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TESTING OF RECURSIVE AND NON-RECURSIVE ALGORITHMS FOR REAL-
TIME PHASOR AND FREQUENCY ESTIMATIONS IN POWER SYSTEMS

By

Dhruv Sharma

A REPORT

Submitted in partial fulfillment of the requirements for the degree of

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In Electrical Engineering

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This report has been approved in partial fulfillment of the requirements for the Degree of
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List of abbreviations

DFT – Discrete Fourier Transform

LSE – Least Square Estimation

DWT – Discrete Wavelet Transform

WLS – Weighted Least Square

LabVIEW – Laboratory Virtual Instrument Engineering Workbench

MRA – Multiresolution Analysis

FIR – Finite Impulse Response

QMF – Quadratic Mirror Filters

c-RIO – National Instruments (NI)[®] Compact RIO Device

ROCOF – Rate of Change of Frequency

NI – National Instruments

C.T. – Current Transformer

P.T. – Potential Transformer

IEEE – Institute of Electrical and Electronics Engineers

ZCM – Zero Crossing Method

PMU – Phasor Measurement Unit

TVE – Total Vector Error

WAMPAC – Wide-Area Monitoring, Protection and Control

SCADA – Supervisory Control and Data Acquisition

EMS – Energy Management System

GPS – Global Positioning System

FACTS – Flexible Alternating Current Transmission System

FPGA – Field Programmable Gate Array

HIL – Hardware-in-the-Loop

RTDS – Real Time Digital Simulator

ADC – Analog-to-Digital Converter

PDC – Phasor Data Concentrator

THD – Total Harmonic Distortion

FE – Frequency Error

Abstract

In this report, steady-state performance of various recursive and non-recursive algorithms are tested according to the test signals given in the IEEE Standard C37.118.1-2011. Phase magnitude and phase angle of the power grid signals have been estimated using Discrete Fourier Transform (non-recursive), Discrete Fourier Transform (recursive), Least Square, and Wavelet Transform Algorithms. Frequency estimation is performed using Discrete Fourier Transform, Weighted Least Square, and Zero Crossing methods. These algorithms are evaluated in LabView software and tested by generating test signals in a Simulink model. Furthermore, Total Vector Error (TVE) is calculated using dynamic test signals as per the IEEE Standard C37.118-2011. Performance of different algorithms are analyzed for various cases and the value of TVE is compared with the permissible error limits given in the standard.

1 Background

1.1 Introduction

Phasor Measurement Units (PMUs) are characterized by their ability to evaluate the phasors. These measured phasors are then utilized in various events such as power system analysis, stability analysis, state estimation and designing of power system protection schemes. Synchrophasors are phasors which are measured at the same instant of time. There is a need for dynamic phasor measurement in power electronics as well. Power electronics-based components like Flexible Alternating Current Transmission System (FACTS) needs the concept of time varying phasors to be used in the analysis. Phasor estimation is significantly used in Wide-Area Monitoring, Protection and Control (WAMPAC) applications in power systems as shown in Figure 1. There is a substantial need for quick and precise estimation of phasors to make an accurate decision related to the stability and dynamics of power systems [6] [8]. As specific method for phasor estimation has not been proposed in the IEEE standard, various methods like the Discrete Fourier Transform (DFT), Least Square Method (LSM) and Discrete Wavelet Transform (DWT) are common in literature for phasor estimation [2] [3].

Figure 1: General Architecture of WAMPAC system

1.2 Literature Review

In the conventional power system paradigm, the Current Transformers (C.T.) and Potential Transformers (P.T.) sense the current and voltages in the system. These signals are then sent via a Supervisory Control and Data Acquisition (SCADA) system to the EMS (Energy Management System), where they are analyzed. If there is a need for any

control actions, then the appropriate signals are sent back to the power system. They can be in the form of tripping or reclosing of circuit breakers.

The main disadvantage of this operating system is that the output data rate of a SCADA system is slow – once in every 4-6 seconds. This is undesirable to capture dynamics of the system. Also, due to unsynchronized clocks at the C.T.s and P.T.s, the data acquired may be at different times. Hence, there is a need for a time synchronized wide area measurement system.

The Research & Development team from Virginia Tech made the first working model of the PMU back in 1988. In 1990's, several manufacturers started making and selling PMUs with add-on features such as Global Positioning System function, Analog-to-Digital converter etc. Thereafter, some of the state estimation applications in power systems started utilizing PMUs [14]. According to one of the studies, to get the complete observability of the whole network, every third or fourth bus should be equipped with a PMU [13]. PMUs were also used to detect the location of the fault using a distinct filtering technique [15]. This method had a 99.9% success rate. Wide area monitoring, protection and control systems used PMU applications for monitoring and control purposes by making the architecture, topology and profile of WAMPAC in power system networks [16]. Some algorithms are proposed for the detection of voltage instability using synchro-phasor data given in [17] [18] [19] [20]. Dynamic tests on PMUs are standardized at the National Institute of Standards and Technology (NIST) and all the results obtained from the tests are published in [21]. Dynamic performance of the PMU was tested by changing the magnitude or frequency of the signal linearly in [22]. In [23], steady-state, dynamic and modulation tests are performed on the PMU as per IEEE

standard C37.118-2005. Total vector error (TVE) are calculated to examine the performance of PMUs as per the IEEE standard. Harmonic rejection test, frequency ramping test, amplitude modulation test, phase magnitude variation test and phase angle variation tests are also performed and the value of TVE are calculated and observed for repeated cases in [24]. A method of extracting signal amplitude, phase and frequency using the Phase-Locked Loop (PLL) is discussed in [26]. In [27], estimator based on Fuzzy if-then rules is used to track down amplitude and phase angle of the signal. Kalman filters are used to estimate the phase as well as the frequency of the signal in [28]. Artificial Neural Network (ANN) based phasor estimation is proposed for higher accuracy and faster response time [29][30].

1.3 IEEE Standard C37.118.1-2011

The very first Synchrophasor standard released by IEEE was IEEE1344-1995. It contained a mixture of a tutorial as well as the requirements. Timing accuracy in that standard was set at 1 μ s. It included the process of calculating the report time. It initially contained the phasor part but added frequency as well as the Rate of change of frequency (ROCOF) part later. The last sample time was usually used to estimate the window length. There was no requirement mentioned for performance. It was not widely implemented as there were a lot of unresolved issues. The second standard released by IEEE was C37.118-2005. In addition to the previous standard, it contained the steady-state tests calculating the total vector error. It also contained the permissible error limits for all the tests. There were no limits mentioned related to the measurement of the response time and accuracy under transient conditions. The recent update to this standard

was IEEE C37.118-2011 [25]. It was split into two parts: C37.118.1-2011 for measurement and C37.118.2-2011 for communication. The C37.118.1-2011 standard is mostly dedicated to testing. It covers two performance classes: Protection (P-Class) for applications requiring fast response and Measurement (M-Class) for greater precision but with a slower reporting speed. Unlike the previous standard, it sets different requirements on performance. Dynamic testing signals are included along with the permissible TVE and other error limits. Measurement bandwidth, tracking, and response time were added under the dynamic testing conditions. Communication messaging was included with most of the common protocols. This standard also includes a latency test for data output delay. Its amendment C37.118.1a-2011 was released soon after which modifies and simplifies the measurement performance of the 2011 standard. The synchrophasor measurement standard C37.118.1-2011 covers two types of performance testing:

1.3.1 Steady-State Testing

These are the conditions where magnitude, frequency, and phase angle of the signal to be tested does not change during the measurement. Magnitude, phase angle, frequency, and ROCOF of the synchronized signal are tested under the steady-state conditions and compared to their corresponding theoretical value. Total Vector Error (TVE) is calculated for all the cases and compared with the permissible limits. All the signal frequency range tests are usually calibrated at three different temperature readings ($T=0^{\circ}\text{C}$, $T=50^{\circ}\text{C}$ and at nominal room temperature $\sim 23^{\circ}\text{C}$)

The error limits for steady-state tests are provided in Appendix B.1.

1.3.2 Dynamic Testing

There are three types of performance tests included in Dynamic Testing:

1.3.2.1 Bandwidth Modulation

Sinusoidal Modulated signals are applied to the balanced three-phase input signal's amplitude and phase angles simultaneously. TVE measurement should be within the error limits. The modulated input signals for three phases are given by:

$$X_a = X_m[1 + k_x \cos(\omega t)] \cos[\omega_0 t + k_a \cos(\omega t - \pi)] \quad (1)$$

$$X_b = X_m[1 + k_x \cos(\omega t)] \cos[\omega_0 t - (\frac{2\pi}{3}) + k_a \cos(\omega t - \pi)] \quad (2)$$

$$X_c = X_m[1 + k_x \cos(\omega t)] \cos[\omega_0 t + (\frac{2\pi}{3}) + k_a \cos(\omega t - \pi)] \quad (3)$$

where,

X_m is the amplitude of the input signal;

ω is the frequency of modulation in rad/sec;

ω_0 is the nominal frequency of the system;

k_x is the amplitude modulation factor; and

k_a is the phase modulation factor.

The permissible range for this test is given in Appendix B.2.

1.3.2.2 System Frequency Ramp

Linear ramp is given to the system frequency and the measurement performance is tested.

The input signals for three phases are given by:

$$X_a = X_m \cos[\omega_0 t + \pi R_f t^2] \quad (4)$$

$$X_b = X_m \cos[\omega_0 t - \left(\frac{2\pi}{3}\right) + \pi R_f t^2] \quad (5)$$

$$X_c = X_m \cos[\omega_0 t + \left(\frac{2\pi}{3}\right) + \pi R_f t^2] \quad (6)$$

where,

X_m is the amplitude of input signal;

ω_0 is the nominal frequency of the system in rad/sec; and

R_f is the frequency ramp rate given in Hz/sec.

The permissible range for this test is given in Appendix B.3.

1.3.2.3 Step Changes in Phase and Magnitude

Balanced three-phase step changes are applied to the magnitude and phase angles of balanced three-phase input signal and the performance is tested. This test is a switch amid two steady-states. The response time of the system, time delay in the measurement and the overshoot is calculated. The equations for balanced three-phase signals are given by:

$$X_a = X_m[1 + k_x f_1(t)] \cos[\omega_0 t + k_a f_1(t)] \quad (7)$$

$$X_b = X_m[1 + k_x f_1(t)] \cos[\omega_0 t - \left(\frac{2\pi}{3}\right) + k_a f_1(t)] \quad (8)$$

$$X_c = X_m[1 + k_x f_1(t)] \cos[\omega_0 t + \left(\frac{2\pi}{3}\right) + k_a f_1(t)] \quad (9)$$

where,

X_m is the amplitude of the input signal;

$f_1(t)$ is the unit step function;

ω_0 is the nominal frequency of the system;

k_x is the magnitude step size; and

k_a is the phase step size.

The limits for this test are the same as the steady-state tests and are given in Appendix B.1.

1.4 Total Vector Error

TVE is used to check the estimation accuracy of the measurement of a signal. The deviation in the input signals have been restricted by the IEEE C37.118.1 standard in a range of operating conditions. This allowable error (deviation) is known as TVE. It

compares the vectoral difference between the compensated phasor and the ideal phasor and is expressed in percentages [2] [6].

$$TVE = \frac{|V_{measured} - V_{ideal}|}{|V_{ideal}|} \times 100\% \quad (10)$$

Possible sources of error include magnitude, angle, frequency and timing. The limit for Magnitude error should be less than 1%. The limit of Phase error should be less than 0.573 deg. and the limit for Time error for a 50 Hz system should be less than 31.8 μs and for 60 Hz system should be less than 26.5 μs . All these limits are used to evaluate the performance in the case of steady-state testing.

IEEE standard defines that a ± 1 Hz/s change of frequency rate must be used to evaluate the dynamic estimation considering a frequency ramp. The TVE must be less than 1% [6].

1.5 Phasor Measurement Unit

A Phasor Measurement Unit or PMU is defined as a device that estimates the phasor, frequency and rate of change of frequency from voltage and/or current signals along with a time tag feature using a GPS clock. The basic phasor measurement blocks are provided in Figure 2.

Figure 2: PMU Operation

The incoming signal from the C.T.s and P.T.s are sampled at various reporting rates. This can be of the order of 10, 12, 15, 20, 30, 60 frames/second. That sampled waveform is sent through various estimation algorithms, details of which form the basis of this project. It is possible to estimate the voltage and current phasors, frequency, ROCOF and the sequence components.

In AC networks, power flow is given by,

$$P = \frac{(V_k \times V_j)}{x_l} \sin(\theta_k - \theta_j) \quad (11)$$

where,

V_k and V_j are the voltages at receiving and sending end respectively;

X_l is the reactance of the interconnected line; and

θ_k and θ_j are the voltage angles at the receiving and sending ends respectively.

During stress conditions, $(\theta_k - \theta_j)$ increases. Hence, we need the measurement of angles as well the magnitudes. The answer to this is Synchrophasor technology.

Figure 3: Synchrophasor Representation

A basic synchrophasor representation is shown in Figure 3 above where,

‘A’ is the magnitude and ϕ is the angular difference between the peak of the phasor signal and the time reference ($t=0$).

The reference signal comes from the GPS at 1 pulse per second. Hence, time synchronization of data across the power system is possible.

2 Algorithms

2.1 Phasor Estimation Algorithms

2.1.1 Discrete Fourier Transform (Non-Recursive)

Figure 4: Basic Sinusoidal Signal

A sinusoid signal can be represented by:

$$V(t) = V_m \cos(\omega t + \phi) \quad (12)$$

where,

V_m = peak amplitude of the phasor;

ϕ = phase angle in radians; and

ω = signal frequency in rad/sec.

When the frequency of the fundamental signal is sampled at N times the nominal frequency (f_o), then the signal can be represented by,

$$X(t) = X_m \cos(2\pi f_0 t + \phi) \quad (13)$$

N samples of this signal X_k : $\{k=0, 1 \dots N-1\}$ are obtained from

$$X_k = X_m \cos(k\theta + \phi) \quad (14)$$

where, $\theta = 2\pi f_0 T/N$

The (N-1) sample is the last sample in phasor estimation. The phasor $X^{(N-1)}$ is given by:

$$X^{N-1} = \frac{2}{N} \sum_{k=0}^{N-1} x_k \cos(k\theta) - j \left(\frac{2}{N} \right) \sum_{k=0}^{N-1} x_k \sin(k\theta) \quad (15)$$

where,

the real part is $X_{re} = \frac{2}{N} \sum_{k=0}^{N-1} x_k \cos(k\theta)$; and

the imaginary part is $X_{im} = \left(\frac{2}{N} \right) \sum_{k=0}^{N-1} x_k \sin(k\theta)$

Thus, the magnitude of the phasor is given by,

$$X_m = \sqrt{(X_{re}^2 + X_{im}^2)} \quad (16)$$

The angle of the phasor is given by,

$$(wt + \phi) = \tan^{-1}(X_{im}/X_{re}) \quad (17)$$

This method is also known as non-recursive method. Explanation is given with reference to Figure 5 below.

Figure 5: Non-Recursive Discrete Fourier Transform

According to Figure 5, the first window of N samples is from $\{n = 0 \dots N-1\}$. Phasor 1 is the phasor estimated from these readings. The second window of N samples is from $\{n=1 \dots N\}$. For estimation of phasor for this window, the previous readings are not considered, and fresh calculations are carried out. Hence, the estimated phasor 2 will be at an angle θ with respect to phasor 1. This is a slow process as it does not use the estimated data of the previous window. All the calculations are performed fresh for each iteration [12].

2.1.2 Discrete Fourier Transform (Recursive)

In this method, a recursive update is made to the old phasor values i.e. phasor estimates from the previous window are taken and updated for a new window. It can be better understood with reference to Figure 6.

Figure 6: Recursive Discrete Fourier Transform

(Fig reference: Phadke, A. G., Thorp, J. S., "Synchronized Phasor Measurements and Their Applications," Springer, 2008)

According to the above figure, the first window of N samples is from $\{n = 0 \dots N-1\}$. Phasor 1 is the phasor estimated from these readings. The second window of N samples is from $\{n=1 \dots N\}$. Phasor 2 is the phasor estimated for these samples. However, as compared to the non-recursive method, here we just take the new sample value. The older samples are taken from the previous window. This makes it much faster than the non-recursive method. However, this method can be unstable as any error in the previous calculations is carried forward. If $(N+k)$ is the latest sample of this window, then the phasor estimation is given by:

$$X^{N+k} = X^{N+k-1} + \left(\frac{2}{N}\right)(x_{N+k} - x_k)e^{-j(k)\theta} \quad (18)$$

where,

$X^{(N+k-1)}$ is the phasor estimated from the previous window.

2.1.3 Least Square Estimation

Least square method is a curve-fitting method. It is the most extensively used and the most eminent method among scholars. This method is very easy to understand. The main advantage of LSE over the other methods is that we can estimate the quantity accurately with fewer readings [9].

A sinusoidal voltage signal can be given:

$$V = V_p \sin(wt + \phi) \text{ or } V = V_p \sin(wt) \cos\phi + V_p \cos(wt) \sin\phi$$

Assuming $V_p \cos\phi = x$ and $V_p \sin\phi = y$,

$$V = x \sin(wt) + y \cos(wt) \quad (19)$$

For k readings ($k=0, 1 \dots N-1$) and writing in matrix form,

$$(\sin(kw\Delta t) \quad \cos(kw\Delta t)) \times \begin{bmatrix} x \\ y \end{bmatrix} = [V_k] \quad (20)$$

By the Least Squares Method,

$$\hat{X} = [A^T A]^{-1} [A]^T [m] \quad (21)$$

where,

$$\hat{X} = \begin{bmatrix} x \\ y \end{bmatrix};$$

$$A = (\sin(kw\Delta t) \quad \cos(kw\Delta t)); \text{ and}$$

$$m = [V_k]$$

2.1.4 Wavelet Transform

For phasor estimation, a 1-D wavelet transform is used for better localization of the signal in terms of frequency and time scale. A mother wavelet function is used which represents the signal in terms of its coefficients [12]. To perceive high frequency data, the wavelet uses short time interval sections while long time sections are used to perceive specific low frequency data. The signal is analyzed using shifted and dilated versions of the selected wavelet function. The continuous wavelet transform is best for time frequency analysis as coefficients are directly in terms of frequency [4].

The main difference between the Wavelet Transform and other algorithms is that the Wavelet uses a waveform instead of a complete sinusoidal signal function. As can be seen in Figure 7, the wavelet uses a time-scale concept instead of a time-frequency region. The Time-Scale concept means that the signal is broken into scaled versions of the mother wavelet either by compressing (small scale) or by stretching (large scale) the signal.

Figure 7: Time-Scale Concept - Wavelet Analysis

The discrete wavelet transform (DWT) is used for estimation as it uses less number of coefficients and no anti-aliasing filter is needed. The DWT of a signal is expressed as two coefficients: approximation coefficients and detail coefficients. As shown in Figure 8, the wavelet transform with MRA (Multiresolution Analysis) is used for real time signal processing by using the mother wavelet function as coefficients of a pair of complementary finite impulse response (FIR) filters. These filters are also called Quadratic mirror filters (QMF's) [4]. The frequency response of these QMF's divides the fundamental signal into high and low frequency elements. Generally, the in-between point is somewhere in the middle of 0 and half the Nyquist Frequency. As can be seen from the Tree Algorithm in Figure 8, the low pass filtered signal coming out from one of the QMF is decimated by 2 and is again given to another QMF pair resulting in the formation of the Tree Algorithm. Wavelet coefficients are used as the coefficients in both low and high pass filters of the QMF pair and this hierarchical system is applied to the wavelet transform method. Low Pass Output of the QMF pair is known as the approximations (A3 in the figure) and High Pass Output of the QMF pair is known as the details (D1, D2, D3 in the figure). Daubechies wavelet family is the most suitable wavelet family in analyzing power system analysis. The phasors of voltage and current

can be estimated by using sinusoidal reference signal of unity amplitude and reference frequency (60Hz) along with the sampled signal to be estimated. Both signals are decomposed into two levels using the db1 mother wavelet. Phasors will be estimated from its coefficient (approximate) [5]. It does not perform well when the signal has a dc offset. For that, a band pass filter is used before estimation.

Figure 8: Tree Algorithm of multiresolution Wavelet Transform

Phasor estimation using CWT should be done for travelling waves. Using (22) we can find the phase of the sampled signal.

$$\theta = \cos^{-1} \frac{(A_{2R1} \cdot A_{2S})}{|A_{2R1}| |A_{2S}|} \quad (22)$$

where,

A_{2R1}= Approximate coefficient of Sinusoidal Reference Signal;

A_{2S}= Approximate Coefficients of Sampled Signal;

$|A_{2R1}|$ = Norm 2 of Sinusoidal Reference Signal; and

$|A_{2S}|$ = Norm 2 of Sampled Signal.

Using the above obtained ' θ ' we can obtain the magnitude of the signal. Then, use this phase angle for the reference sinusoidal signal and find its coefficients again using the same wavelet. Taking the norm of the new coefficients, use (23) to find the magnitude.

The magnitude is given by:

$$Y = \frac{|A_{2S}|}{|A_{2R2}|} \quad (23)$$

where,

A_{2R2} = Norm 2 with new phase of Reference Signal.

2.2 Phasor Estimation of Off-Nominal Frequency Signals

All the above algorithms work perfectly for nominal frequency inputs i.e. 60 Hz inputs.

However, when we input off-nominal frequency inputs, we get erroneous results.

When a signal with off-nominal frequency (60.5Hz) is sent through the DFT algorithm, we see deviations (chattering) in the estimated phasor magnitude. Hence there is a need for compensation. Compensation logic is provided for the input signal to extract the 60 Hz component [12] [8].

We multiply the input signal by multipliers – ‘P’ and ‘Q’ given by:

$$P = \left\{ \frac{\sin \frac{N(w-w_0)\Delta t}{2}}{N \sin \frac{N(w-w_0)\Delta t}{2}} \right\} e^{j(N-1)\frac{(w-w_0)\Delta t}{2}} \quad (24)$$

$$Q = \left\{ \frac{\sin \frac{N(w+w_0)\Delta t}{2}}{N \sin \frac{N(w+w_0)\Delta t}{2}} \right\} e^{j(N-1)\frac{(w-w_0)\Delta t}{2}} \quad (25)$$

The compensated phasor is given by:

$$X'_r = P X_{\epsilon}^{jr(w-w_0)\Delta t} + Q X^* e^{-jr(w+w_0)\Delta t} \quad (26)$$

This compensated phasor is then sent through the estimation algorithms.

2.3 Frequency Estimation Algorithms

Frequency is being widely used in power system analysis applications. The accuracy and flexibility of the frequency depends on the algorithms used. The algorithms are chosen based on the real signal environment. Recently, many algorithms including time domain as well as time-frequency combined methods are used widely in various areas.

2.3.1 Discrete Fourier Transform Method

Frequency estimation using Discrete Fourier Transform is performed using an iterative method. This method also requires moderate computation. Two orthogonal filters have been used assuming the fundamental frequency to be equal to the nominal frequency. The gains derived from the filters are used to calculate the phase angle of the frequency measurement. Using the DFT, the frequency estimation is done accurately within 20ms.

The phase angle of the frequency is calculated using

$$\theta_n = \tan^{-1} \left(\frac{Vi}{Vr} \right) \quad (27)$$

where,

Vi – Imaginary frequency component; and

Vr – Real frequency component.

For the next sample, θ_{n+1} is calculated and the estimated frequency is calculated as,

$$\hat{f} = \left(\frac{\theta_{n+1} - \theta_n}{\frac{2\pi}{f_s}} \right) \quad (28)$$

where,

f_s – Sampling frequency.

The estimated frequency would be equal to the nominal frequency if orthogonal filters are designed using the fundamental frequency. If the estimated frequency is not equal to the nominal frequency, then the filters are designed using the estimated frequency as the nominal frequency and the process is repeated [7].

2.3.2 Recursive Least Square Method

The Recursive Least Square Method (RLS) is used for frequency estimation of instantaneous power system. RLS is utilized owing to a faster convergence rate compared to other frequency estimation methods. Because of the higher convergence rate, the ability to perform complex computations is lost. The coefficients are recursively found to minimize using the weighted least square [10].

In this method the following equation is used for frequency estimation:

$$f = \{\cos^{-1}(n/2)\}/(2\pi t_s) \quad (29)$$

where,

t_s is the sampling time-period;

n is the weighting factor and is given by the equation:

$$n v(n) = v(n - 1) + v(n + 1) \quad (30)$$

where,

$v(n)$, $v(n+1)$ and $v(n-1)$ are the current, previous and next samples considered.

The above equation was chosen as it has better precision and reduces noise [10].

2.3.3 Zero Crossing Method

The zero-crossing method is used in frequency estimation since it resolves the inconsistency between the number of samples and the accuracy and time of frequency measurements. This method is simple, has low computation, is fast, and has real-time performance. Only 3 data samples are utilized to estimate frequency in this method. This method also reduces the impact of noise [11]. This method is fast but not that accurate.

The following formula is used to estimate the frequency:

$$2\pi f t_s = \cos^{-1}(\Sigma x(n+2)x(n) + \Sigma x(n)x(n))/(2\Sigma x(n+1)x(n)) \quad (31)$$

where,

n – iterative count;

f – power frequency; and

t_s is the sampling time.

3 Implementation

In this project, test signals are created in Simulink for testing the steady-state and dynamic performance of different algorithms as per the IEEE standard C37.118.1-2011 for the estimation of Phase Magnitude and Frequency. The signal samples are saved in a text file for each case. In LabView, two files are created: an FPGA bitfile for exchange of input and output signals and the main file for the execution of the main algorithms. The text files are given as input to the LabView main file. This signal is sent to different algorithms for phasor and frequency estimations and the Estimated Magnitude is given to the output block i.e. to the Trip Write block. This output is then read with the help of Trip Read block in the FPGA bitfile and the output data is then written to the Output module of the c-RIO Hardware. The NI 9263 module is used for Analog Output. The output module is connected to the Oscilloscope to see the estimated phase magnitude.

Figure 9: Flowchart of Project

3.1 Case Studies

3.1.1 Steady-State Testing for Phasor Estimation

Steady-state performance is evaluated by testing the algorithms with IEEE standard test signals as per IEEE C37.118.1-2011. In case of magnitude deviation, 10 units is assumed as the reference condition and a DC offset of ± 2 units is given to the input signal. This condition is given to satisfy the deviation range of 80% to 120% of the rated signal. In case of phase-angle deviation, steady-state performance is tested for a range of $\pm$ radians. For evaluating steady-state performance in case of frequency deviation, nominal frequency of 60 Hz is kept as reference and a range of ± 2 Hz disturbance is given to the input signal conditions.

All the algorithms are tested for the above conditions to evaluate the steady-state performance and the results are discussed below:

3.1.1.1 DFT (Non-Recursive)

A 48-point (48 samples/cycle) Discrete Fourier Transform algorithm is developed in LabView. The performance is tested for all three types of deviation from the fundamental signal and the results are compared with the permissible limits. The model is given in Appendix A.1 and the test results for all the cases are given below.

3.1.1.1.1 Deviation in Magnitude

Figure 10: Magnitude Deviation DFT (NR) Results

3.1.1.1.2 Deviation in Phase Angle

Figure 11: Phase angle Deviation DFT (NR) Results

3.1.1.1.3 Deviation in Frequency

Figure 12: Frequency Deviation DFT (NR) Results

3.1.1.2 DFT (Recursive)

A 48-point Discrete Fourier Transform (Recursive) algorithm is developed in LabView. The performance is tested for all three types of deviation from the fundamental signal and the results are compared with the permissible limits. The model is given in Appendix A.2 and the test results for all the cases are given below.

3.1.1.2.1 Deviation in Magnitude

Figure 13: Magnitude Deviation DFT (R) Results

3.1.1.2.2 Deviation in Phase Angle

Figure 14: Phase angle Deviation DFT (R) Results

3.1.1.2.3 Deviation in Frequency

Figure 15: Frequency Deviation DFT (R) Results

3.1.1.3 Least Square Estimation

A 48-point Least Square Estimation algorithm is developed in LabView. The performance is tested for all three types of deviation from the fundamental signal and the results are compared with the permissible limits. The model is given in Appendix A.4 and the test results for all the cases are given below.

3.1.1.3.1 Deviation in Magnitude

Figure 16: Magnitude Deviation LS Results

3.1.1.3.2 Deviation in Phase Angle

Figure 17: Phase Angle Deviation LS Results

3.1.1.3.3 Deviation in Frequency

Figure 18: Frequency Deviation LS Results

3.1.1.4 Wavelet Transform

The LabView model for the Wavelet Transform method is developed using 'db2' at Level 6. The performance is tested for all three types of deviation from the fundamental signal and the results are compared with the permissible limits. The model is given in Appendix A.5 and phasor estimates are shown below.

3.1.1.4.1 Deviation in Magnitude

Figure 19: Magnitude Deviation Wavelet Results

3.1.1.4.2 Deviation in Phase Angle

Figure 20: Phase angle Deviation Wavelet Results

3.1.1.4.3 Deviation in Frequency

Figure 21: Frequency Deviation Wavelet Results

3.1.2 Dynamic Testing for Phasor Estimation

Dynamic Performance is evaluated by conducting the Frequency Ramp Test on all the above-mentioned algorithms and the value of TVE is estimated and compared with the permissible limits set in the IEEE Standard C37.118.1-2011.

3.1.2.1 DFT (Non-Recursive)

The LabView model for the Discrete Fourier Transform is given in Appendix A.1.

The performance is tested by inputting a frequency ramp in the fundamental frequency signal and the results are compared with the permissible limits. The results for the same are given in Figure 22.

Figure 22: Frequency Ramp DFT (NR) Results

3.1.2.2 DFT (Recursive)

The LabView model for the Discrete Fourier Transform is given in Appendix A.2

The performance is tested by inputting a frequency ramp in the fundamental frequency signal and the results are compared with the permissible limits. The results for the same are given in Figure 23.

Figure 23: Frequency Ramp DFT (R) Result

3.1.2.3 Least Square Estimation

The LabView model for this algorithm is given in Appendix A.4.

The performance is tested by inputting a frequency ramp in the fundamental frequency signal and the results are compared with the permissible limits. The results for the same are given in Figure 24.

Figure 24: Frequency Ramp LS Result

3.1.2.4 Wavelet Transform

The LabView model for Wavelet Transform is given in Appendix A.5.

The performance is tested by inputting a frequency ramp in the fundamental frequency signal and the results are compared with the permissible limits. The results for the same are given in Figure 25.

Figure 25: Frequency Ramp Wavelet Result

3.1.3 Phasor Estimation of Off-Nominal Frequency Signals

The model for Phasor Compensation is given in A.3. The result can be seen in Figure 26 below. It is seen that for an input signal of 10 V, the estimated magnitude after the compensation is observed to be exactly 10 V and there is no chattering involved in the result.

Figure 26: Phasor Compensation Results

3.1.4 Frequency Estimation

Frequency of the input signal is estimated using three different algorithms. The input signal has a magnitude of 10V and a fundamental frequency of 60 Hz, and the frequency output is estimated. Frequency Error is calculated using the formula:

$$\text{Frequency Error} = |\text{Nominal Frequency} - \text{Estimated Frequency}|$$

3.1.4.1 Discrete Fourier Transform Method

The model for this method along with the DFT phasor estimation model is shown in Appendix A.1.

The results for this algorithm are given along with the phasor estimation results in 3.1.1.1.

3.1.4.2 Recursive Least Square Method

The model for this method along with the Least Square phasor estimation model is shown in Appendix A.4.

The results for frequency estimation using RLS are given in 3.1.1.3.

3.1.4.3 Zero Crossing Method

The Last method used for estimating the frequency is the Zero Crossing method. The model for this method is given in Appendix A.6.

The results are given in Figure 27.

Figure 27: Frequency Estimation Zero Crossing Result

3.2 Results

3.2.1 Steady-State Performance

The results for all the algorithms are compared with the permissible Total Vector Value and are given in Table 1 below. The results of all the algorithms for magnitude deviation tests are within the permissible TVE limits. The least square method gave the best result with TVE value well within 0.001%. For phase deviation tests, wavelet transform is the most reliable option with TVE value of 0.034%. The results for frequency deviation tests showed that the algorithms still have some scope for improvement in terms of estimation accuracy with only wavelet transform method (0.209%) giving the error value well within the limits.

Table 1: Steady-State Performance Evaluation

Tests Conducted	Permissible TVE Limit (%)	DFT (Non-Recursive) TVE (%)	DFT (Recursive) TVE (%)	Least Square TVE (%)	Wavelet TVE (%)
Magnitude Deviation	1	0.013	0.016	0.008	0.860
Phase Angle Deviation	1	0.147	0.155	0.151	0.034
Frequency Deviation	1	5.010	5.009	5.012	0.209

3.2.2 Dynamic Performance

The results for all the algorithms are compared with the permissible Total Vector Value and are given in Table 2 below. The dynamic performance of least square method is the most optimal with the TVE value as low as 0.002%. Wavelet transform method gave the TVE value of 1.34% so this method can be improved a little to get the error value less than 1%.

Table 2: Dynamic Performance Evaluation

Tests Conducted	Permissible TVE Limit (%)	DFT (Non-Recursive)	DFT (Recursive)	Least Square	Wavelet
Ramp in Frequency	1	0.105	0.158	0.002	1.340

3.2.3 Frequency Estimation

The Results for the Frequency Estimation tests are given in Table 3 below. The Discrete Fourier Transform method for frequency estimation gave the most accurate result with an error of around 0.023 Hz. Zero crossing method is the quickest in estimating the frequency but gave an error of 0.648 Hz.

Table 3: Frequency Estimation Results Comparison

Tests Conducted	Nominal Frequency	DFT Method	RLS Method	Zero Crossing
Frequency Estimation	60 Hz	60.023 Hz	59.901 Hz	59.352 Hz
Frequency Error		0.023 Hz	0.099 Hz	0.648 Hz

4 Conclusion and Future Scope

4.1 Observations

The main conclusions from this report can be highlighted as following:

1. Least square method is the most accurate method in the case of magnitude deviation while wavelets came out to be the best in the other two cases of steady-state testing.
2. In the case of dynamic testing, least square is the most accurate algorithm with a TVE value near about 0.002%.
3. The recursive least square method took the least time in estimating the frequency.
4. The least square estimation method for phasor estimation is the fastest. It uses just a fraction of a cycle for estimation i.e. it does not require full window frame for estimation. Hence, it is most commonly used in relaying applications (P-Class PMU).
5. Except the case of deviation in nominal frequency, the steady-state performance of most of the algorithms are up to the mark. The value of TVE came out within the permissible limits when tested against the IEEE standard test signals. However, the performance against the frequency deviation tests showed that the algorithms still have room for improvement.
6. The major advantage of using non-recursive methods is that any error does not propagate throughout the entire method as the calculations are repeated for each iteration.

7. For frequency deviation test conditions in the phasor estimation part, Wavelet Transform should be preferred.
8. The Discrete Fourier Transform method is the most accurate in case of frequency estimation giving least value of frequency error.
9. Zero Crossing Method for frequency estimation is very fast but not that accurate.

4.2 Recommendation for Future Work

The following recommendations are suggested for future scope of this project:

1. OPAL-RT could be used to generate IEEE standard test signals. Generating test signals using a real time simulator will give an actual idea of how an algorithm is behaving. Hardware-in-the-Loop (HIL) testing can be performed after complete Hardware implementation.
2. Output from the c-RIO hardware onto the oscilloscope contains noise. It needs to be tuned.
3. The other tests related to the transient conditions like bandwidth modulation and step changes in magnitude could be evaluated to get a better picture of the reliability of these algorithms.
4. SEL Relay's inbuilt PMU algorithm could be tested using IEEE standard test signals. The performance of inbuilt algorithms can be compared with the algorithms build in LabView.

5. Algorithms based on Kalman Filter method can also be used for the dynamic phasor estimation as it modifies the modelling process of state space to decrease the error bound and its computational requirement is low as well.
6. Available open source PMU algorithms can be studied, and their performance can be compared.
7. The algorithms can be improved to get more accurate and improved results in cases of deviation in frequency of the incoming signal.
8. Performance evaluation of frequency estimation algorithms could be evaluated by calculating the frequency error (FE) and Rate of change of frequency error (RFE) and comparing FE, RFE values with the limits given in IEEE standard.

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A Appendix (Model Screenshots)

A.1 Discrete Fourier Transform (Non-Recursive)

Figure A.1 1: DFT LabView Model (Non-Recursive)

A.2 Discrete Fourier Transform (Recursive)

Figure A.2 1: DFT LabView Model (Recursive)

A.3 Off-Nominal Frequency Compensation Phasor Estimation

Figure A.3 1: DFT Off-Nominal Correction LabView Model

A.4 Least Square Estimation

Figure A.4 1: Least Square Method LabView Model

A.5 Wavelet Transform

Figure A.5 1: Wavelet Transform LabView Model

A.6 Frequency Estimation (Zero Crossing Method)

Figure A.6 1: Frequency Estimation Zero Crossing Method LabView Model

B Appendix (Tables)

B.1 Steady State Tests

Table B 1: Steady-State Measurement Requirements

Influence Quantity	Reference Condition	Minimum Range over which PMU is within TVE limit			
		P-Class		M-Class	
		Range	Max TVE (%)	Range	Max TVE (%)
Signal Frequency Range	$F_{\text{nominal}} (f_0)$	± 2.0 Hz	1	± 2.0 Hz for $F_s < 10$, $\pm F_s/5$ for $10 \leq F_s < 25$, ± 5.0 Hz for $F_s \geq 25$	1
Signal Magnitude – Voltage	100% rated	80% to 120% rated	1	80% to 120% rated	1
Signal Magnitude – Current	100% rated	10% to 200% rated	1	10% to 200% rated	1
Phase Angle	Constant or slowly varying angle	$\pm \pi$ radians	1	$\pm \pi$ radians	1
Harmonic Distortion	$< 0.2\%$ (THD)	1%, each harmonic up to 50 th	1	10%, each harmonic up to 50 th	1
Signal Frequency	$F_{\text{nominal}} (f_0)$	± 2.0 Hz	Max FE (%)	± 2.0 Hz for $F_s < 10$, $\pm F_s/5$ for $10 \leq F_s < 25$, ± 5.0 Hz for $F_s \geq 25$	Max FE (%)
			0.005Hz		0.005Hz

B.2 Bandwidth Measurement

Table B 2: Bandwidth Measurement Requirements

Modulation Level	Reference Condition	Minimum Range over which PMU is within TVE limit			
		P-Class		M-Class	
		Range	Max TVE (%)	Range	Max TVE (%)
$k_x=0.1, k_a=0.1$ rad	100% rated signal magnitude, $F_{\text{nominal}} (f_0)$	Modulation frequency 0.1 to lesser of $F_s/10$ or 2 Hz	3	Modulation frequency 0.1 to lesser of $F_s/5$ or 5 Hz	3
$k_x=0, k_a=0.1$ rad	100% rated signal magnitude, $F_{\text{nominal}} (f_0)$		3		3
Signal Frequency	$F_{\text{nominal}} (f_0)$	$F_s > 20$	Max FE (%)	$F_s > 20$	Max FE (%)
			0.06Hz		0.3Hz
		$F_s \leq 20$	Max FE (%)	$F_s \leq 20$	Max FE (%)
			0.01Hz		0.06Hz

B.3 Frequency Ramp Performance

Table B 3: Frequency Ramp Testing Requirements

Test Signal	Reference Condition		Minimum Range over which PMU is within TVE limit			
			P-Class		M-Class	
		Ramp Rate (R _r)	Ramp Range	Max TVE (%)	Ramp Range	Max TVE (%)
Linear Frequency Ramp	100% rated signal magnitude, & f _{nominal} (f ₀) at start or some point during the test	±1.0 Hz/sec	±2 Hz	1	Lesser of ± (F _s /5) or ± 5 Hz	1
Linear Frequency Ramp	100% rated signal magnitude and 0 radian base angle	Transition Time	Max FE (%)		Max FE (%)	
		±2/F _s for start and end of ramp	0.01Hz		0.005Hz	

C Appendix (Hardware/Software Used)

C.1 NI c-RIO

It is a software designed embedded controller owned by National Instruments having a processor and user-programmable FPGA populated with one or more conditioned I/O modules. It is used for real-time industrial control and monitoring applications. The I/O modules provide direct sensor connectivity and specialty functions. The Compact-RIO Controller is a rugged, reliable, high-performance, industrial-grade embedded controller with industry-standard certifications. It is ideal for applications that need waveform acquisition, high-speed control or signal processing, hardware algorithm acceleration, hardware reliable tasks, or unique timing and triggering.

Figure 28: NI c-RIO 9068 controller

(Fig reference: <http://www.ni.com/white-paper/14602/en/>)

The NI cRIO-9068 combines a dual-core processor, a reconfigurable FPGA, and eight slots for C Series I/O modules within one chassis. These controllers are only compatible with National Instruments C Series I/O Modules. I/O modules are hot swappable (can be connected/disconnected while the unit is powered up). The Full Development System version of LabVIEW does not come with the modules needed to program the c-RIO. The Real-Time module and FPGA modules have to be purchased separately and installed with LabVIEW for programming the hardware. The programming is done on a host PC running the windows operating system and is deployed on the c-RIO via ethernet.

C.2 LabVIEW

Laboratory Virtual Instrument Engineering Workbench (LabVIEW) is systems engineering software for applications that require test, measurement, and control with rapid access to hardware and data insights. It is a system-design platform and development environment for a visual programming language from National Instruments. It offers a graphical programming approach that helps in visualizing every aspect of the application, including hardware configuration, measurement data, and debugging. This visualization makes it simple to integrate measurement hardware from any vendor, represent complex logic on the diagram, develop data analysis algorithms, and design custom engineering user interfaces. It was originally released just for the Apple Macintosh in 1986, but now it is widely used on a variety of operating systems for data acquisition, instrument control, and industrial automation.