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DESIGN AND OPTIMIZATION OF AN ELECTROMAGNETIC RAILGUN

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AN ELECTROMAGNETIC RAIL GUN

By

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A REPORT

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Abstract

Railgun is a future weapon for shooting long-range targets with high accuracy. Railgun is an economical weapon as it does not use any kind of propellant for shooting a projectile. It uses high impulsive current to generate the necessary force required to hit the target. The US Navy has been planning to implement railgun system on ships as their long-range shooting weapon. Railgun needs to be charged up to a certain voltage level to generate the necessary force to shoot the projectile to achieve the specified target location. It is very important that minimal voltage is required for shooting the projectile to hit the target accurately as it minimizes the losses occurring in the railgun system. This project focuses on solving this challenge by minimizing the railgun's initial voltage required to shoot a projectile for hitting the target with high accuracy. It also focuses on minimizing the copper losses occurring in the system. In this project, a novel method has been generated which solves both challenges.

In this project, a model of the discharging circuit of the railgun and a model of projectile's trajectory with and without air drag have been generated using Simulink. Then both of these models are merged using MATLAB. Once this model was complete, it simulates the discharge of the capacitor voltage into the railgun system to achieve the necessary acceleration required to launch a projectile with a certain velocity. It also simulates the trajectory of a projectile. In this project, an optimization technique has been implemented using Fmincon to optimize the shooting accuracy to hit the target with minimum copper losses. This has been achieved by generating a code which optimally chooses the initial input voltage required for discharging and the launch angle at which the railgun is kept. At the end, higher than 97.5 % accuracy has been achieved in hitting the target while reducing the copper loss by more than 6 % for both the cases that were modeled.

Acknowledgments

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Nomenclatures

R'	Railgun resistance per unit length
L'	Railgun inductance per unit length
R_c	Resistance of the conductor
L_c	Inductance of the conductor
$R_R(x)$	Total railgun resistance
$L_R(x)$	Total railgun inductance
I_L	Current passing through railgun
V_c	Capacitor voltage
l_c	Length of the conductor
l_R	Length of the rail
l_{PR}	Length of the projectile
h_c	Height of the conductor
h_R	Height of the rail
w_R	Width of the rail
A_c	Area of the conductor
A_R	Area of the rail
ρ	Resistivity of the material
μ_0	Permeability of free space
μ_r	Relative permeability
x	Position of the projectile on the rails
v	Velocity of the projectile
c	Capacitance value of the capacitor
F_e	Force produced by electric circuit
F_t	Total force acting on the projectile
μ_s	Friction coefficient
F_n	Force due to friction
θ	Angle at which railgun is placed/Angle of launch
m	Mass of the projectile
g	Acceleration due to gravity
a	Acceleration of the projectile due to total force acting on it
F_x	X-axis component of force on the projectile
F_y	Y-axis component of force on the projectile
v_x	X-axis component of the velocity of the projectile
v_y	Y-axis component of the velocity of the projectile
x	X-axis component of the position of the projectile
y	Y-axis component of the position of the projectile
C_d	Drag coefficient
F_{dx}	X-axis component of force on the projectile due to air drag

F_{dy}	Y-axis component of force on the projectile due to air drag
a_{dx}	X-axis component of acceleration of the projectile due to air drag
a_{dy}	Y-axis component of acceleration of the projectile due to air drag
x_{ref}	Set target location
x_{end}	Achieved target location
$W_{Cu_{loss}}$	Weight of Copper loss minimization term
W_{to}	Weight of target offset minimization term
Cu_{loss}	Copper loss happening in the railgun system

1. Introduction

Conventional guns use high pressure to accelerate a projectile by applying pressure inside a barrel [1]. Such high pressure is generated using the expansion of gases through combustion, but due to expansion limits of gas achieving high velocity becomes a constraint. Because of such limitations, projectile shot from conventional guns can barely reach velocity up to 1300 m/s [2]. By using electromagnetic rail gun velocity as high as 20,000 - 50,000 m/s can be achieved theoretically, while practically range of 6000 – 7000 m/s has been achieved [3]. Railgun's such ability makes it desirable future weapon for shooting long range targets.

The US Navy has been working on implementing the railgun technology in the battlefield for a long time [4]. In 1992 the US Army started a Focused Technology program with the University of Texas. For this program, The Center of Electro-Mechanics (CEM-UT) was created to do research on the electromagnetic launchers such as the electromagnetic railgun [5]. Electromagnetic launchers have very diverse applications. They can be used from launching loads as small as a projectile to loads as large as a space shuttle [6]. Because of such flexibility in its configuration and its parameters, electromagnetic launchers are getting very popular. Still, there exist some areas in which improvements are needed as launcher's efficiency is quite low. Research has been done in areas such as pulsed power supply technology, high impulsive current switching, arc quenching at the barrel's end, projectile design as well as increase in the life of railgun for overcoming those challenges [7].

The electromagnetic railgun is one such electromagnetic launcher which consists of two parallel rails kept at some distance. In this distance, a projectile is kept which completes the electrical circuit. The whole electromagnetic railgun consists of parts such as energy storage unit, input power supply and launching mechanism which consists of two rails and a projectile [8]. Independent research has been done for all this subsection of the railgun. Railgun works on a very basic electromagnetic law called Lorentz Force Law [9]. According to Lorentz Force Law, a force will be generated as per the current flows through the circuit, which is explained in detail in section 2. In railgun, high impulsive current is provided by the input power supply to achieve the required force. For generating this high impulsive current, different techniques have been implemented to date and research has been done to efficiently supply the impulsive current to the railgun system.

Usually, a capacitor is used as a source of power. It is discharged to generate the impulsive waveform of current required for the railgun. But there exist other more efficient ways to supply impulsive current to the rail gun system. Research has been done to increase the efficiency of the input power supply. Pulse forming networks seems to be the solution to this low efficiency of the railgun system [10]. Design and optimization of the pulsed power supply system was studied by Ju Lan, in which different parameters of the pulsed power supply were weighted and desired performance was achieved [11]. This gave the inspiration to achieve the desired performance by optimizing different parameters for this project.

High current passes through the railgun system because of which high copper loss occurs in the railgun that results into decrease in the efficiency of the railgun. As railgun is a long-range shooting weapon, it must provide necessary force to the projectile by means of electrical energy to hit the target. Therefore, it is required to formulate a control scheme to minimize the losses while hitting the target accurately. In this project, a novel optimization strategy has been formulated to reduce the copper loss occurring in the system while hitting the target within set accuracy. To achieve this goal, first railgun's dynamics have been modeled using Ordinary Differential Equations (ODE) in Simulink. Then, models of projectile's trajectory with and without air drag have been created using Simulink. These models are merged using MATLAB and a complete model of railgun's discharging circuit including projectile's trajectories is generated.

Here optimization function includes two terms that can be seen in section 4. The railgun system is a nonlinear system, which makes the optimization function a nonlinear function. Fmincon function was used in a similar type of nonlinear optimization problem for optimization of the energy transfer efficiency [12]. It implements a sequential quadratic algorithm to give the desired optimum output by choosing the input variables within set bounds and changing those input variables until the most optimum solution is found. Therefore, in this project, Fmincon function has been used in the MATLAB code to minimize the copper loss while hitting the target within allowed offset limits.

2. Railgun Theory

An electromagnetic railgun is used for accelerating projectiles with the use of electromagnetic energy. It consists of two rails set parallelly to each other with some distance between the rails. A projectile is kept in that gap between the rails, which forms a closed electrical circuit. Current with high amplitude is passed through the circuit to create the force required to move the projectile. It works on a very basic electromagnetic concept called the Lorentz Force Law, which can be given by

$$F = Il \times B. \tag{1}$$

where, F is the force acting on the projectile, I is the current passing through the projectile, l is the length of the projectile and B is the magnetic field generated by the current. This concept can be visualized using Figure 1. As it can be seen, current passes through the rails as the direction shown in Figure 1. Because of the current flow, magnetic field is generated in the downward direction. Based on the Lorentz's Force Law, due to this magnetic field and current, the projectile experiences force in the outward direction. Thus, the projectile moves forward because of that force which can be seen in Figure 1. Even if the current's direction is flipped, it will produce the force in outward direction only because the magnetic field generated will be switched to the upward direction.
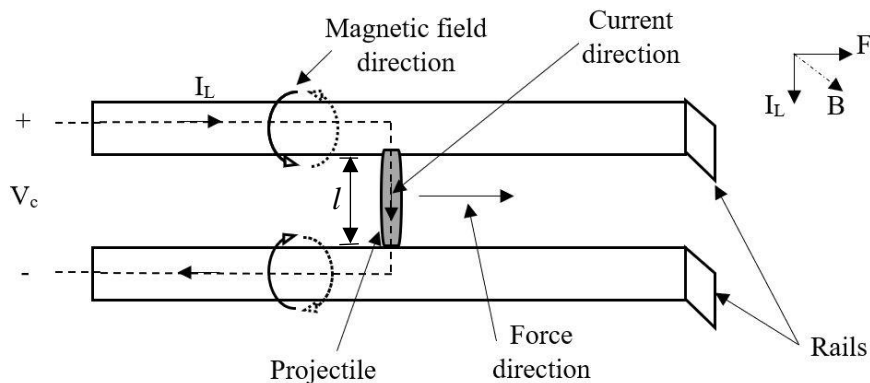
Figure 1: Visualization of the Lorentz Force Law in the Railgun

As the current passing through the rails will be very high, they need to be designed carefully. Because of such high current, even the rails experience a large force which tries to push the rails away from each other. Therefore, rails need to be fixed at their positions with the help of some mechanism to withstand that force. Even there exist few ways to increase the magnetic field that gets developed because of the current. One of the ways is to use railgun core with high magnetic permeability.

High impulsive current requires capacitors to provide energy to the railgun. Capacitors are charged and then discharged through discharging circuit. A power supply is connected to the discharging circuit through a switch, which controls when to discharge the capacitors. Based on requirement number of capacitors are connected in parallel while charging to form large capacitor bank. Once those capacitors are fully charged, the switch is closed and those parallelly charged capacitors are discharged in series to form a pulse of current. There exist different ways to generate such pulses of current. One of them is by using a pulse power supply. Pulse power supply is used in which one can generate multiple pulses of current and can also control the trigger timing of different pulses. This way energy loss can be minimized, and higher magnitude of the current can be obtained.

3. Modeling and Simulation

In this section railgun's working is modeled using ODEs. Once the working of the railgun is modeled, then the trajectories of the projectile's flight are modeled by using ODEs. After obtaining all the ODEs mathematically, those equations are simulated in a software called Simulink to obtain accurate time-based performance. In all the models, variables are initialized using a MATLAB file. After initializing those variables, they are called in the Simulink model. It is discussed in detail in following subsections.

3.1 Modeling and Simulation of the Discharging Circuit

The main purpose of this project is to optimize the initial value of the charging voltage to reach a certain target with minimum copper loss in the discharging circuit. For that, it is assumed that capacitor will be charged up to the voltage required to meet the project's purpose by the charging circuit. Modeling and simulation of the charging circuit is excluded from the scope of the project as the focus is given to the working of the discharging circuit. To understand the working of the discharging circuit, all the component's values are designed according to their market availability and then modeled using ODEs. Discharging circuit consists of a capacitor, which is assumed to be charged up to a necessary voltage level. That capacitor is further connected to a switch, which controls when to discharge that capacitor. That switch is further connected to the rails using conductors. As the current impulse will have very large values, conductors need to be properly designed and isolated. Those conductors will have resistance (R_c) and inductances (L_c) which can be seen in Figure 2.
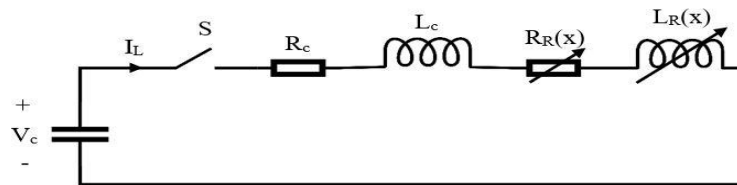
Figure 2: Discharging circuit of the Railgun

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It can be seen from Figure 2 that $R_R(x)$ and $L_R(x)$ are varying. It can be understood by Figure 3.
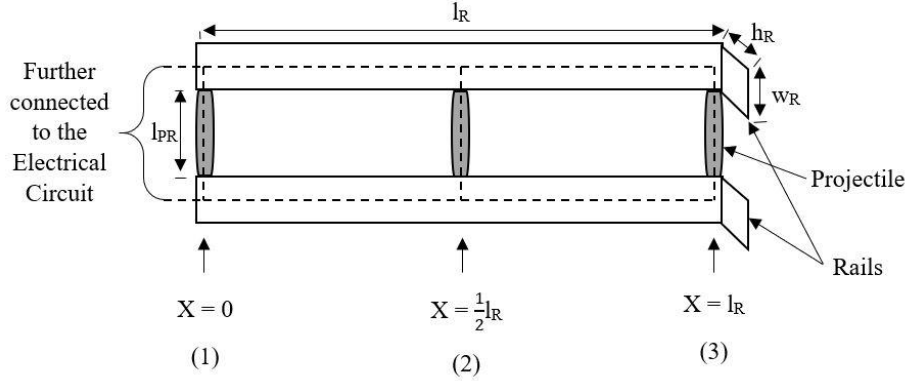
Figure 3: Visualization of change in resistance and induction of the Railgun

Figure 3 shows the configuration of the railgun. It contains two parallel rails separated at some distance. It has projectile between that gap which can be seen in Figure 3. Projectile completes the electric circuit and forms a closed loop for the current to pass. Dotted line represents the electric path of the current. As it can be seen from Figure 3, three positions have been considered to understand the change in resistance and inductances. For position 1, the rail resistance and inductance will be smallest as current is just passing through the starting point of rails, which is at X=0 and the projectile. This can be seen by dotted lines passing through the projectile at X=0. Relationship of resistance and inductance to this position can be seen in the (6) and (9). As the energy will get transferred from the capacitor, projectile will start moving forward from position 1 to position 2. It can be seen from Figure 3 that the current has to pass through half of the rail, because of that resistance and inductances will keep on increasing until the projectile reaches position 3. At that time both values will be maximum. To include this change in resistance and inductance values, resistance gradient (R') and inductance gradient (L') have been included in the equations while modeling the working of railgun. Following equations are used for building the model of the discharging circuit. All the variables names are listed in nomenclature section.

The resistance of the conductor can be found by

$$R_c = \frac{\rho l_c}{A_c}.$$

(2)

To understand how these equations are constructed in Simulink, Simulink model of (2) is shown below. Simulink models of rest of the equations are listed in Appendix A.1.
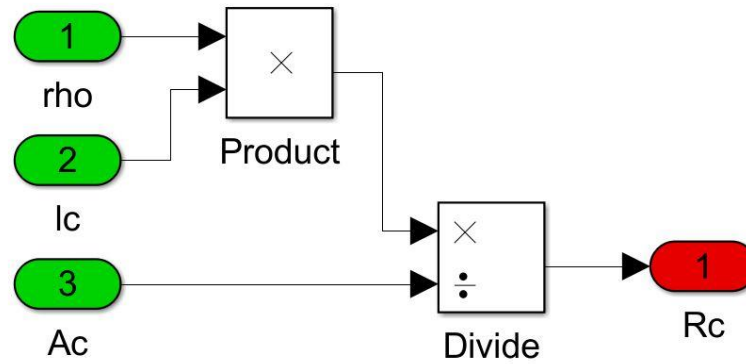
Figure 4: Rc subsystem

The inductance of the conductor can be given by

$$L_c = \frac{\mu_0 \mu_r (l_c(l_{PR} + w_R))}{h_c}.$$

(3)

Total railgun resistance can be found by

$$R_R(x) = R'(l_{PR} + 2x).$$

(4)

where, R' is the railgun resistance per unit length, which can be found by

$$R' = \frac{d}{dx}\left(\frac{\rho(l_{PR} + 2x)}{A_R}\right) = \frac{2\rho}{A_R}.$$

(5)

Substituting (5) into (4), the total railgun resistance is

$$R_R(x) = \frac{2\rho}{A_R}(l_{PR} + 2x).$$ (6)

Total railgun inductance can be found by

$$L_R(x) = L'(2x).$$ (7)

where L' is the railgun inductance per unit length, which can be found by

$$L' = \frac{d}{dx}\left(\frac{\mu_0\mu_r(2x)(l_{PR} + w_R)}{h_R}\right) = \frac{2\mu_0\mu_r(l_{PR} + w_R)}{h_R}.$$ (8)

Substituting (5) into (4), the total railgun inductance is

$$L_R(x) = \frac{4\mu_0\mu_r(l_{PR} + w_R)}{h_R}x.$$ (9)

Now, applying KVL to the electric discharging circuit shown in figure 2, following equations are derived:

$$\frac{dI_L}{dt} = \frac{1}{(L_c + L_R)}[V_c - I_L(R_c + R_R + L'v)].$$ (10)

and,

$$\frac{dV_c}{dt} = \frac{-I_L}{c}.$$ (11)

(10) and (11) are the ODEs, which are used to simulate time-based analysis of the railgun discharging circuit. By integrating both the equations V_c and I_L can be found. Simulink model of both equations are shown below.
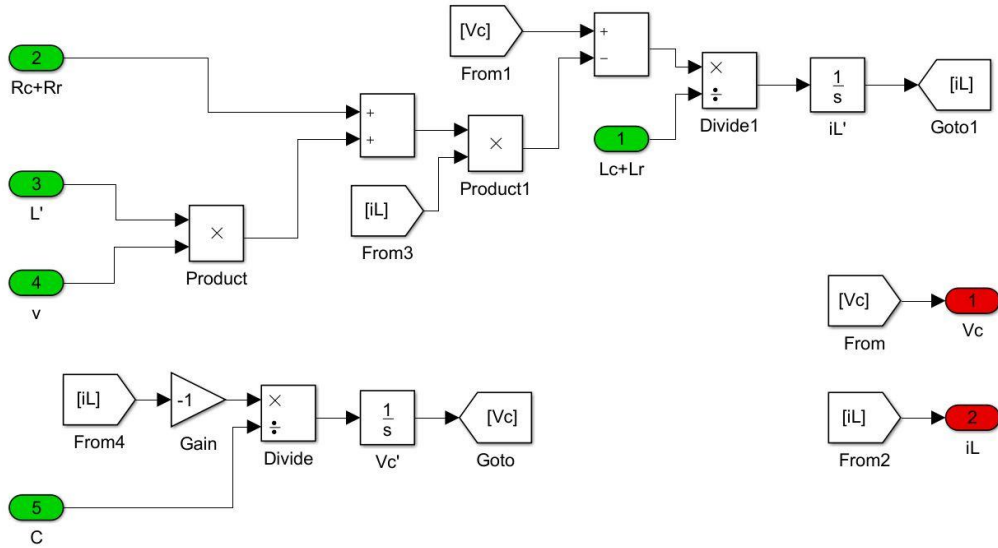
Figure 5: Vc and I_L subsystem

Once I_L is known, the force produced by the electric circuit can be found by

$$F_e = \frac{1}{2}L'I_L{}^2. \tag{12}$$

As the projectile will be sliding forward, it will experience friction on both the sides of the rail. Therefore, friction force has to be considered which is working on the opposite side of the force produced by the electric circuit. Although the point of contact of the projectile with the rails is assumed to be very less. Therefore, friction force acting on the projectile will be very less compared to electric force.

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The friction force is given by

$$F_n = mgsin\theta. \tag{13}$$

The total force acting on the projectile after considering friction force will be

$$F_t = F_e - \mu_s F_n = \frac{1}{2}L'I_L{}^2 - \mu_s mgsin\theta. \tag{14}$$

From this total force, acceleration can be found by

$$a = \frac{F_t}{m} = \frac{1}{m}\left[\frac{1}{2}L'I_L{}^2 - \mu_s mgsin\theta\right]. \tag{15}$$

As acceleration is,

$$\frac{dv}{dt} = a. \tag{16}$$

By integrating ODE given in (16), the velocity of the projectile can be found.
As velocity is,

$$\frac{dx}{dt} = v. \tag{17}$$

By integrating ODE given in (17), projectile's position on the rail can be found. Copper loss can be found by

$$Cu_{loss} = I_L^2(R_c + R_r). \tag{18}$$

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Using all these equations listed above, a Simulink model is created. The Simulink model gives the instantaneous value of the current, voltage, force acting on the projectile, acceleration of the projectile, velocity of the projectile, copper loss due to the discharge as well as the position of the projectile on the railgun. The Simulink model is shown in Figure below which is divided into 2 parts. As the whole model is very big, it is further divided into subfigures which are included in appendix A.1.
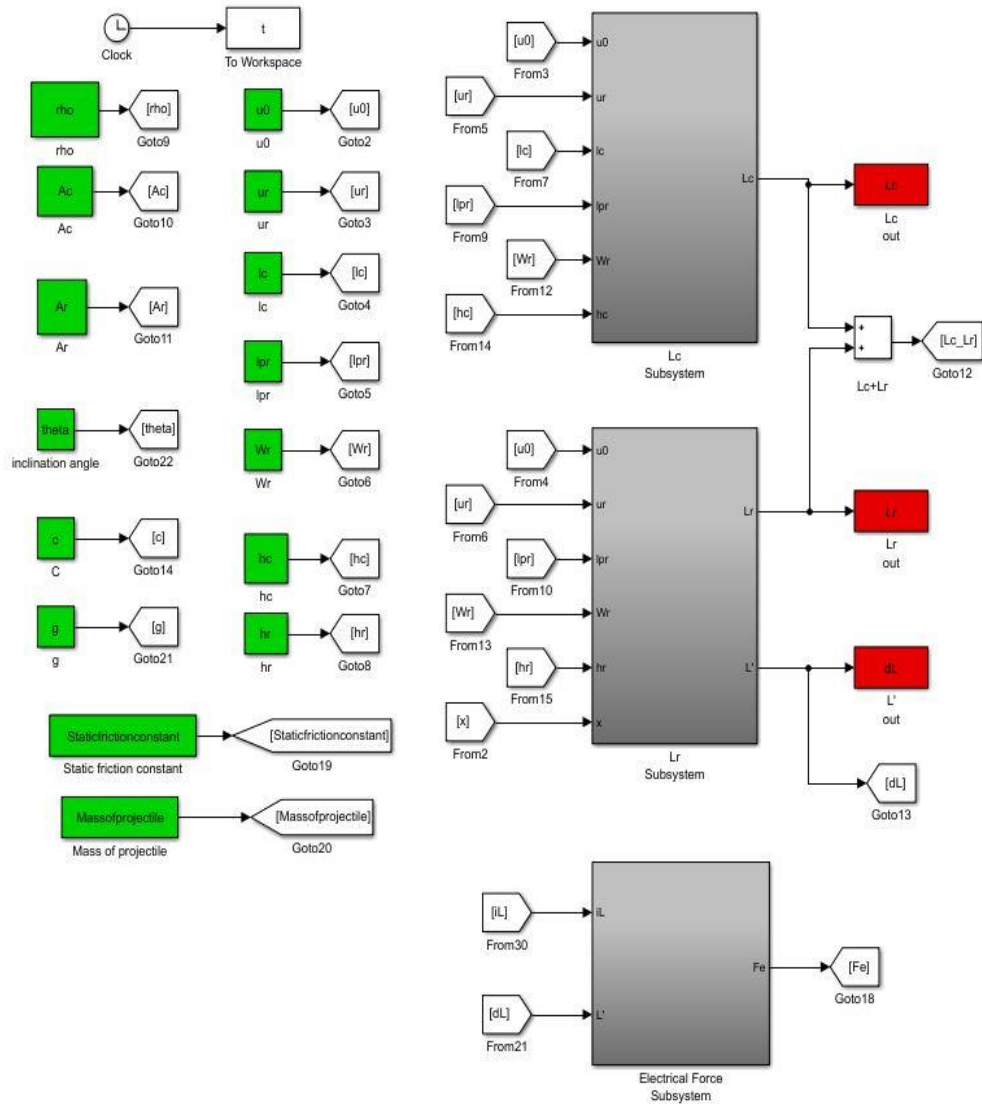
Figure 6: Model of the discharging circuit part 1
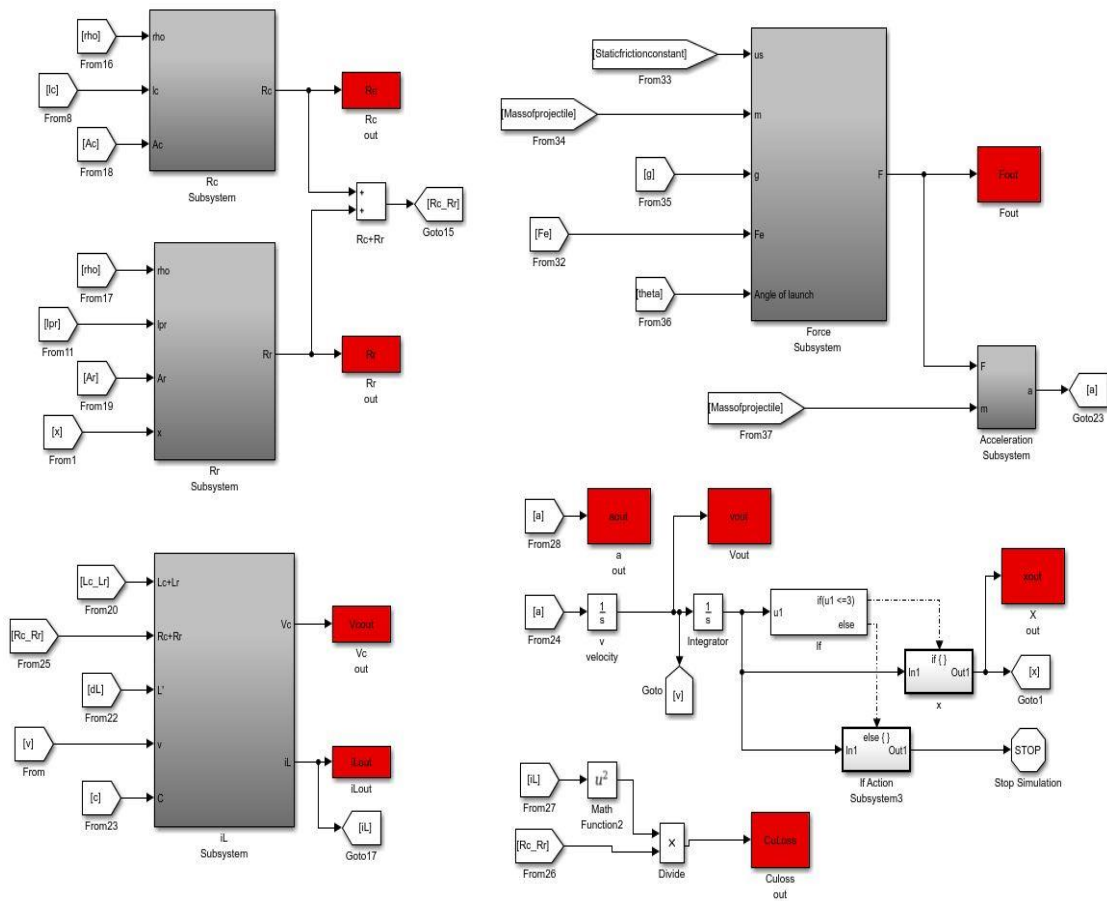
Figure 7: Model of the discharging circuit part 2

As it can be seen from Figure 6 and Figure 7, the whole model has several subsystems. All the blocks that are colored green, are the input values taken from the MATLAB script and all the blocks that are colored red are output values taken out from the Simulink model to the MATLAB script. Those values which are taken out from the Simulink model are further analyzed and used in different Simulink models.

3.2 Modeling and Simulation of the Trajectories

There have been two types of cases considered for modeling the trajectories. One is the ideal case, which is the trajectory of the projectile without any opposing forces while the other case includes opposing force acting on the projectile during flight. In this case, the force due to air drag is considered. Both the cases are modeled separately using Simulink which are discussed in following subsections.

3.2.1 Projectile's ideal trajectory (without any opposing forces)

To obtain a better understanding of the projectile's trajectory, ODE models are obtained using force equations. As once the projectile leaves railgun's endpoint, there exists no electrical force acting on it and as this case includes ideal condition, there exists no force acting on the projectile in opposite direction as well. If the force is divided into two components, one in x-axis component and another in y-axis component, forces can be written as per following equations.

- X-axis component of the force can be given by

$$F_x = 0. \tag{19}$$

therefore,

$$m\frac{dv_x}{dt} = 0. \tag{20}$$

therefore,

$$\frac{dv_x}{dt} = 0. \tag{21}$$

- Y-axis component of the force can be given by

$$F_y = -mg. \tag{22}$$

If F_y is expanded as per (20), m will get canceled on both the side, therefore

$$\frac{dv_y}{dt} = -g. \tag{23}$$

(21) and (23) are modeled using Simulink. Once x and y-axis components of velocity are found, they are integrated to get x and y-axis components of projectile's position. This can be seen in following Figure.

Figure 8: Model for finding projectile's ideal trajectory (without opposing forces)

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3.2.2 Projectile's trajectory with air drag

Same as per the ideal projectile trajectory case, even in this case ODEs are obtained for a better understanding of the instantaneous movement of the projectile. Even in this case, the projectile will not experience any electric force once it leaves rail's endpoint. But in this case opposing forces are considered once the projectile leaves rail's endpoint. One of the main factor affecting the force acting on the projectile is air drag. Therefore, forces in x-axis and y-axis directions are calculated by taking air drag into account and they can be given by the following equations:

- X-axis component of the force can be given by

$$F_x = -F_{dx}. \tag{24}$$

Here, the negative sign explains F_{dx} is in the opposing direction of F_x. Here, F_{dx} is the force acting on the projectile in x-axis direction due to air drag, which can be further expanded as

$$F_{dx} = ma_{dx}. \tag{25}$$

therefore,

$$a_{dx} = \frac{F_{dx}}{m} = \frac{AC_d\rho(v_0^2 cos\theta)}{2m}. \tag{26}$$

Here, a_{dx} is the x component of the acceleration due to air drag in the opposite direction. $(v_0^2 cos\theta)$ is the x component of the velocity due to air drag in opposite direction. Here, V_0 is dependent on $v_x \ and \ v_y$ based on

$$V_0 = \sqrt{V_x^2 + V_y^2}. \tag{27}$$

Here, θ is also dependent on v_x and v_y based on

$$\theta = tan^{-1}\left(\frac{v_y}{v_x}\right). \tag{28}$$

Substituting (28) into (26), it gives x component of the acceleration due to air drag in the opposite direction, which is given by

$$a_{dx} = \frac{AC_d\rho\left(v_0^2 cos\left(tan^{-1}\left(\frac{v_y}{v_x}\right)\right)\right)}{2m}. \tag{29}$$

From (29),

$$\frac{dv_x}{dt} = -\frac{AC_d\rho\left(v_0^2 cos\left(tan^{-1}\left(\frac{v_y}{v_x}\right)\right)\right)}{2m}. \tag{30}$$

Now,

• Y-axis component of the force can be given by

$$F_y = -F_{dy}. \tag{31}$$

where, F_{dy} is the force acting on the projectile in y-axis direction due to air drag, which can be further expanded as

$$F_{dy} = ma_{dy} + mg. \tag{32}$$

here,

$$a_{dy} = \frac{AC_d\rho(v_0^2 sin\theta)}{2m}.$$ (33)

Here, a_{dy} is the y-axis component of the acceleration due to air drag. $(v_0^2 sin\theta)$ is the y component of the velocity due to air drag in opposite direction. Here, V_0 is dependent on v_x and v_y based on (27) and θ is also dependent on v_x and v_y based on (28). Substituting (28) and (33) into (32), it gives y component of the acceleration due to air drag in the opposite direction, which is given by

$$F_{dy} = m\left(\frac{AC_d\rho\left(v_0^2 sin\left(tan^{-1}\left(\frac{v_y}{v_x}\right)\right)\right)}{2m}\right) + mg.$$ (34)

From (34),

$$\frac{dv_y}{dt} = -\frac{AC_d\rho\left(v_0^2 sin\left(tan^{-1}\left(\frac{v_y}{v_x}\right)\right)\right)}{2m} - g.$$ (35)

(27), (30) and (35) are modeled using Simulink. Once x and y-axis components of velocity are found, they are integrated to get x and y-axis components of projectile's position. This can be seen in following Figures.
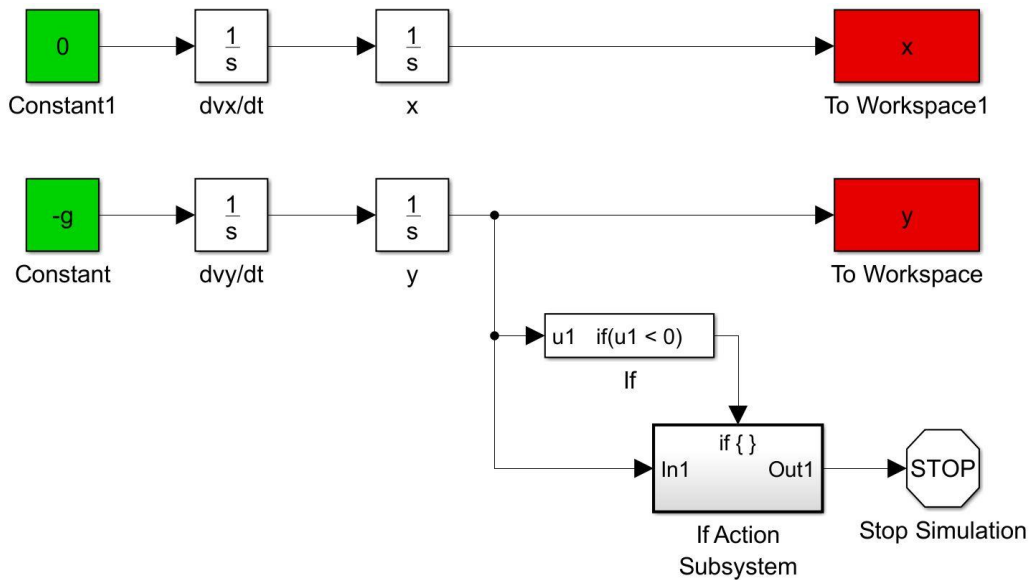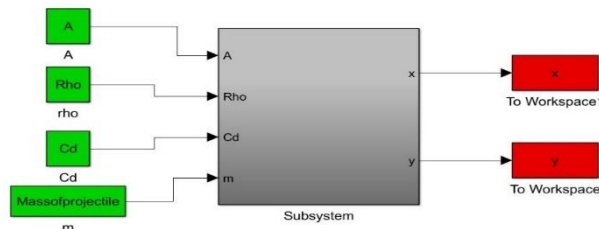

Figure 9: Model of the projectile trajectory with air drag part 1

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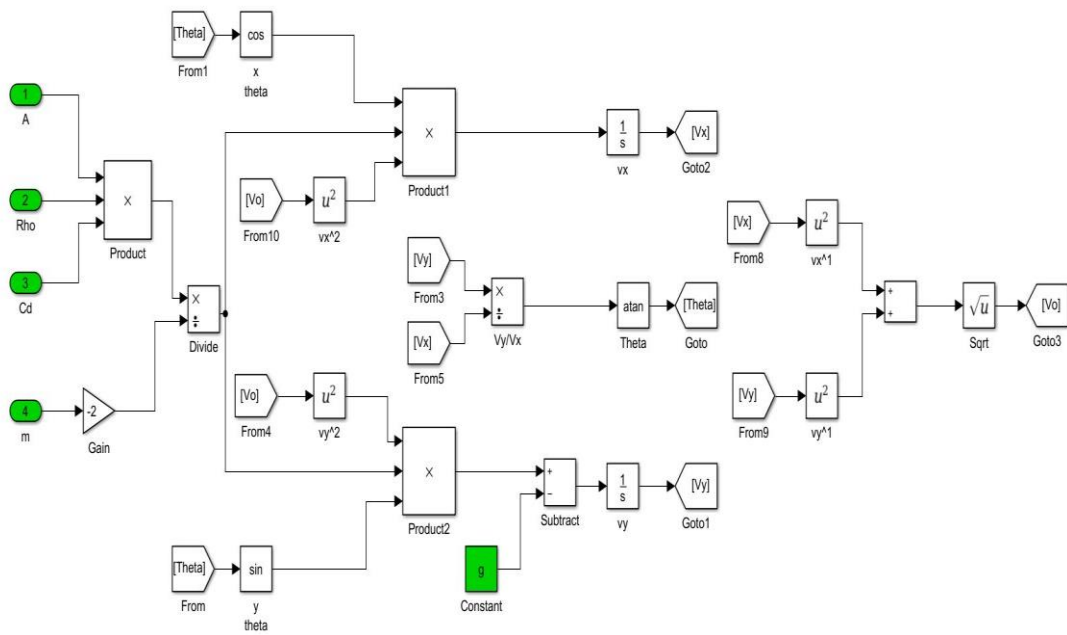
Figure 10: Model of the projectile trajectory with air drag part 2
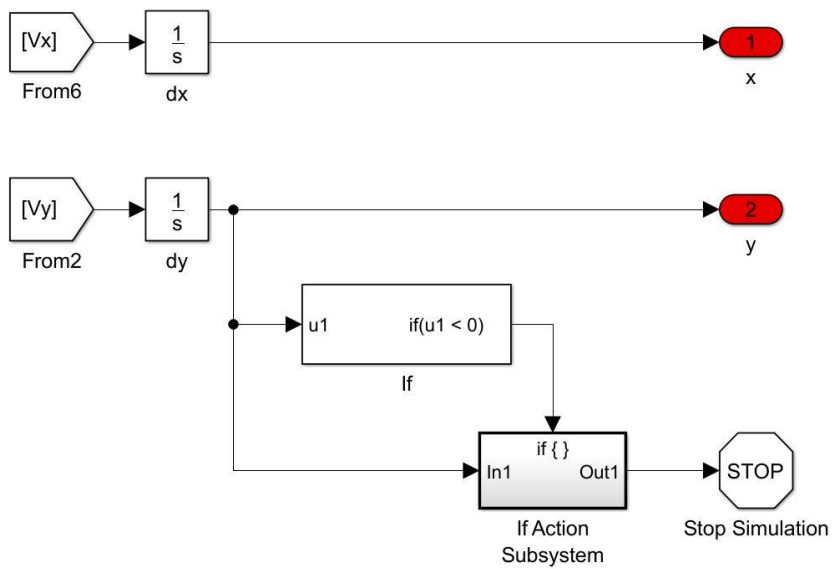
Figure 11: Model of the projectile trajectory with air drag part 3

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3.3 Complete model of the railgun's system including projectile's trajectory

Once the model of railgun's discharging circuit and the model of projectile's trajectory is ready, both the models are combined into one model to get the railgun's discharging characteristics along with the projectile's trajectory. This information is further used to optimize different parameters for achieving the goal of this project which is explained in next section. Both the Simulink models are combined using MATLAB script. As the projectile leaves the endpoint of the railgun, the velocity it had at that moment is saved and used as the starting point to find initial x-axis and y-axis coordinates of velocity for the trajectory model. The same configuration is used for both types of trajectories. MATLAB code for complete railgun model including projectile's trajectory without air drag is included in Appendix A.2 and MATLAB code for complete railgun model including projectile's trajectory with air drag is included in Appendix A.3. Results of both the models are discussed in following subsections.

3.3.1 Results obtained from the complete model of the railgun's system including projectile's trajectory excluding air drag

As it can be seen from Appendix A.2, a MATLAB code is generated to launch projectile using railgun's discharging circuit. In this case, air drag is excluded from the force calculations. For shooting the projectile, the capacitor is charged to 75 V and the railgun is kept at 45-degree angle to reach maximum distance. All the initial values used for the code are specified in the MATLAB code given in Appendix A.2. Projectile's trajectory excluding air drag is simulated and the result is shown in Figure 12. As it can be seen in Figure 12 that the projectile travels 964.5 m and then hits the ground. Projectile's trajectory has a parabolic curve because Ideal trajectory has been considered.
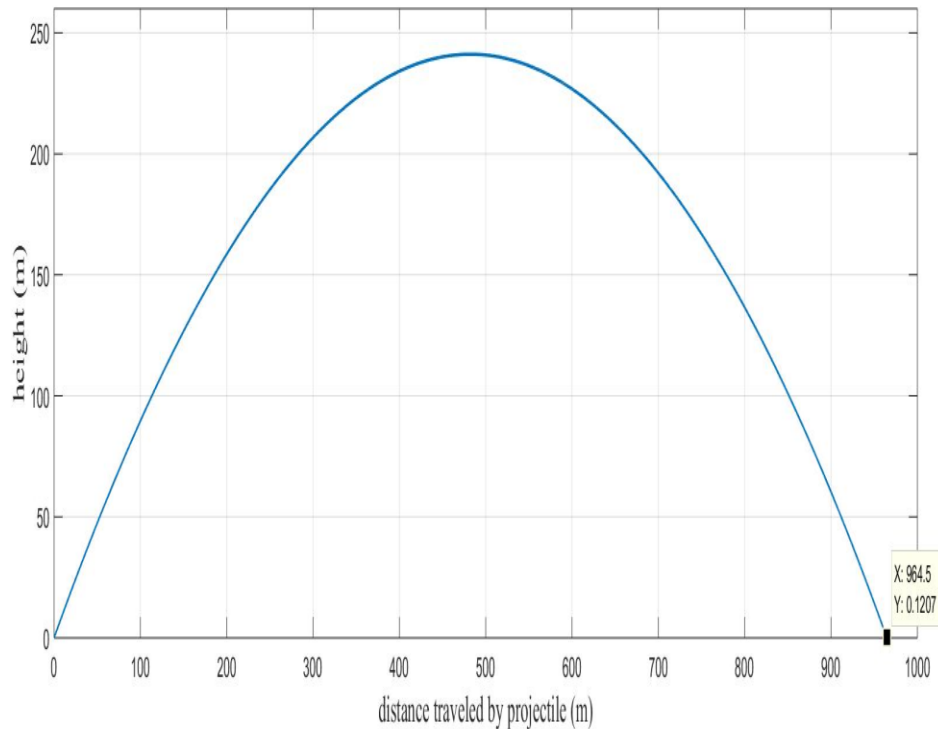
Figure 12: Projectile's trajectory excluding air drag

The behavior of the railgun's discharging circuit can be understood by Figures 13 and 14. Figure 13 contains characteristics of (a) discharging current $[I_L]$, (b) discharging voltage $[V_c]$ and (c) position of the projectile on the rails $[x]$ while Figure 14 contains (a) acceleration of the projectile $[a]$ and (b) velocity of the projectile $[v]$. As it can be seen from these results, it only takes 0.04 s for the projectile to leave the rails. Simulation of the railgun discharging circuit stops after the projectile leaves rails as there will not be any energy transfer once projectile leaves the rails. As it can be seen from the Figure 13, it needs a current impulse with very high amplitude to give a projectile enough acceleration. Rail's length is 3 m. Therefore, from Figure 13 it can be seen how the projectile starts from 0 m and travels 3 m in just 0.04 s. As it can be seen in Figure 14, the projectile gains speed as high as 97.28 m/s in just 0.04 s.

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Figure 13: plots of (a) discharging current, (b) discharging voltage and (c) position of the projectile on rails for projectile's trajectory without air drag
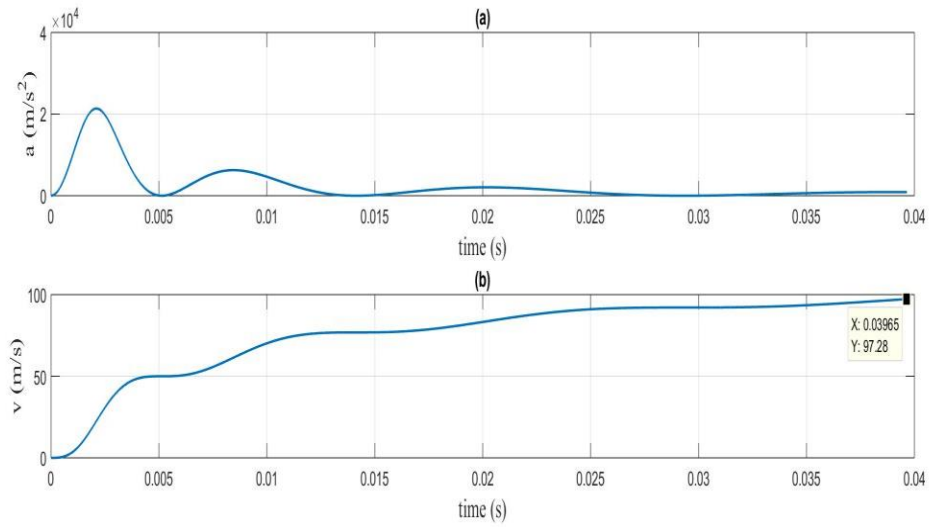
Figure 14: plots of (a) acceleration of the projectile, (b) velocity of the projectile for projectile's trajectory without air drag

3.3.2 Optimized results obtained from the complete model of the railgun's system including projectile's trajectory excluding air drag using analytical approach

In section 4, optimization is done for hitting the target location accurately with minimum losses. It is important to check whether the optimization technique is giving the optimized results. Therefore, an analytical approach has been used to obtain results which are close to optimum results. For this approach, values of Vinit and Theta have been changed manually and the complete model of the railgun including projectile's trajectory excluding air drag has been run with each of these values. As air drag is excluded in this model projectile's trajectory will follow ideal parabolic trajectory. In ideal trajectory maximum distance is achieved when theta is kept as 45 degrees. Target location has been chosen to be 500 m away from the railgun's location. After hit and trial method Vinit's value is kept as 54.5 V. With these values of Vinit and Theta projectile lands at location which is at 498.2406 m away from the railgun's location, which is quite close to 500 m. Further accuracy can be obtained using optimization function given in section 4. Cu loss occurring in the railgun's system is 942.5996 W with these values of Vinit and Theta. These results can be obtained using MATLAB code given in Appendix A.2. Now results have been tabulated below by changing values of Vinit and Theta.

Table 1. Results obtained from the railgun system with projectile's trajectory excluding air drag using analytical approach

Serial No.	Vinit (V)	Theta (Degree)	Xend (m)	Mean (Cu Loss) (W)
1	54.5	45	498.2406	942.5996
2	54.5	40	490.8782	942.7421
3	54.5	50	490.4826	942.6068
4	65	45	706.7792	1329
5	65	40	696.2527	1329
6	65	50	695.8523	1329
7	75	45	964.6071	1814.6
8	75	40	949.9797	1814.6
9	75	50	949.7712	1814.6

3.3.3 Results obtained from the complete model of the railgun's system including projectile's trajectory including air drag

A MATLAB code is generated for combining the railgun's discharging circuit with projectile's trajectory model. This can be seen in Appendix A.3. Following results include air drag. Therefore, it can be seen that for reaching the same target more force is required. For obtaining that force, the capacitor is charged to 565 V, which was charged only to 75 V for the ideal case without air drag. Railgun is kept at 45-degree angle. As it can be seen in Figure 15, projectile travels 964.6 m. Its trajectory is not parabolic as air drag is present.
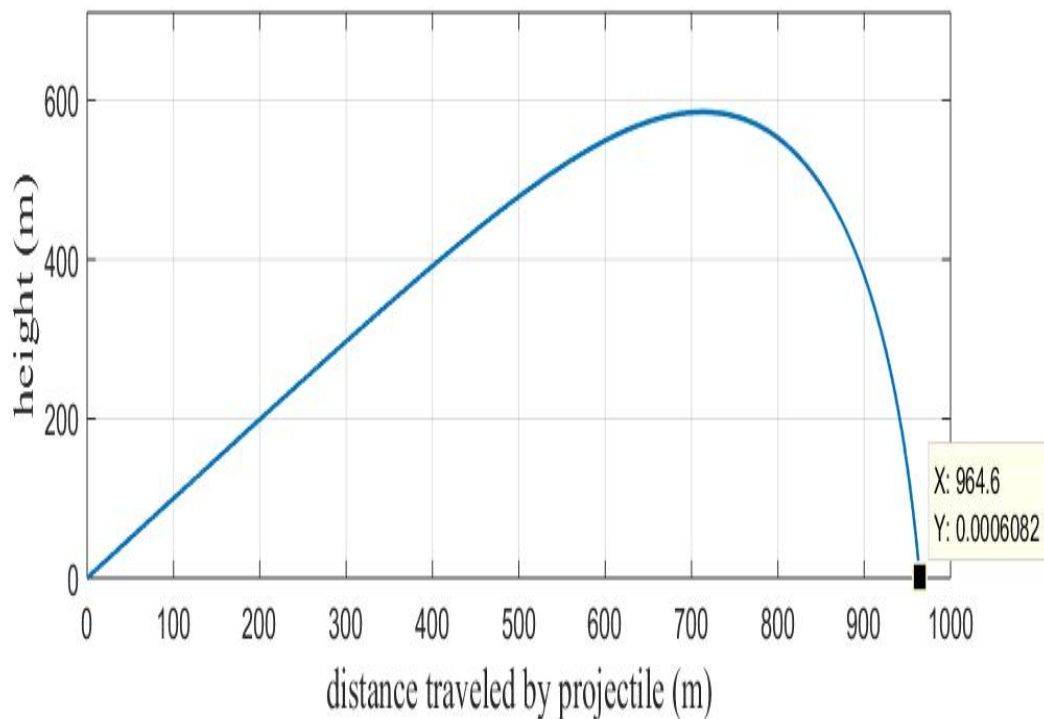
Figure 15: Projectile's trajectory including air drag

To achieve the initial velocity needed to reach a certain target location, the capacitor is discharged from a certain voltage level. The time-based analysis has been done using the code generated in Appendix A.3. Its behavior can be seen in Figures 16 and 17.
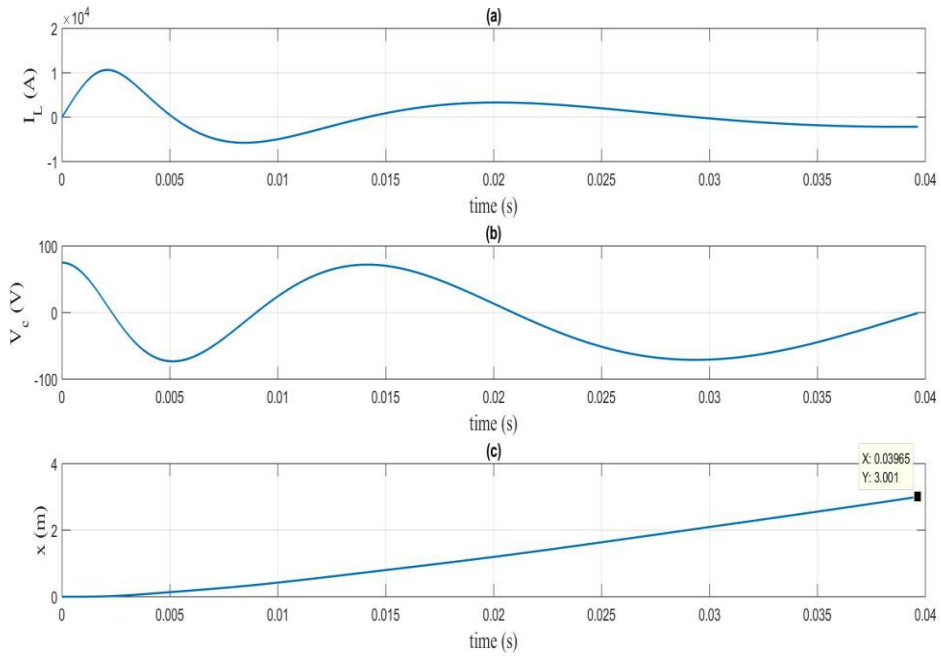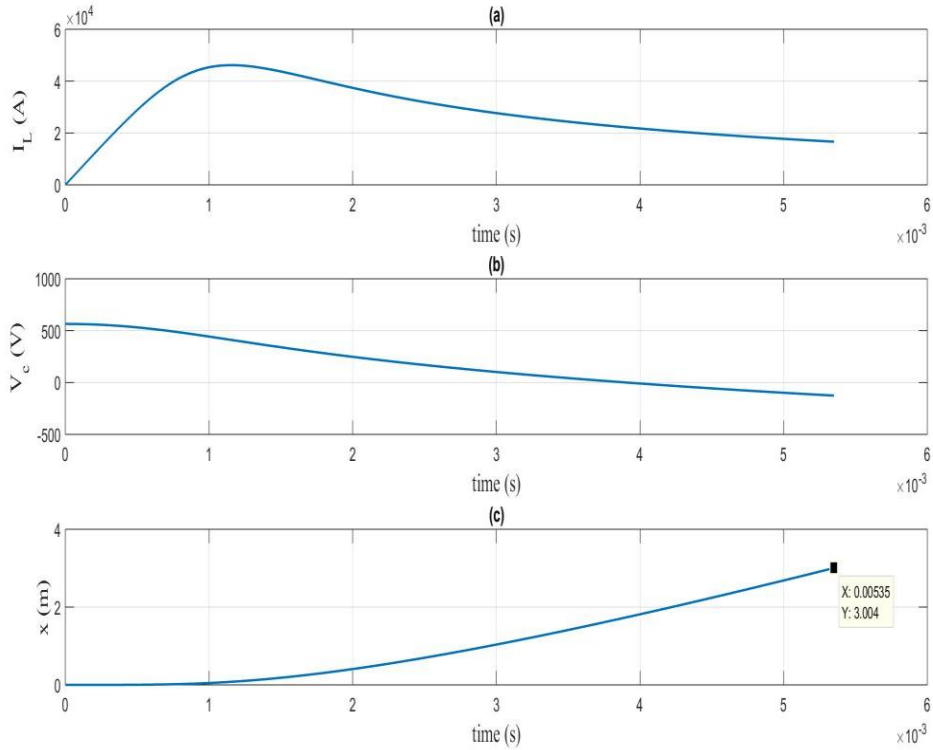

Figure 16: plots of (a) discharging current, (b) discharging voltage and (c) position of the projectile on rails for projectile's trajectory with air drag

As it can be seen from Figure 16, projectile starts from the beginning of the rail and reaches the end of the rail which is 3 m away in just 0.006 s. It can also be seen that it takes a larger current impulse to launch a projectile to reach a certain target when air drag is considered. Therefore, the capacitor is charged to a higher value compared to the ideal case.
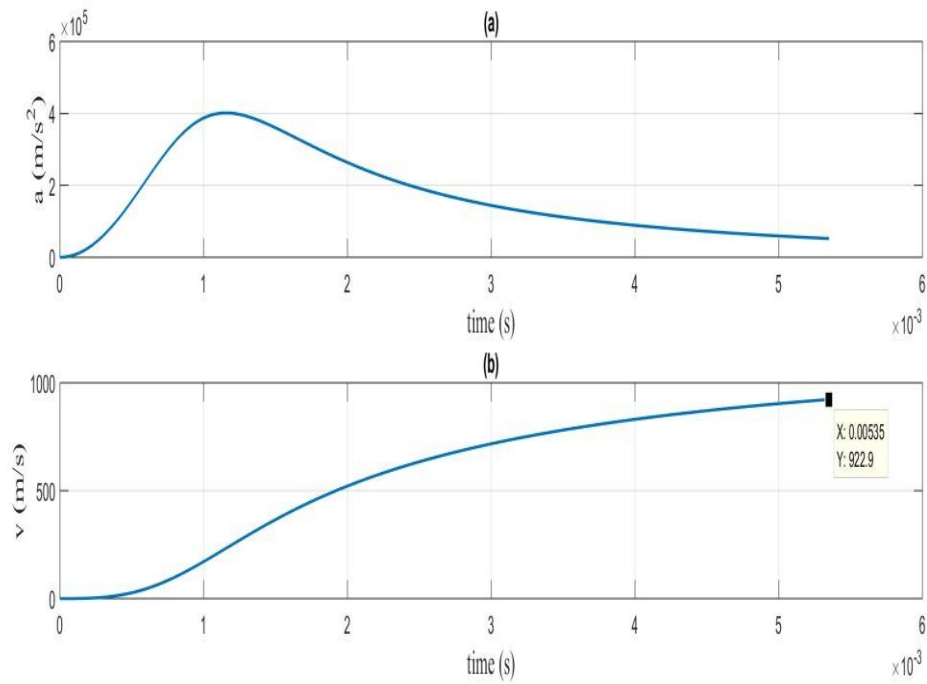
Figure 17: plots of (a) acceleration of the projectile, (b) velocity of the projectile for projectile's trajectory with air drag

As it can be seen from Figure 17, very high acceleration is achieved by the projectile because of the high discharging current that can be seen in Figure 16. Projectile gains speed of 922.9 m/s in a very less duration. It only takes 0.006 s for the projectile to leave the rails.

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3.3.4 Optimized results obtained from the complete model of the railgun's system including projectile's trajectory including air drag using analytical approach

Similar to section 3.3.2, an analytical approach has been used to obtain results which are close to optimum results for the complete model of the railgun's system including projectile's trajectory including air drag. Using hit and trial method values of Vinit and Theta have been changed manually and results close to optimized result have been obtained. As projectile's ideal trajectory is parabolic, maximum distance can be achieved when Theta is kept as 45 degrees. Therefore, it is assumed that even with air drag maximum distance can be achieved when Theta is kept as 45 degrees. Now similar to section 3.3.2, target location has been chosen to be 500 m away from the railgun's location. After using hit and trial method, finally Vinit's value is kept as 125 V. With these values of Vinit and Theta projectile lands at location which is at 502.2444 m away from the railgun's location, which is quite close to 500 m. Further accuracy can be obtained using optimization function given in section 4. Cu loss occurring in the railgun's system is 5286.7 W with these values of Vinit and Theta, which is higher compared to the Cu loss found for Railgun's system with ideal trajectory. These results can be obtained using MATLAB code given in Appendix A.3. Now results have been tabulated below by changing values of Vinit and Theta.

Table 2. Results obtained from the railgun system with projectile's trajectory including air drag using analytical approach

Serial No.	Vinit (V)	Theta (Degree)	Xend (m)	Mean (Cu Loss) (W)
1	125	45	502.2444	5286.7
2	125	35	527.9703	5288.9
3	125	25	514.5080	5288.9
4	365	45	857.1343	61426
5	365	35	938.8034	61426
6	365	25	969.7646	61426
7	565	45	964.5561	137575
8	565	35	1065	137575
9	565	25	1111.9	137575

4　Optimization and Results

After obtaining the complete model of the railgun with both the trajectories, optimization was done to achieve the purpose of hitting the target accurately with minimum losses. To achieve this optimization, Fmincon function was used. Fmincon is used for finding a suitable value of input variables to achieve minimum/maximum value of an objective function. An objective function may be designed for minimizing the value of the objective's component or maximizing the value of the objective's component. For reaching the goal of that objective function Fmincon runs the script several times till it finds the optimum values of input parameters that satisfies the objective function. For that bounds might be kept on the input parameters so that input values will be in permittable limits. Initial values of the input parameters can vary the results of the optimization process. Therefore, initial values are guessed carefully as it can speed up the process of optimization.

For this project, optimization function consists of two components. One is the square of the difference between set target location and achieved target location. An objective function is designed to minimize this error. Because of this component, after optimization process, the projectile should hit the target location precisely. Another part of optimization function contains the mean power loss term. Power loss considered for this project is copper loss occurring in the railgun system. It can be found by (18). Average value of this copper loss found from (18) is found using MATLAB script and that is used as the second component of the optimization function. An objective function is designed such that it will minimize this power loss component as well. The objective function can be expressed as,

$$\min_{V_{init},\theta} J. \tag{36}$$

where the objective function J is,

$$J = \underbrace{W_{to}(x_{ref} - x_{end})^2}_{\substack{Target\ offset \\ minimizing\ term}} + \underbrace{W_{Cu_{loss}}(mean(Cu_{loss}))^2}_{\substack{Copper\ loss \\ minimizing\ term}}. \tag{37}$$

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Therefore, the objective is to

$$\min_{V_{init},\theta} \left[W_{to}\left(x_{ref} - x_{end}\right)^2 + W_{Cu_{loss}}\left(mean(Cu_{loss})\right)^2 \right]. \tag{38}$$

The function given in (37) has been implemented using a MATLAB script. Fmincon has been used for optimization. A function has been created to achieve the optimization objective given in (38). Bound on V_{init} is from 0 to 1000 V and bound on θ is from 0 to 90 degrees. It can be seen in Appendix A.4. As there exist two components in the objective function. Weightage must be given to each component based on the importance of that objective function's component. As the copper loss minimization term has high weightage, to compensate that high weight should be kept in the target offset minimization term. Once an equilibrium is found between both the terms, then the weight of one term is changed slightly to meet the required performance parameters.

In this project, the end goal is to hit the set target location within defined accuracy limit by minimizing the copper loss. For that two cases are considered. In one case projectile's ideal trajectory is considered and in second case air drag is included into projectile's trajectory. Optimization has been implemented for both the cases. As discussed above, only weights of the target offset minimization term have been changed to different values and the results have been tabulated in Table 3 and Table 4.

Table 3. Optimization result for the railgun system with projectile's trajectory excluding air drag (Target = 500 m)

Target offset error margin = ±5 m (97.5% Accuracy)						
Serial No.	W_{to}	$W_{Cu_{loss}}$	Vinit (V)	Theta (Degree)	Error (xref-xend) (m)	Mean (Cu Loss) (W)
1	0.85×10^2	1×10^{-2}	56.0186	54.9499	0.3155	1006.6
2	0.2×10^2	1×10^{-2}	55.478	53.306	1.3936	985.3972
3	0.1×10^2	1×10^{-2}	54.523	46.9134	2.3913	943.6048
4	0.06×10^2	1×10^{-2}	54.4905	42.9614	3.1634	942.1552
5	0.05×10^2	1×10^{-2}	54.4141	44.9414	3.8299	938.6119
6	0.044×10^2	1×10^{-2}	54.3962	45.1969	4.2829	937.7597
7	0.0429×10^2	1×10^{-2}	54.3778	45.5026	4.808	936.8919

Table 3 contains the ideal case in which there exists no opposing forces while projectile is travelling in the air. As it can be seen in the Table 3, if the weight on target offset minimization term is more, the projectile will hit the target very accurately, but railgun will also have high Cu losses. As end goal also focuses on minimization of Cu loss, it is only possible to achieve that goal if there exists some allowable offset from target location. Therefore, for this case 97.5% accuracy have been chosen. Target location has been chosen as 500 m. As the chosen accuracy is 97.5%, the achieved target location should not be more than 5 m away from the set target location. Therefore, weights of target offset minimization term has been changed such that the achieved target location stays in permissible range. Results obtained by changing weights are listed in Table 3. As it can be seen in Table 3, 1st entry is very accurate as it hits exactly at set target location. But it also has the highest copper loss which is as high as 1006.6 W. Weight of target offset minimizing term have been decreased gradually and it can be seen that the target offset error keeps on increasing while Cu loss keeps on decreasing. The most optimum solution is found in Last entry in Table 3, which shows that the copper loss is now 936.8919 W while target offset error is still within 5 m. If weight is decreased further, then it decreases Cu loss but achieved target location is not within permissible tolerance. Further Cu loss reduction can be achieved by reduction of weights, but accuracy of the railgun has to be chosen accordingly.

As it can be seen from Table 3, when there exists the least error in the target offset, the initial voltage required is maximum and as the target offset error increases initial voltage required keeps on decreasing. Projectile's trajectory has been plotted for the first and the last entry in Table 3, which is given in Figure 18. As it can be seen in Figure 18 that both the cases reach within 5 m of the set target which is 500 m.

From Table 1 and Table 3, it can be seen that results obtained using analytical method is close to the results obtained using optimization function for hitting a target at 500 m. Similarly, from Table 2 and Table 4, it can be seen that results obtained using analytical method is close to the results obtained using optimization function for hitting a target at 500 m. But for the second case, the Cu loss found using analytical method is greater then the Cu loss found by optimization function. It is because for projectile's trajectory including air drag, maximum distance can be reached by keeping the launch angle around 32 degrees rather than keeping the launch angle around 45 degrees. Therefore, more voltage is required for reaching the same distance.
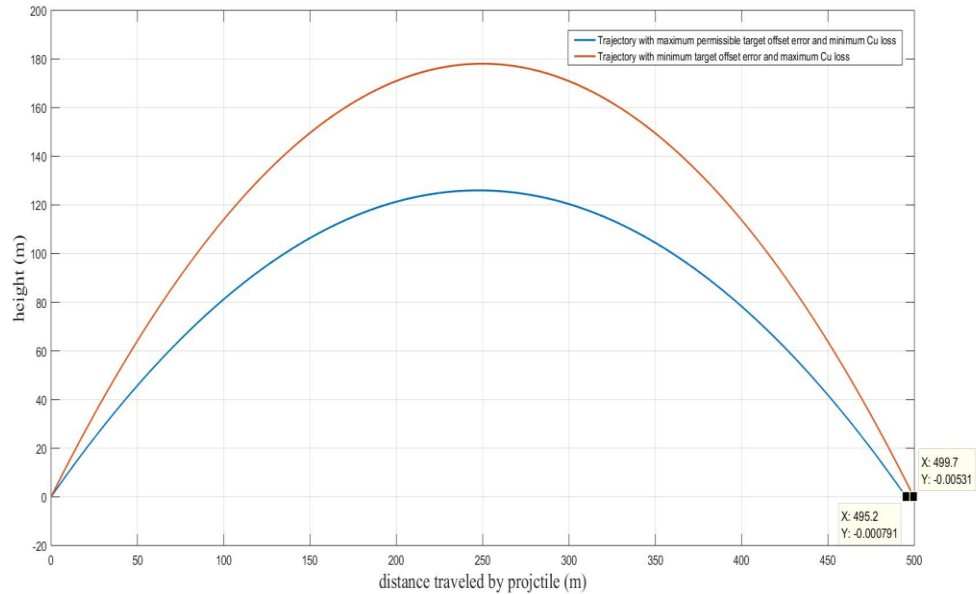
Figure 18: Optimized projectile's trajectory excluding air drag

Similarly, the same process has been implemented on the railgun system including air drag in projectile's trajectory. A MATLAB code, as well as a MATLAB function has been created just as the previous section, which can be seen in Appendix A.5. Objective function has been same as the previous one and can be given by (36) but bound on V_{init} is now from 0 to 300 V instead of 0 to 1000 V. After running that optimization code using Fmincon results have been tabulated in Table 4. As it can be seen from Table 4, it shows the same characteristic as Table 3. Similar to results from Table 3, Cu loss decrease as the weight on target offset error term is decreased. Even in this case accuracy has been chosen to be more than 97.5%. Therefore, the projectile has to land within 5 m distance from the set target location. It can be seen from Table 4 that the Cu loss will be maximum which is 4643.2 W when the projectile lands closest to the target location. As accuracy is 97.5% weight of target offset term has been decreased and the results have been tabulated in Table 4. As it can be seen from Table 4, the last entry has the most optimum result. It has the minimum Cu loss possible with 97.5% accuracy in hitting the set target location, which is 4345 W. Similar to the previous case, Cu loss can still be reduced but then accuracy has to be sacrificed.

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Table 4. Optimization result for the railgun system with projectile's trajectory including air drag (Target = 500 m)

Serial No.	W_{to}	$W_{Cu_{loss}}$	Vinit (V)	Theta (Degree)	Error (xref-xend) (m)	mean (Cu Loss) (W)
7						

Serial No.	W_{to}	$W_{Cu_{loss}}$	Vinit (V)	Theta (Degree)	Error (xref-xend) (m)	mean (Cu Loss) (W)
1	45.97×10^2	1×10^{-2}	119.3422	39.696	0.2842	4643.2
2	7.1×10^2	1×10^{-2}	117.7078	30.4555	1.5158	4483.5
3	6.9×10^2	1×10^{-2}	117.4092	30.9615	2.0001	4455.9
4	5.5×10^2	1×10^{-2}	117.0129	32.4604	2.2201	4420.4
5	3.5×10^2	1×10^{-2}	116.6202	33.0021	3.3766	4384.7
6	2.5×10^2	1×10^{-2}	116.4941	31.7132	4.5651	4373.6
7	2.1×10^2	1×10^{-2}	116.1624	32.9538	4.9935	4345

Target offset error margin = ±5 m (97.5% Accuracy)

It can be seen from Table 4 that initial voltage required is maximum when there exists the least error between set target location and achieved target location. The initial voltage keeps on decreasing as the target offset error increases. Projectile's trajectory has been plotted for the first and the last entry in Table 4, which is given in Figure 19.
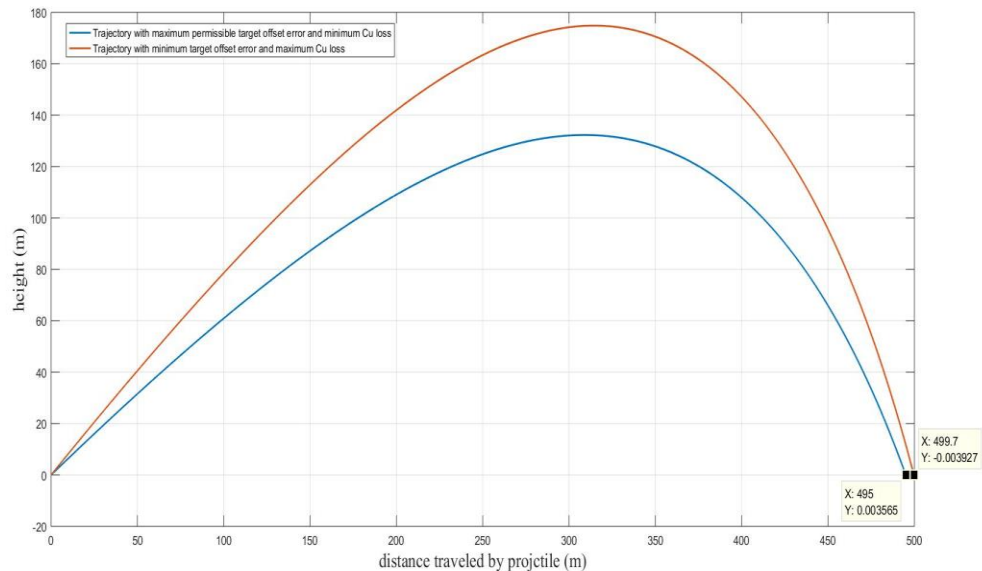

Figure 19: Optimized projectile's trajectory including air drag

5 Conclusion and Future Work

5.1 Conclusion

The end goal of this project was to minimize the Cu loss while hitting the set target accurately within permissible offset margin. Here, two cases have been considered. One is the ideal case where there exists no air drag so the projectile has an ideal parabolic trajectory. While in the other case, air drag has been considered so the trajectory will not be parabolic. For both the cases accuracy has been chosen to be 97.5%. As the set target is kept 500 m, this means that the projectile must land within ±5 m from the set target. An optimization code was generated to achieve this end goal. Fmincon function was used for optimization. As Fmincon gives local minimum/maximum values, multiple starting points were chosen to find the most optimum results. It can be seen from the results given in section 4 that the optimization code is working properly, and it is giving desired results.

For minimizing the copper loss weights have been multiplied with both the terms in objective function given by (37). As it can be seen from both Table 3 and Table 4, when the value of W_{to} is maximum then the error between set target location and achieved target location is minimum but the Cu loss is maximum. As the value of W_{to} decreases, the target offset error increases but at the same time, the Cu loss decreases. It can be seen from the tables 3 and 4 that the last entries in both the tables have minimum Cu loss within 97.5% accuracy in hitting the exact target location. Further Cu loss can be reduced but the accuracy has to be sacrificed. Therefore, a balance between both these quantities has to be made.

As it can be seen from Table 3 that in ideal case, just within 5m margin in target offset, Cu loss reduces from 1006.6 W to 936.8919 W. Which means that the Cu loss has been reduced by almost 69.7081 W. This shows that the Cu loss has been minimized by almost 6.25% while still staying in allowable target offset margin. Similarly, in case 2 which considers air drag during projectile's travel, it can be seen from Table 4 that Cu loss reduces from 4643.2 W to 4345 W. Which means that Cu loss has been reduced by almost 298.2 W. This shows that the Cu loss has been minimized by almost 6.42%. Thus, using this optimization code further Cu loss can be minimized if the accuracy is reduced from 97.5%. Hence, it can be seen that the increase of efficiency is a tradeoff between railgun's accuracy and desired reduction of the Cu loss occurring in the railgun.

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5.2 Future Work

In this project, optimization has been achieved by varying only two parameters (V_{init} and θ) because of the complexity of the optimization. Cu loss can be further reduced by minimizing the resistance in the system. For that, optimization of the resistance can be done by varying length and width of the rails as well as of the conductors. Next step of this project can be of implementing large capacitor banks rather than using just one capacitor. Further, charging circuit can also be included in the model of the railgun.

Once all of these are added in the circuit, next step can be of implementing pulse forming networks as the input power supply and then optimizing various parameters by optimally choosing related linked parameters. For example, optimization can be done of the efficiency of the railgun system by applying multiple pulses while the projectile is moving within the barrel. Further, optimization of the number of pulses as well as the timing of the pulses can be done for pulse forming network to get maximum efficiency. Similarly, optimization can be done by choosing the value of the capacitance and number of capacitors used for the discharge. Thus, there exist many possible directions in which optimization can be achieved for the railgun. As rail gun system is still under development, it has a vast possibility for improving its efficiency.

6   Reference List

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A. Appendix

A.1 Simulink model of the discharging circuit
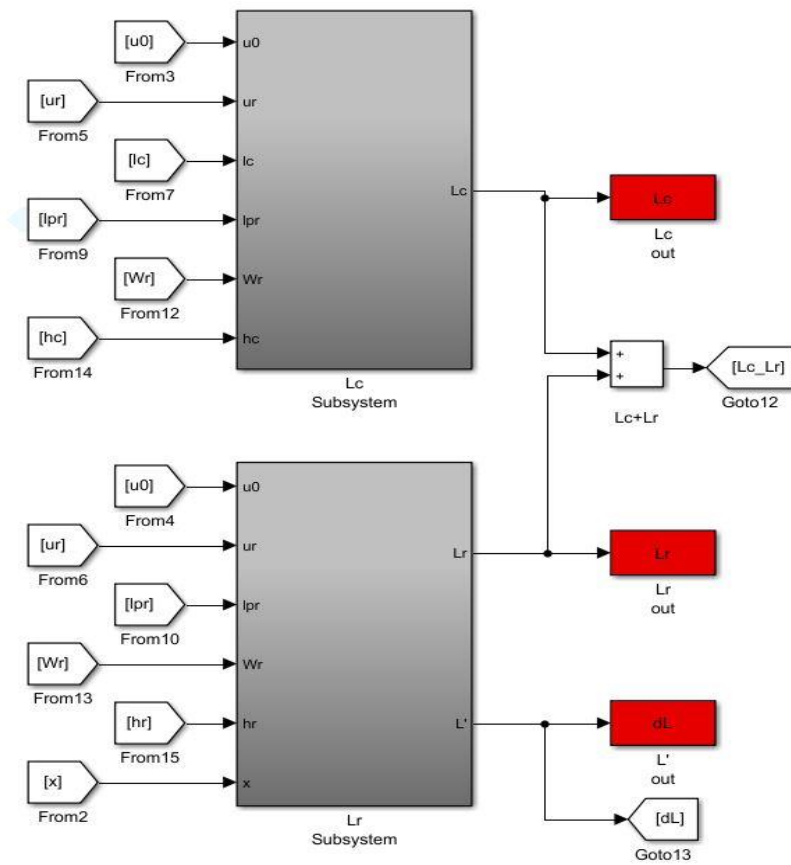
This section includes figures of the whole Simulink model of the discharging circuit. As it is a very large and complex system, it is broken down into many subsections so that it can be understood easily.
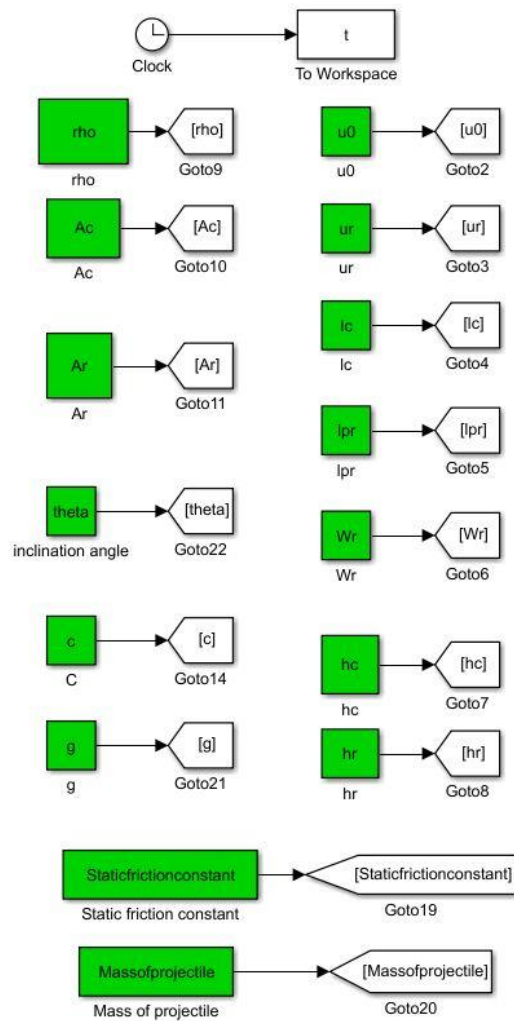
A.1.1: Simulink model of discharging circuit part 1

A.1.2: Simulink model of discharging circuit part 2
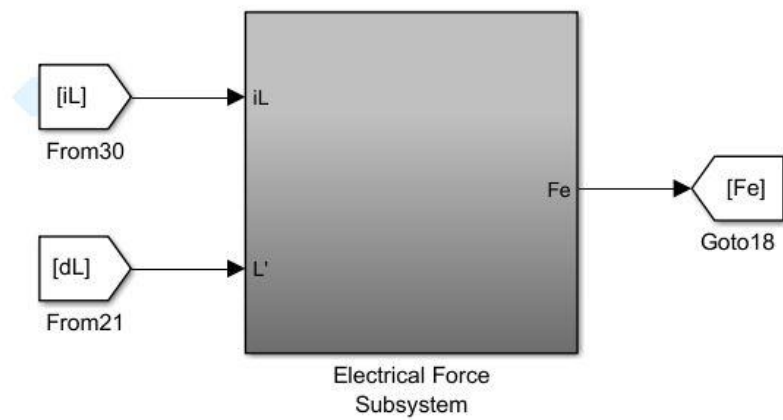
A.1.3: Simulink model of discharging circuit part 3

48
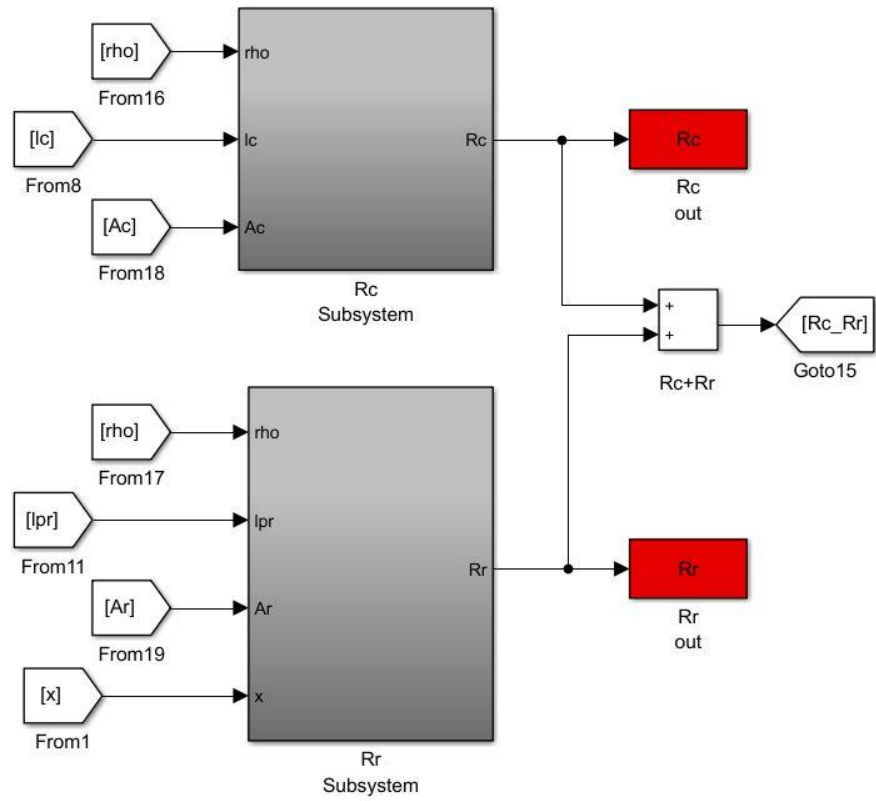
A.1.4: Simulink model of discharging circuit part 4
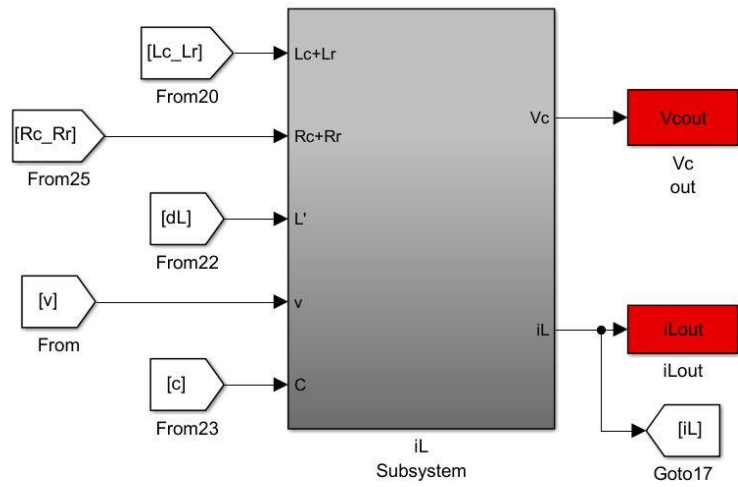
A.1.5: Simulink model of discharging circuit part 5

49
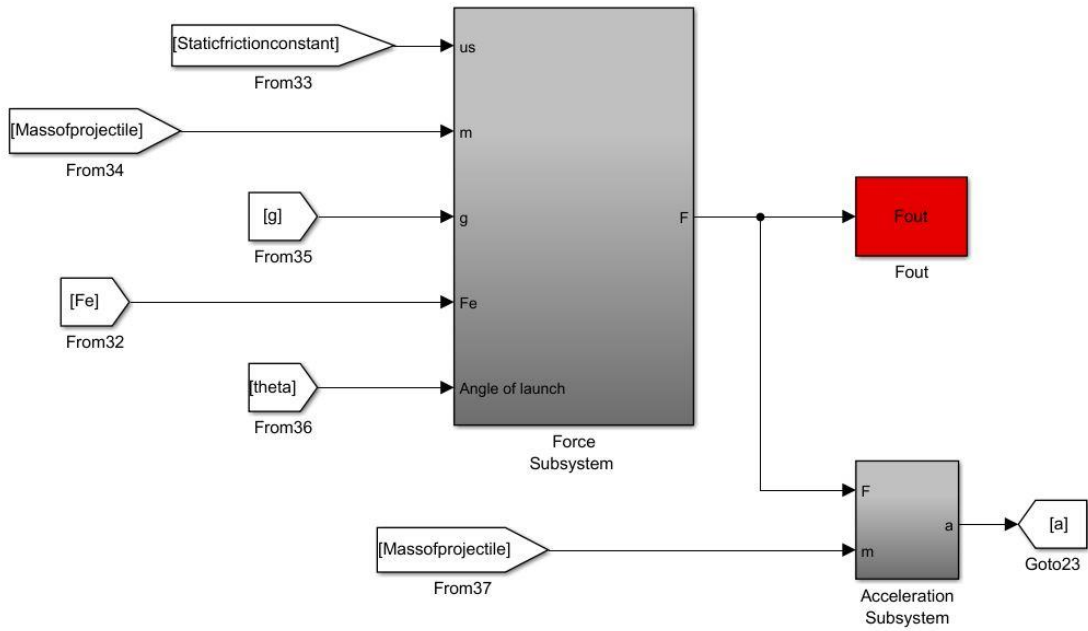
A.1.6: Simulink model of discharging circuit part 6
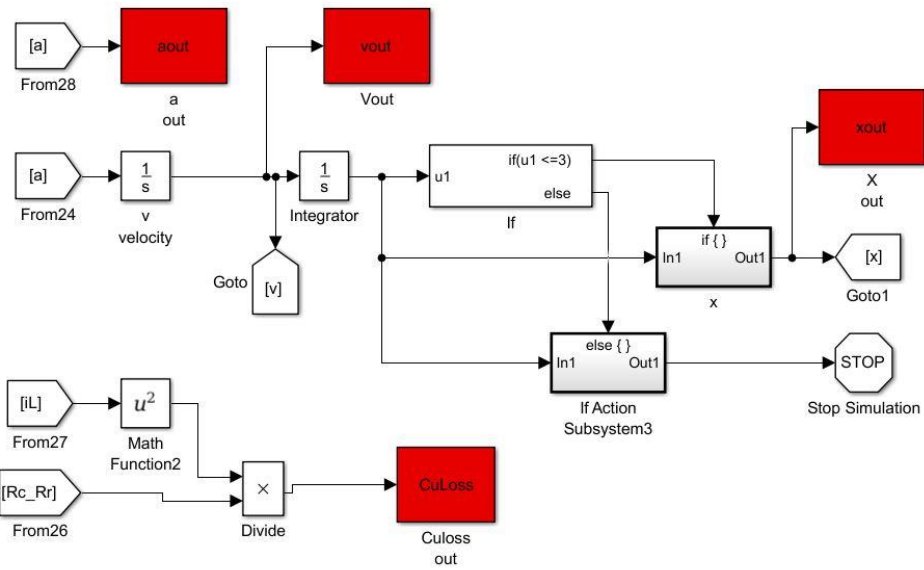
A.1.7: Simulink model of discharging circuit part 7

50

All the subparts are further broken down into following figures.

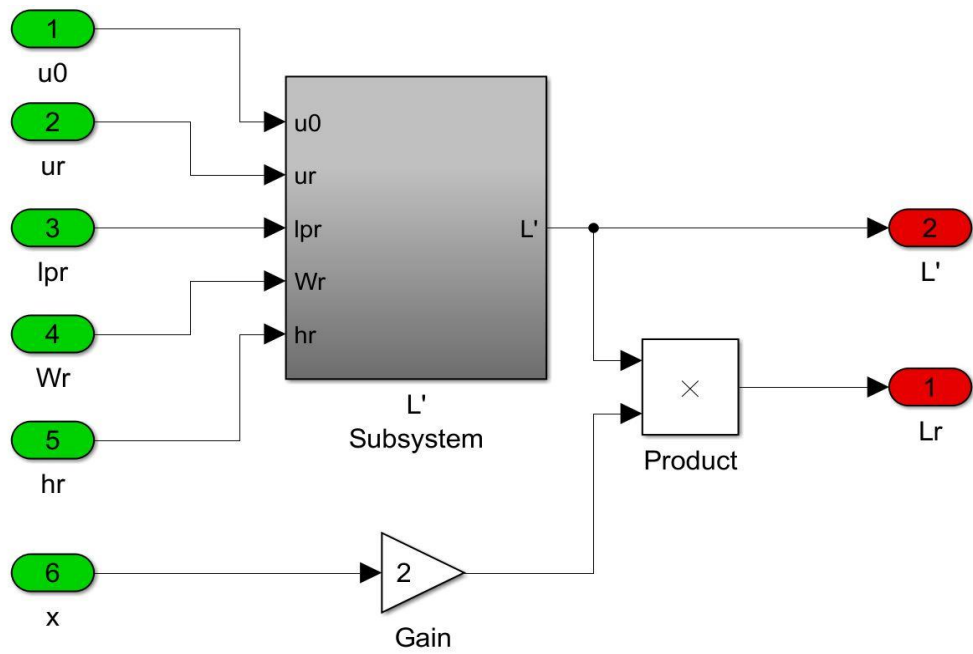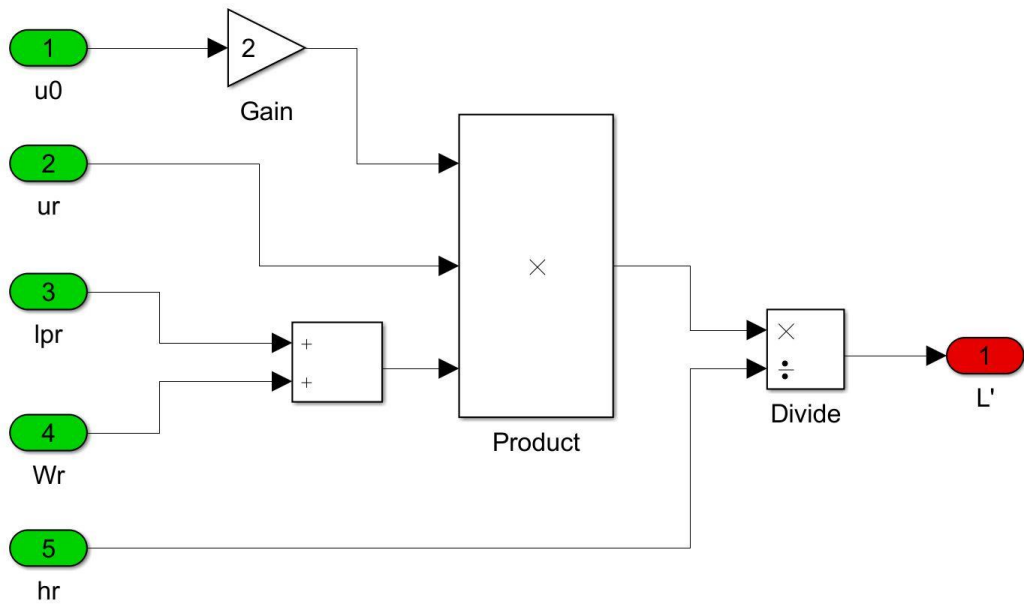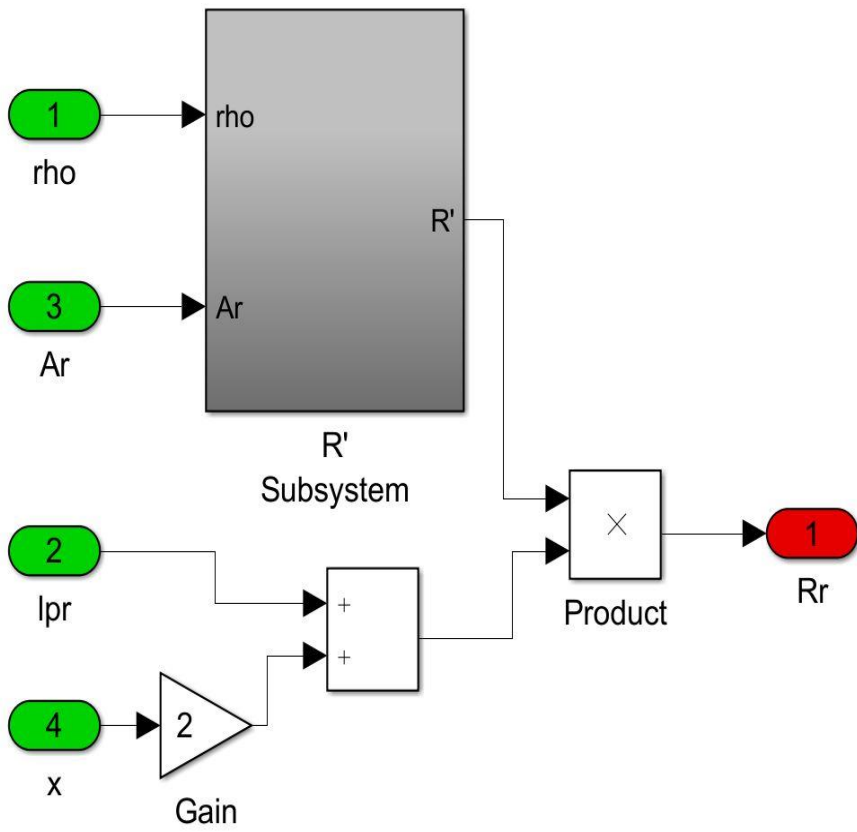
A.1.8: Lr subsystem

A.1.9: L' subsystem

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A.1.10: Rr Subsystem
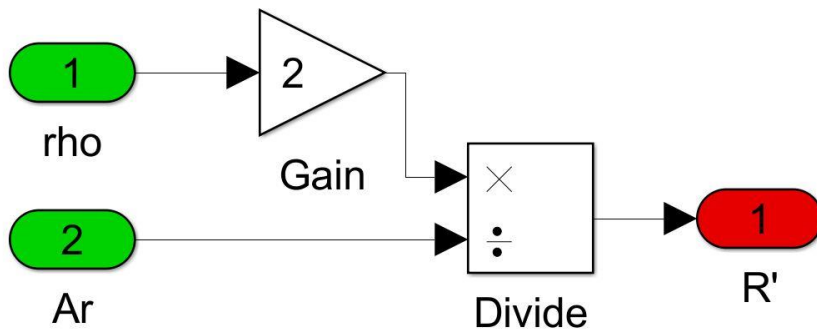
A.1.11: R' Subsystem
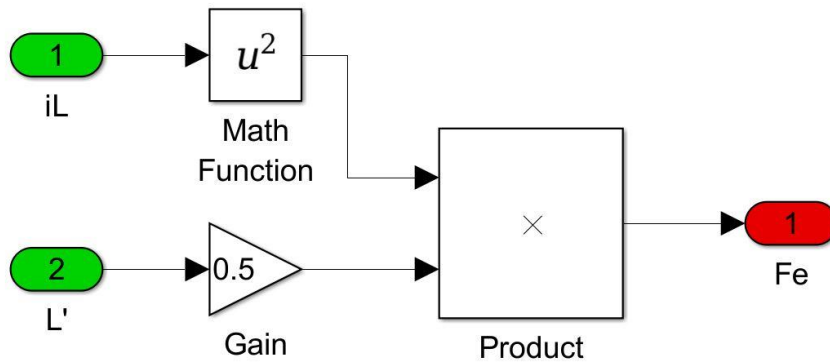
A.1.12: Fe Subsystem
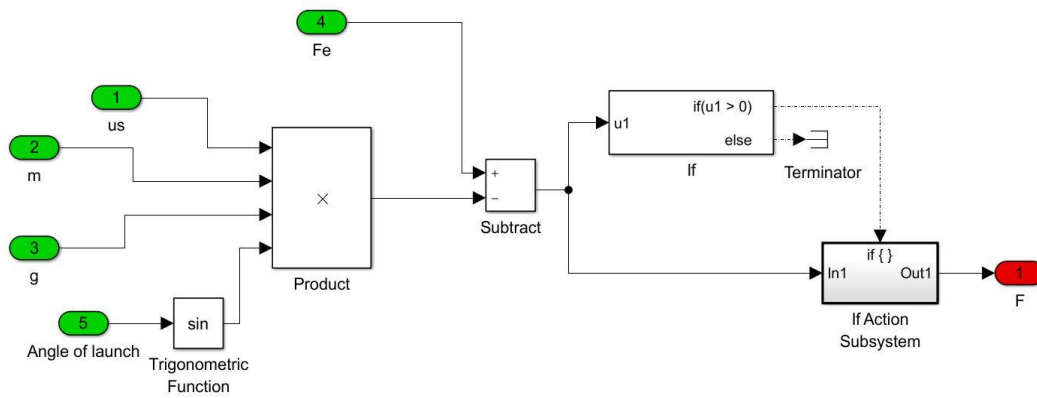
A.1.13: Ft Subsystem
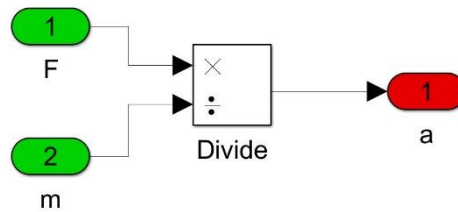
A.1.14: Acceleration Subsystem

53
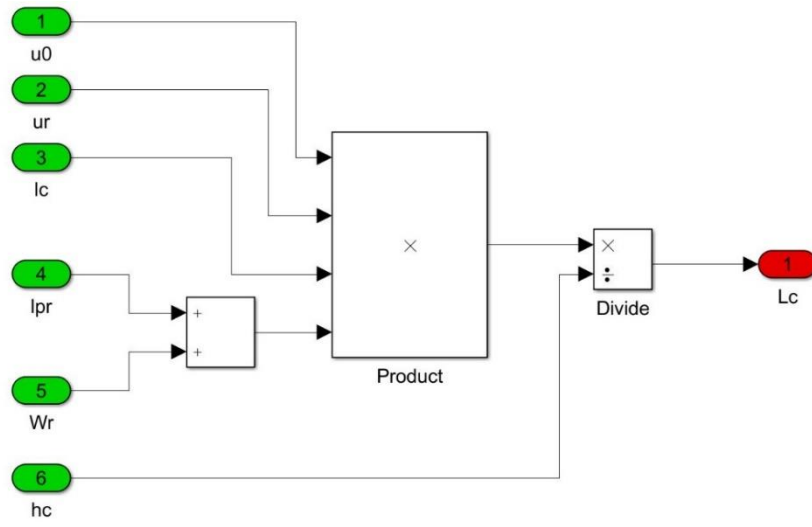
A.1.15: Lc Subsystem

A.2 Complete model of the railgun excluding air drag in projectile's trajectory

The MATLAB code for the complete model of the railgun excluding air drag in projectile's trajectory is given in this section. All the figures containing results have been discussed in the section 3.3.1.

```matlab
clc;
clear all;
close all;

% Variables used for Discharging circuit of the railgun

global Vinit u0 ur lc lpr Wr hc hr rho Ac Ar c Staticfrictionconstant
global Massofprojectile g theta Vx0 Vy0

Vinit = 75;                      % Charging voltage
u0 = 4*3.14*10^(-7);             % permeability
ur = 1;                          % Relative permeability
lc = 0.5;                        % Length of circuit
lpr = 0.05;                      % Length of projectile
Wr = 0.1;                        % Width of rail
hc = 0.01;                       % Height of circuit
hr = 0.01;                       % Height of rail
rho = 1.68*10^(-8);              % Resistivity of conductor
Ac = (3.14*(1)^2/4)*(10^(-4));   % Area of circuit
Ar = 0.001;                      % Area of rail
c = 2.2*10^(-1);                 % Capacitance

Staticfrictionconstant = 0.5;    % Static friction constant
Massofprojectile = 0.1;          % Mass of projectile
g = 9.81;                        % Gravitational acceleration
theta = 45*pi/180;               % Angle of launch

% Code for Discharging circuit of railgun

sim('Final_Railgun_discharging_model')

% Plots of discharging circuit important parameters

figure(1)

subplot(3,1,1)
plot(t,iLout,'Linewidth',1.5)
```

```matlab
set(gca,'fontsize',12)
title('(a)')
xlabel('time (s)')
ylabel('I_{L} (A)')
grid on

subplot(3,1,2)
plot(t,Vcout,'Linewidth',1.5)
set(gca,'fontsize',12)
title('(b)')
xlabel('time (s)')
ylabel('V_{c} (V)')
grid on

subplot(3,1,3)
plot(t,xout,'Linewidth',1.5)
set(gca,'fontsize',12)
title('(c)')
xlabel('time (s)')
ylabel('x (m)')
grid on

figure(2)

set(gca,'fontsize',12)

subplot(2,1,1)
plot(t,aout,'Linewidth',1.5)
set(gca,'fontsize',12)
title('(a)')
xlabel('time (s)')
ylabel('a (m/s^2)')
grid on

subplot(2,1,2)
plot(t,vout,'Linewidth',1.5)
set(gca,'fontsize',12)
title('(b)')
xlabel('time (s)')
ylabel('v (m/s)')
grid on
```

```matlab
% Code for projectile trajectory withour air drag

V = vout(end) ;                % velocity of the projectile at start

vx0 = V*cos(theta);            % x-axis component of velocity at start
vy0 = V*sin(theta);            % y-axis component of velocity at start
```

```matlab
sim('Projectile_trajectory_without_air_drag')

% Plot of the trajectory without air drag

figure(3)
plot(x,y,'Linewidth',1.5)
set(gca,'fontsize',12)
xlabel('distance traveled by projectile (m)')
ylabel('height (m)')
ylim([0 260])
grid on

Target_location = x(end)
Cu_loss = mean(CuLoss)
```

```
Target_location =

   964.6071


Cu_loss =

   1.8146e+03
```

A.3 Complete model of the railgun including air drag in projectile's trajectory

The MATLAB code for the complete model of the railgun including air drag in projectile's trajectory is given in this section. All the figures containing results have been discussed in the section 3.3.2.

```matlab
clc;
clear all;
close all;

% Variables used for Discharging circuit of the railgun

global Vinit u0 ur lc lpr Wr hc hr rho Ac Ar c Staticfrictionconstant
global Massofprojectile g theta Vx0 Vy0 A Cd Rho Wpr

Vinit = 565;                    % Charging voltage
u0 = 4*3.14*10^(-7);            % permeability
ur = 1;                         % Relative permeability
lc = 0.5;                       % Length of circuit
lpr = 0.05;                     % Length of projectile
Wr = 0.1;                       % Width of rail
hc = 0.01;                      % Height of circuit
hr = 0.01;                      % Height of rail
rho = 1.68*10^(-8);             % Resistivity of conductor
Ac = (3.14*(1)^2/4)*(10^(-4));  % Area of circuit
Ar = 0.001;                     % Area of rail
c = 2.2*10^(-1);                % Capacitance

Staticfrictionconstant = 0.5;   % Static friction constant
Massofprojectile = 0.1;         % Mass of projectile
g = 9.81;                       % Gravitational acceleration
theta = 45*pi/180;              % Angle of launch

% Code for Discharging circuit of railgun

sim('Final_Railgun_discharging_model')

% Plots of discharging circuit important parameters

figure(1)

subplot(3,1,1)
plot(t,iLout,'Linewidth',1.5)
```

```matlab
set(gca,'fontsize',12)
title('(a)')
xlabel('time (s)')
ylabel('I_{L} (A)')
grid on

subplot(3,1,2)
plot(t,Vcout,'Linewidth',1.5)
set(gca,'fontsize',12)
title('(b)')
xlabel('time (s)')
ylabel('V_{c} (V)')
grid on

subplot(3,1,3)
plot(t,xout,'Linewidth',1.5)
set(gca,'fontsize',12)
title('(c)')
xlabel('time (s)')
ylabel('x (m)')
grid on

figure(2)

set(gca,'fontsize',12)

subplot(2,1,1)
plot(t,aout,'Linewidth',1.5)
set(gca,'fontsize',12)
title('(a)')
xlabel('time (s)')
ylabel('a (m/s^2)')
grid on

subplot(2,1,2)
plot(t,vout,'Linewidth',1.5)
set(gca,'fontsize',12)
title('(b)')
xlabel('time (s)')
ylabel('v (m/s)')
grid on
```

```matlab
% Variables used for projectile trajectory with air drag
Wpr = 0.1;                      % Width of the projectile
A = 0.005;                      % Area of the projectile
Rho = 1.225;                    % Density of air
Cd = 0.1;                       % drag coefficient of bullet
```

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```matlab
% Code for projectile trajectory with air drag

v = vout(end);                  % velocity of the projectile at start

vx0 = V*cos(theta);             % x-axis component of velocity at start
vy0 = V*sin(theta);             % y-axis component of velocity at start
```

```matlab
sim('Projectile_trajectory_with_air_drag')
```

```matlab
% Plot of the trajectory with air drag

figure(3)
plot(x,y,'Linewidth',1.5)
set(gca,'fontsize',12)
xlabel('distance traveled by projectile (m)')
ylabel('height (m)')
ylim([0 710])
grid on

Target_location = x(end)
Copper_Loss = mean(CuLoss)
```

```
Target_location =

   964.5561


Copper_Loss =

   1.3757e+05
```

A.4 Optimization code for the railgun system with projectile's trajectory excluding air drag

The following section consists of MATLAB code used for optimizing the objective function given in (37). Fmincon is used for optimization. A function is created to call Fmincon to optimize the model parameters. MATLAB code and MATLAB function used for optimization of the railgun system excluding air drag is given below.

```
clc;

global Vinit u0 ur lc lpr Wr hc hr rho Ac Ar c Staticfrictionconstant
global Massofprojectile g theta xref xend Vx0 Vy0 CuLoss

% Code for optimizing initial voltage, Launch Angle and CuLoss for railgun
% for reaching certain target position considering Ideal Conditions

% Variables used for Discharging circuit of the railgun

Vinit0 = [75];
Vinit = vinit0;                  % Charging voltage
u0 = 4*3.14*10^(-7);             % permeability
ur = 1;                          % Relative permeability
lc = 0.5;                        % Length of circuit
lpr = 0.05;                      % Length of projectile
Wr = 0.1;                        % Width of rail
hc = 0.01;                       % Height of circuit
hr = 0.01;                       % Height of rail
rho = 1.68*10^(-8);              % Resistivity of conductor
Ac = (3.14*(1)^2/4)*(10^(-4));   % Area of circuit
Ar = 0.001;                      % Area of rail
c = 2.2*10^(-1);                 % Capacitance

Staticfrictionconstant = 0.5;    % Static friction constant
Massofprojectile = 0.1;          % Mass of projectile
g = 9.81;                        % Gravitational acceleration

theta0 = [75];
theta = theta0;                  % Angle of launch

xref = 5*10^2;                   % Target location

% Code for optimization using Fmincon
```

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```
problem = createOptimProblem('fmincon',...
               'objective',@optimize_objective_theta_vinit_CuLoss_wo_ad,...
               'x0',[Vinit0,theta0],...
               'lb',[0,0],...
               'ub',[1000,90]);

ms = MultiStart

[z(:),exitflag,output,solutions] = run(ms,problem,3);
[z(:)] = fmincon(problem);

% Optimized parameters

Vinit = z(1);
theta = z(2);
error = xref-xend;
Copper_Loss = mean(CuLoss);
```

The MATLAB function created for optimization using the objective function given in (36) is given below.

```
Function J = optimize_objective(z)

global Vinit u0 ur lc lpr Wr hc hr rho Ac Ar c Staticfrictionconstant
global Massofprojectile g theta xref xend Vx0 Vy0 CuLoss


Vinit = z(1);
theta = z(2)*(pi/180);


sim('Final_Railgun_discharging_model')
V = vout(end) ;                 % velocity of the projectile at start
Vx0 = V*cos(theta);             % x-axis component of velocity at start
Vy0 = V*sin(theta);             % y-axis component of velocity at start
sim('Projectile_trajectory_without_air_drag')
xend = x(end);


% Optimization objective

J = Wto*(xref-xend)^2 + WCuloss*(mean(CuLoss))^2;
```

A.5 Optimization code for the railgun system with projectile's trajectory including air drag

In this section MATLAB code is given for optimizing the objective function that is given by (37). Optimization has been implemented using Fmincon. A function is created similar to the previous section to optimize the model parameters using Fmincon. A MATLAB code as well as MATLAB function used for optimization of the railgun system including air drag is given below.

```matlab
clc;

% Code for optimizing initial voltage, Launch Angle and CuLoss for railgun
% for reaching certain target position considering air drag

% Variables used for Discharging circuit of the railgun

global Vinit u0 ur lc lpr Wr hc hr rho Ac Ar c Staticfrictionconstant
global Massofprojectile g theta Vx0 Vy0 A Cd Rho xref xend CuLoss Wpr

Vinit0 = [75];                   % Charging voltage
Vinit = Vinit0;
u0 = 4*3.14*10^(-7);             % permeability
ur = 1;                          % Relative permeability
lc = 0.5;                        % Length of circuit
lpr = 0.05;                      % Length of projectile
Wr = 0.1;                        % Width of rail
hc = 0.01;                       % Height of circuit
hr = 0.01;                       % Height of rail
rho = 1.68*10^(-8);              % Resistivity of conductor
Ac = (3.14*(1)^2/4)*(10^(-4));   % Area of circuit
Ar = 0.001;                      % Area of rail
c = 2.2*10^(-1);                 % Capacitance

Staticfrictionconstant = 0.5;    % Static friction constant
Massofprojectile = 0.1;          % Mass of projectile
g = 9.81;                        % Gravitational acceleration

% Variables used for projectile trajectory with air drag
Wpr = 0.1;                       % Width of the projectile
A = 0.005;                       % Area of the projectile
Rho = 1.225;                     % Density of air
Cd = 0.1;                        % drag coefficient of bullet

theta0 = [70];
theta = theta0;                  % Angle of launch
```

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```
xref = 5*10^2;                  % Target location
```

```
% Code for optimization using Fmincon

problem = createOptimProblem('fmincon',...
            'objective',@optimize_objective_theta_vinit_CuLoss_w_ad,...
            'x0',[Vinit0,theta0],...
            'lb',[0,0],...
            'ub',[300,90]);

ms = MultiStart

[z(:),exitflag,output,solutions] = run(ms,problem,3);
[z(:)] = fmincon(problem);

% Optimized paraeters

Vinit = z(1);
theta = z(2);
error = xref-xend;
Copper_Loss = mean(CuLoss);
```

The MATLAB function generated to find the optimization using (36) as the objective function is given by,

```
Function J = optimize_objective(z)

global Vinit u0 ur lc lpr Wr hc hr rho Ac Ar c Staticfrictionconstant
global Massofprojectile g theta xref xend Vx0 Vy0 CuLoss
```

```
Vinit = z(1);
theta = z(2)*(pi/180);
```

```
sim('Final_Railgun_discharging_model')
V = vout(end) ;                 % velocity of the projectile at start
Vx0 = V*cos(theta);             % x-axis component of velocity at start
Vy0 = V*sin(theta);             % y-axis component of velocity at start
sim('Projectile_trajectory_with_air_drag')
xend = x(end);
```

```
% Optimization objective

J = W_to*(xref-xend)^2 + W_CuLoss*(mean(CuLoss))^2;
```