forall j \in \{1, \dots, n_{acdc}\}, \\
& \mu_{j,min} \leq \mu_j \leq \mu_{j,max}, \forall j \in \{1, \dots, n_{dc}dc\}, \\
& \gamma_{j,min} \leq \gamma_j \leq \gamma_{j,max}, \forall j \in \{1, \dots, n_{acdc}\}, \\
& P_{j,min} \leq P_{Gj} \leq P_{j,max}, \forall j \in \{1, \dots, n\}, \\
& Q_{j,min} \leq Q_{Gj} \leq Q_{j,max}, \forall j \in \{1, \dots, n_{ac}\}, \tag{7.23}
\end{aligned}$$

(7.23) is solved using the SSIP method introduced in Chapter 6. Note that the equality constraints are complex and should be broken apart into real and imaginary parts in application. Also, note that the objective function for each player takes the generation at all buses into account. The goal is to allow all inverters to independently work towards the same objective. The key issue to note here again is that since the constraints will differ, the solutions at each bus too will differ except for in a few trivial cases. These differences in solutions can result in an unstable or undesired operating point in the actual system. The main concern here is that each converter is not overloaded. To alleviate this issue at inverter l , the reference voltage angle of each inverter is adjusted after optimization

using the overload feedback control

$$\delta_{adj,l} = k_i \int (\min(P_{inv,l} - P_{Imin,l}, 0) + \max(P_{inv,l} - P_{Imax,l}, 0)) dt, \quad (7.24)$$

where k_i is the integral gain, $P_{inv,k}$ is the total per-unit power out of the ac side of inverter l , $P_{Imin,l}$ is the minimum per-unit power out of the ac side of inverter l , and $P_{Imax,l}$ is the maximum per-unit power out of the ac side of inverter l . For the example in this section, $k_i = -0.1$. $\delta_{adj,l}$ is added to the reference value δ_l determined from (7.23) in order to maintain operation within inverter active power constraints. The parameters of Fig. 6.1 are as earlier defined. Only the load is altered. The reason that the load is defined as it is in Table 7.5 is to intentionally create local overloads at each inverter output over the duration of the simulation. That is, the local load of each inverter at some point is higher than its capability during at least one time interval for each inverter.

7.4.1 Simulation Results

As can be seen in Fig. 7.28b, without applying overload feedback, the inverter angle are all fixed at zero. The angle adjustment with (7.24) applied are shown in Fig. 7.28a. These angle adjustment allow the load to be distributed among the inverters to avoid overloading. The active power output of the inverters is shown in Fig. 7.29. The dashed line is the upper active power limit of the rectifiers. It can be seen in Fig. 7.29b that without application of overload feedback, the inverters become severely overloaded throughout the simulation. This is because each inverter is supplying the totality of its own local load. This overload is also reflected in the 15 kW generators as can be seen in Fig. 7.30b. With application of overload feedback, the inverter outputs and, in turn, the generator outputs are maintained within the desired operating ranges as can be seen in Fig. 7.29a and Fig. 7.30a respectively.

The game theoretical optimization approach alone does not yield desirable results alone. Since the constraints are coupled and the current state of the system is not globally known, inconsistencies among local optimizers lead to undesired results. Certainly, if load information were globally available to each player, it would be possible to yield desirable results directly using the game theoretical approach. In fact, if it is possible to take into account all constraints at once, the problem effectively becomes the centralized problem of Chapter 6.

Also, the voltage magnitudes of all of the inverters are equal (not shown). This is the reason

that there is no load sharing without application of overload feedback. This calls into question application of game theory to this problem overall. A more efficient alternative would be to simply set some nominal operating point for the inverters and adjust based upon (7.24) or some other perturbation method to drive the system back into the desired range. The game theoretical method of optimization is more suited towards problems in which there are either no constraints or where the constraints are known to all players, which goes against the premise of operating without a communication infrastructure.

(a) Inverter angles with application of 7.24

(b) Inverter angles without application of 7.24

Figure 7.28: Inverter voltage angles. In (a) angle adjustments are made to avoid overloading of the inverter. In (b) the game theory method is applied without this correction.

(a) Inverter power output with application of 7.24

(b) Inverter power output without application of 7.24

Figure 7.29: Inverter power output. In (a) angle adjustments are made to avoid overloading of the inverter. In (b) the game theory method is applied without this correction.

(a) Source power output with application of 7.24

(b) Source power output without application of 7.24

Figure 7.30: Source power output. In (a) angle adjustments are made to avoid overloading of the inverter. In (b) the game theory method is applied without this correction.

7.5 Conclusion

In this chapter, methods for decentralized control of power flow were introduced. The primary controls introduced were POSA and overload feedback. These methods were effective in driving overloaded power sources back within their desired operating ranges. The secondary control was constrained game theoretical optimization. For the game theoretical optimization, since there were no communication available, only local data were able to be utilized by each player. Therefore, even though each player had the same objective, the constraints for each player differed due to informational limitations. This resulted in each local optimizer solving a different optimization problem. Thus, the optimization results for each player were in direct conflict resulting in undesired operating conditions within the actual system. The only foreseeable way to remedy this problem directly would be to add communication to allow information about local loads to be available to all players. In which case, it would be more ideal to simply use the centralized optimization methods of Chapters 5 and 6.

Chapter 8

Conclusions and Future Work

8.1 Conclusion

In this dissertation, methods for centralized and decentralized power flow management were introduced. The squared slack interior point method was introduced to solve the general hybrid ac/dc optimal power flow problem. This methodology assumes a centralized control structure. Where there is not communication present, disallowing the utilization of a centralized optimization, it was proposed that non-cooperative constrained game theoretical optimization be utilized with adjustments being made after the optimization process has completed to compensate for inconsistencies among local solutions. Methods for compensation introduced in this dissertation were the perturb and observe squeeze algorithm (POSA) and the overload feedback control of (7.24).

The squared slack interior point method was found to provide reasonably fast and accurate results when applied to the dynamic simulation of a hybrid ac/dc grid. Simulation results were found to coincide with the optimization results within the desired tolerance of 0.01% with short deviations during transient events. In addition, this method allows inequality constraints to be held to the same standard as the equality constraints. That is, assuming the problem is not ill-defined, if the error tolerance for the optimization process is ϵ_{tol} , it can be guaranteed upon convergence that the inequality constraints will also be satisfied within this same error tolerance. This method was also used to solve the constrained non-cooperative game theory problem. Results for the local optimizations of the constrained non-cooperative game theory problem were rapid. However, inconsistencies due to the incomplete system load information of each local controller resulted in overloading of

sources. In order to circumvent this issue, POSA was applied to two algebraically modeled pure ac systems and the overload feedback method (7.24) was applied to a dynamic hybrid ac/dc grid model. For all three cases, the overloaded components were able to be rapidly driven back within their desired operating ranges.

Overall, the application of constrained game theoretical optimization proved not to be useful where information on remote loads was not available. This lack of information caused the computed solution to always be non-representative of the actual state of the system. Game theoretical optimal optimization was effectively a waste of resources since its application would require a microcontroller or computer to calculate the result at each bus.

There are two possible cases to consider. If communication is available, a centralized control method such as the proposed squared slack interior point method should be applied to find the best operating point. Then, if communication is lost, either Part 2 of Algorithm 3 or (7.24) should be applied until communication is restored in order to avoid overload of components and potential cascaded breaker tripping. The second case to consider is that in which there is no centralized communication system is available at any time so that communication is never available. If all sources are in parallel on a single bus, standard droop control would be sufficient. However, if this is not the case, then an alternative is to set each bus voltage to its nominal value. This can lead to immediate overload of some sources. Again, either Part 2 of Algorithm 3 or (7.24) may be applied to drive these sources back within their desired ranges. In either case, time-overcurrent relays should be set such that they allow for these temporary overloads to be taken into account.

Additionally, if there is no communication infrastructure present or communications are lost, rectifiers may be adversely affected. Standard control methods such as feedback linearization of direct power control may not be able to provide a stabilize the the dc output voltage or maintain unity power factor out of the ac source without complete load information. An alternative was proposed in section 4.2 using the Hamiltonian surface shaping and power flow control method. This method was found to accurately track the reference voltage and maintain unity power factor over large load steps. The HSSPFC method was also found to perform better than both feedback linearization and direct power control in terms of overshooting the reference voltage. Also, it HSSPFC was comparable to both methods in terms of transient duration despite the fact the for both feedback linearization and direct power control load information was directly utilized and no load was utilized for the HSSPFC method. Overall, the HSSPFC method was found to be more versatile than the methods to which it

was compared. That is, the proposed HSPFC method works the same regardless of whether load or interconnection data are available for the VSR to which it is applied.

8.2 Future Work

In Chapter 6, the squared slack interior point optimization method was introduced. In this dissertation, the optimal power flow computation was limited to a system where the ac portion was balance 3-phase. However, it is frequently the case that there is unbalanced loading among phases. Also in the average value models developed in this dissertation, switching losses were not taken into account. For a higher level of accuracy, a frequency dependent loss function should be introduced into the objective function. In future work, both unbalanced loading and switching losses will be taken into account. In addition to these omissions, the optimization algorithm was developed and introduced without an explicit analysis of convergence. In future work, a concrete proof needs to be developed to determine convergence criteria. Also, the feasibility of the result of Algorithm 2 was established. However first order optimality was not. In future work, a goal will be to establish an analytical proof for the optimality of the computation provided by Algorithm 2. Lastly, the squared slack interior point requires matrix inversion at each iteration. A BFGS approximation of the Hessian is already utilized in order to speed up computation time. However, for larger systems matrix inversion will substantially degrade performance of the algorithm. To alleviate this potential issue the goal will be to manipulate the algorithm so that the BFGS approximation of the inverse Hessian may be used. This will completely remove matrix inversion from the algorithm which will drastically reduce computation time for larger systems.

Also, the control and optimization methods proposed in this dissertation were demonstrated in dynamic and algebraic simulations. However, in future work, the goal will be to test these controls and optimizations on actual hardware. These hardware tests will be used to further validate the controls and optimizers presented in this dissertation.

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Appendix A

Electric Grid Model Builder For MATLAB/Simulink and RT-LAB

A.1 Introduction

This software is intended to be used for dynamic simulation of electric grids containing any combination of ac and dc components. This software streamlines the process of creating electric grid models in Simulink by programmatically constructing models based solely upon input parameters. There is no need to drag and drop blocks to create a Simulink model. The component models are as detailed in Chapter 2. This quick start guide explains to the user how to manipulate an already existing Excel spreadsheet to create different models. It is required that the user is familiar with editing sheets in Excel and the user at least know how to open MATLAB and run an already existing script (m-file). In the next section, a sample Excel file is detailed using the example of Fig. 6.1 with the addition of pulse dc loads at buses 12, 15, and 18.

A.2 Excel File Format

In this section a detailed example is used to describe how to edit an Excel file in order to create a hybrid ac/dc grid model. Images of the necessary sheets are shown and descriptions of cell entries are given for clarity.

Model Parameters

The first sheet titled 'Model Parameters' is shown in Fig. A.1. This sheet contains system-wide parameters used for the Simulation. There are 3 types of model possible. This depends upon the choice of cell B9. 'dynamic (Simulink)' and 'dynamic (OPAL-RT)' are chosen to construct dynamic models to run in Simulink and on OPAL-RT respectively. 'power flow only' is chosen to run a script to calculate the steady state optimal power flow without constructing a Simulink model.

	A	B	C
1			Unit
2	Start Time	0	minutes
3	End Time	2	minutes
4	Step Size	0.1	milliseconds
5	Number of Data Points	1000000000	
6	Decimation	1	
7	Solver Method	ode3 (Bogacki-Shampine)	
8	Solver Mode	Normal	
9	Model Type	dynamic (Simulink)	
10	Optimizer Update Period	5	seconds
11	Optimizer Iterations	100	
12	Optimizer Error Tolerance	0.0001	
13	tau	0.95	
14	Nominal AC Line-to-Line Bus voltage	380	V
15	System Base Power	10	kW
16	ac grid base frequency (Hz)	60	
17	power loss objective factor	1	
18	objective factor	0	
19	log data	yes	
20			
	Model Parameters	Bus Data	Line Connections

Figure A.1: 'Model Parameters' sheet.

- Sheet Title: '**Model Parameters**'
- B2: Simulation Start time
- C2: Simulation Start time unit
- B3: Simulation Stop time
- C3: Simulation Start time unit
- B5: Maximum number of store data points per output
- B6: Decimation. A decimation of N means every Nth data value is stored
- B7: Solver Mode for Simulink Model

- B8: Solver Method for Simulink Model. Fixed Step size only.
- B9: Solver Mode for Simulink Model
 - dynamic (Simulink): Dynamic simulink model with average mode converters
 - dynamic (OPAL-RT): Dynamic simulink model with average mode converters. Formatted for immediate use on OPAL-RT.
 - power flow only: Steady state optimal power flow calculation
 - steady state: does nothing
- B10: Optimizer update period (magnitude). This is the time period between optimal power flow (OPF) calculations
- C10: Optimizer update period (unit)
- B11: Optimizer maximum iterations. OPF calculation will stop is this number of iterations is reached.
- B12: OPF error tolerance. Note that this is per-unit.
- B13: $\tau \in (0, 1)$. This will be referred to as the bound rate. Larger values result in faster solutions, but possible degraded convergence and vice versa.
- B14: Nominal line-to-line ac voltage (Magnitude). If only dc components are present, the base dc voltage may be used here.
- C14: Nominal line-to-line ac voltage (unit)
- B15: System Base Power (magnitude)
- C15: System Base Power (unit)
- B16: System-wide base ac frequency (Hz)
- B17: scale for power loss objective in OPF formulation
- B18: Scale of objective function in OPF formulation
- B19: Choose whether to log model output.

Bus Data

The sheet titled 'Bus data' is shown in Fig. A.2. This sheet contains parameters associated with each bus.

	A	B	C	D
1	bus	active (W)	reactive (VAr)	capacitance (C)
2	1	0	0	0
3	2	0	0	0
4	3	0	0	0
5	4	0	0	0
6	5	0	0	0
7	6	0	0	0
8	7	0	0	0.0002
9	8	0	0	0.0002
10	9	0	0	0.0002
11	10	0	0	0.0001
12	11	0	0	0
13	12	0	0	0.0016
14	13	0	0	0.0001
15	14	0	0	0
16	15	0	0	0.0016
17	16	0	0	0.0001
18	17	0	0	0
19	18	0	0	0.0016
20				

E	F	G	H	I	J	K	L	M	N
ac/dc	Vbase	Vmin	Vmax	Pmin	Pmax	Qmin	Qmax	objective	type
ac	220	220	220	1500	15000	-15000	15000	1*P^2	3-phase ac generator
ac	220	220	220	1500	15000	-15000	15000	1*P^2	3-phase ac generator
ac	220	220	220	1500	15000	-15000	15000	1*P^2	3-phase ac generator
ac	220	220	220	1500	15000	-15000	15000	1*P^2	3-phase ac generator
ac	220	220	220	1500	15000	-15000	15000	1*P^2	3-phase ac generator
ac	220	220	220	1500	15000	-15000	15000	1*P^2	3-phase ac generator
ac	220	209	231	0	0	-30000	30000	0*P^2	capacitor
ac	220	209	231	0	0	-30000	30000	0*P^2	capacitor
ac	220	209	231	0	0	-30000	30000	0*P^2	capacitor
dc	518.4	518.4	518.4	3300	3300	0	0	0*P^2	pv array
dc	569.2	569.2	569.2	-15000	15000	0	0	1*P^2	battery
dc	750	750	750	0	0	0	0	0*P^2	capacitor
dc	518.4	518.4	518.4	3300	3300	0	0	0*P^2	pv array
dc	569.2	569.2	569.2	-15000	15000	0	0	1*P^2	battery
dc	750	750	750	0	0	0	0	0*P^2	capacitor
dc	518.4	518.4	518.4	3300	3300	0	0	0*P^2	pv array
dc	569.2	569.2	569.2	-15000	15000	0	0	1*P^2	battery
dc	750	750	750	0	0	0	0	0*P^2	capacitor

Figure A.2: 'Bus Data' sheet.

- Column A: Bus number
- Column B: Active load power (W)
- Column C: Reactive load power (VA)
- Column D: Capacitance (F)

- if ac, per-phase line-to-neutral
- if ac generator, not used
- if dc capacitor, line to ground
- if PV array, capacitance of PV array output capacitor
- if battery, no used
- Column E: ac or dc bus?
 - if ac, line-to-neutral
 - if dc capacitor, line to ground
- Column G: Lower voltage limit (V)
- Column H: Upper voltage limit (V)
- Column I: Lower Active power generation limit (W)
- Column J: Upper Active power generation limit (W)
- Column K: Lower Reactive power generation limit (VA)
- Column L: Upper Reactive power generation limit (VA)
- Column M: Objective function
 - V, δ , P, and Q represent the voltage magnitude, voltage angle active power and reactive power respectively. The objective function may be defined as any function recognized by MATLAB.
- Column N: voltage source type
 - 3-phase generator: synchronous 3 phase diesel generator
 - PV array
 - battery
 - capacitor: may be either 3 phase ac or dc depending upon entry in column E

Line Connections

The sheet titled 'Line Connections' is shown in Fig. A.3. This sheet contains parameters pertaining to connections between buses. Buses may be connected via lines or power electronics interfaces. For the ac lines, the values are per-phase. For the dc lines, the values represent full current loop (sending and returning combined).

	A	B	C	D	E	F	G	H	I
1	from	to	R	L	type	ac/dc #	dc/dc #	kp	ki
2	7	8	0.01	0.0001	ac	0	0	0	0
3	8	9	0.01	0.0001	ac	0	0	0	0
4	9	7	0.01	0.0001	ac	0	0	0	0
5	12	7	0.1	0.001	inverter	1	0	0	0.1
6	15	8	0.1	0.001	inverter	2	0	0	0.1
7	18	9	0.1	0.001	inverter	3	0	0	0.1
8	1	12	0.1	0.001	rectifier	4	0	0	0.1
9	2	12	0.1	0.001	rectifier	5	0	0	0.1
10	3	15	0.1	0.001	rectifier	6	0	0	0.1
11	4	15	0.1	0.001	rectifier	7	0	0	0.1
12	5	18	0.1	0.001	rectifier	8	0	0	0.1
13	6	18	0.1	0.001	rectifier	9	0	0	0.1
14	12	10	0.1	0.001	buck	0	1	0	0.1
15	11	12	0.1	0.001	boost	0	2	0	0.1
16	15	13	0.1	0.001	buck	0	3	0	0.1
17	14	15	0.1	0.001	boost	0	4	0	0.1
18	18	16	0.1	0.001	buck	0	5	0	0.1
19	17	18	0.1	0.001	boost	0	6	0	0.1
20									
	◀ ▶ ...	Bus Data	Line Connections	PV Array Parameters	Battery Par				

Figure A.3: 'Line Connections' sheet.

- Sheet Title: 'Line Connections'
- Column A: sending end of line
- Column B: receiving end of line
- Column C: line resistance (per-phase for ac)
- Column D: line inductance (per-phase for ac)
- Column E: connection type
 - ac: ac line
 - dc: dc line
 - inverter

- rectifier
 - boost: boost converter
 - buck: buck converter
- Column F: ac/dc converter index. Must start at 1 and be incremented by 1. This includes both inverters and rectifiers
 - Column G: dc/dc converter index: Must start at 1 and be incremented by 1. This includes both boost and buck converters.
 - Column H: converter proportional gain
 - Column I: converter integral gain

PV Array Parameters

The sheet titled 'PV Array Parameters' is shown in Fig. A.4. This sheet contains PV array datasheet parameters. Parameters for any commercially produced PV array should have these values readily available.

	A	B	C	D	E	F	G	H	I	J	K	L
1	ac bus	Pn(W)	Voc,stc(V)	Isc,stc(A)	Vmpp(V)	Impp(A)	NOCT(°C)	$\alpha_{isc}(\%/^{\circ}\text{C})$	$\alpha_{voc}(\%/^{\circ}\text{C})$	Ns	Np	C(uF)
2	10	330	45.6	9.39	36.8	8.84	45	0.1	-0.37	15	1	100
3	13	330	45.6	9.39	36.8	8.84	45	0.1	-0.37	15	1	100
4	16	330	45.6	9.39	36.8	8.84	45	0.1	-0.37	15	1	100

Figure A.4: 'PV Array Parameters' sheet.

- Sheet Title: 'PV Array Parameters'
- Column A: Bus number
- Column B: Nominal cell power (W)
- Column C: Open circuit voltage (OCV) of cell at standard test conditions (V)
- Column D: Short circuit current of cell at standard test conditions (A)
- Column E: Maximum power point (MPP) voltage of cell (V)
- Column F: MPP current of cell (V)
- Column G: Nominal operating cell temperature (°C)

- Column H: Short circuit temperature coefficient ($\%/^{\circ}C$)
- Column I: open circuit temperature coefficient ($\%/^{\circ}C$)
- Column J: Number of cells in series
- Column K: Number of cells in parallel
- Column L: Output capacitor capacitance (μF)

Battery Parameters

The sheet titled 'Battery Parameters' is shown in Fig. A.5. This sheet contains parameters for the battery pack. 4 battery chemistries are available to choose from.

	A	B	C	D	E	F
1	bus	type	Vnom(V)	Ah rating	effic.(%)	SOC(0)
2	11	Li-Ion	518	620	99.5	0.8
3	14	Li-Ion	518	620	99.5	0.8
4	17	Li-Ion	518	620	99.5	0.8

Figure A.5: 'Battery Parameters' sheet.

- Sheet Title: 'Battery Parameters'
- Column A: Bus number
- Column B: Battery Type
 - Lead-Acid
 - Li-Ion
 - NiCd: Nickel-Cadmium
 - NiMH: Nickel-Metal Hydride
- Column C: Nominal battery voltage (V)
- Column D: Battery Ah rating (Ah)
- Column E: Battery efficiency (%)
- Column F: Initial state of charge

Generator Parameters

The sheet titled 'Generator Parameters' is shown in Fig. A.6. The diesel generator is modeled as earlier described.

	A	B	C	D	E	F
1	ac bus	P(poles)	J(kg m ²)	Bm(Nm.s/rad)	Rs(ohms)	K
2	1	4	1.662	0.1	0.087	40
3	2	4	1.662	0.1	0.087	40
4	3	4	1.662	0.1	0.087	40
5	4	4	1.662	0.1	0.087	40
6	5	4	1.662	0.1	0.087	40
7	6	4	1.662	0.1	0.087	40

G	H	I	J	K	L	M	N	O	P	Q	R
T1	T2	T3	T4	T5	T6	Tmin	Tmax	Td	Pm0	rated kW	kDelta
0.01	0.02	0.2	0.25	0.009	0.0384	0	1.1	0.024	0	50	0.01
0.01	0.02	0.2	0.25	0.009	0.0384	0	1.1	0.024	0	50	0.01
0.01	0.02	0.2	0.25	0.009	0.0384	0	1.1	0.024	0	50	0.01
0.01	0.02	0.2	0.25	0.009	0.0384	0	1.1	0.024	0	50	0.01
0.01	0.02	0.2	0.25	0.009	0.0384	0	1.1	0.024	0	50	0.01
0.01	0.02	0.2	0.25	0.009	0.0384	0	1.1	0.024	0	50	0.01

Figure A.6: 'Generator Parameters' sheet.

- Column A: Bus number
- Column B: Number of poles
- Column C: Overall inertia (kgm^2)
- Column D: Linear bearing friction coefficient (Nms/rad)
- Column E: Generator stator resistance (Ω)
- Column F: Speed controller gain
- Column G: Diesel engine time constant T_1
- Column H: Diesel engine time constant T_2
- Column I: Diesel engine time constant T_3
- Column J: Diesel engine time constant T_4
- Column K: Diesel engine time constant T_5
- Column L: Diesel engine time constant T_6
- Column M: Lower torque limit (pu)

- Column N: Upper torque limit (pu)
- Column O: Combustion time delay T_d (s)
- Column P: Initial diesel engine power output (pu)
- Column Q: Diesel engine rated power output (kW)
- Column R: angle controller gain

Environmental Conditions

The sheet titled 'Environmental Conditions' is shown in Fig. A.7. This sheet contains a profile for the environmental conditions. The temperature and irradiance are required by the PV array as earlier detailed.

	A	B	C
1	time(s)	irradiance (W/m ²)	temperature (°C)
2	0	750	25
3	30	750	25
4	60	750	25
5	90	750	25
6	120	750	25
7	150	750	25
8	180	750	25
9	210	750	25
10	240	750	25
11	270	750	25
12	300	750	25
	Environmental Conditions		

Figure A.7: 'Environmental Conditions' sheet.

- Sheet Title: 'Environmental Conditions'
- Column A: Time (s)
- Column B: Irradiance (W/m^2)
- Column C: Temperature ($^{\circ}C$)

Active Load Profile

The sheet titled 'Active Load (kW)' is shown in Fig. A.8. This is the ac 3-phase balanced active load profile.

	A	B	C	D
	time(s)			
1	bus	0	40	80
2	1	0	0	0
3	2	0	0	0
4	3	0	0	0
5	4	0	0	0
6	5	0	0	0
7	6	0	0	0
8	7	15	0	0
9	8	30	40	60
10	9	17	30	30

Active Load (kW)

Figure A.8: 'Active Load (kW)' sheet.

- Sheet Title: 'ActiveLoad (kW)'
- Column A : AC Bus Number
- Row 1: Time(s)
- Inside Table: AC active load (kW)

Reactive Load Profile

The sheet titled 'Reactive Load (kVar)' is shown in Fig. A.9. This is the ac 3-phase balanced reactive load profile.

	A	B	C	D	E	F
1	time(s) bus	0	30	60	90	120
2	1	0	0	0	0	0
3	2	0	0	0	0	0
4	3	0	0	0	0	0
5	4	0	0	0	0	0
6	5	0	0	0	0	0
7	6	0	0	0	0	0
8	7	0	0	0	0	0
9	8	0	0	0	0	0
10	9	0	0	0	0	0

Figure A.9: 'Reactive Load (kVAr)' sheet.

- Sheet Title: 'ReactiveLoad (kVAr)'
- Column A : AC Bus Number
- Row 1: Time(s)
- Inside Table: AC reactive load (kVAr)

DC Load Profile

The sheet titled 'DC Load (kW)' is shown in Fig. A.10. This is the load profile for the loads at the dc buses.

	A	B	C	D	E
1	time(s) bus	0	60	120	180
2	10	0	0	0	0
3	11	0	0	0	0
4	12	0	0	0	0
5	13	0	0	0	0
6	14	0	0	0	0
7	15	0	0	0	0
8	16	0	0	0	0
9	17	0	0	0	0
10	18	0	0	0	0

Figure A.10: 'DC Load (kW)' sheet.

- Sheet Title: 'DCLoad (kVAr)'

- Column A : AC Bus Number
- Row 1: Time(s)
- Inside Table: dc load (kW)

Pulse Load: Power

At each bus, a pulse load may be added in addition to the dc load profile mentioned in the prior subsection.

The sheet titled 'Pulse Load (kW)' is shown in Fig. A.10. This sheet contains the peak pulse power profile for the pulse load.

	A	B	C	D	E	F
1	time(s) bus	0	30	60	90	120
2	12	5	10	2	3	4
3	15	3	7	2	6	8
4	18	0	3	7	9	0

Figure A.11: 'Pulse Load (kW)' sheet.

- Sheet Title: 'Pulse Load (kW)'
- Column A : AC Bus Number
- Row 1: Time(s)
- Inside Table: Pulse dc load (kW)

Pulse Load: Duty Cycle Profile

The sheet titled 'Duty Cycle' is shown in Fig. A.12. This sheet contains the duty cycle profile for the pulse load.

	A	B	C	D	E	F
1	time(s) bus	0	30	60	90	120
2	12	0.5	0.3	0.5	0.2	0.25
3	15	0.5	0.3	0.5	0.2	0.25
4	18	0.5	0.3	0.5	0.2	0.25

Figure A.12: 'Duty Cycle' sheet.

- Sheet Title: 'Duty Cycle'
- Column A : AC Bus Number
- Row 1: Time(s)
- Inside Table: Duty Cycle

Pulse Load: Pulse Period

The sheet titled 'Pulse Period (s)' is shown in Fig. A.13. This sheet contains the period of the pulse load. This values is fixed throughout the simulation, but will be variable in later versions of this software.

	A	B	C
1	bus	period(s)	
2	12	1	
3	15	1	
4	18	1	

Figure A.13: 'Pulse Period' sheet.

- Sheet Title: 'Pulse Period (s)'
- Column A: AC Bus Number
- Column B: Pulse period (s)

A.3 Running the Model Builder

Once the Excel file has been created as describe in the previous section, the model can be constructed by running the file 'main.m'. When the file is run, the user will be asked to choose an Excel file. The user prompt is shown in Fig. A.14.

Figure A.14: File Selection Prompt.

The file used for the example described in the prior section is 'acdc18bus.xlsx'. This file is double-clicked as usual to select the file. After the file is selected, text of the form shown in Fig. A.15. The model will start to be constructed immediately after this full message appears. The time the message takes to be constructed will depend upon the size of the model. The top level of the model is shown in Fig. A.16. The block 'sm_grid' contains the system model along with the centralized optimizer and the block 'sm_scopes' contains all scopes. The layout of the grid model inside the 'sm_grid' block is shown in Fig. A.17. Without getting into the specifics details of each subsystem, it can be seen that the blocks are automatically aligned in an organized manner. The contents of 'sm_scopes' are shown in Ref. A.18. The option of cell B9 in the 'Model Parameters sheet is changed to dynamic (OPAL-RT) so the block is immediately ready for use in RT-LAB. This is why the OpComm block is present in Fig. A.17.

```

Command Window
Reading data from Excel file ...
File read complete!
Elapsed time is 3.932911 seconds.
fx >> |

```

Figure A.15: Command window message after file has been selected.

Figure A.16: Top level of new model constructed by model builder.

Figure A.17: Contents of sm_grid block. This block contains the full grid model along with the hybrid ac/dc optimizer.

Figure A.18: Contents of sc_scopes block. This block contains all scopes.

A.4 Conclusion

This software package is still in development. However, in its current state, it allows for various models to be developed within minutes by simple editing Excel files. In future versions of this software, it is planned to add IO functionality to the OPAL-RT option. Also, the linearized diesel engine model will be replaced with a more representative and flexible nonlinear diesel engine model using typical datasheet parameters. In addition full switching models will also be made available in later versions of this model builder.