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Fluctuation Properties of Precipitation. Part II: Reconsideration of the Meaning and Measurement of Raindrop Size Distributions

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ABSTRACT

For M drop size categories, rain is frequently viewed simply as the superposition of M, statistically independent Poisson-distributed drop fluxes each described by its own mean concentration. Implicit in such a Poissonian model is the assumption of uncorrelated counts among the drops. However, it is well known that drop size distributions are the result of the processes of collision, coalescence, and breakup, which should lead to correlations.

This inconsistency is resolved in this work. Using 1-min disdrometer measurements, two-point cross-correlation functions are used to show that drop counts at different sizes are correlated rather than independent. Moreover, it is argued that it is more appropriate to characterize rain statistically as a doubly stochastic Poisson process (Poisson mixture) among a collection of M correlated random variables (fluxes) each having its own probability distribution of unpredictable (random) mean values and its own coherence time, τ_M.

It is also shown that a drop size distribution has a characteristic coherence time, τ. It is then argued that in order to preserve the purity of a size distribution of interacting drops, τ must be equivalent to the shortest τ_M. For sampling intervals much shorter than τ and when the observation time, T, is less than τ, the drop counts remain correlated and the drop size distribution assumes the definition of a collection of physically interacting drops. On the other hand, when $T \gg \tau$, the drop counts decorrelate and the concept of the drop size distribution changes to a formal relation among the M observed drop concentrations averaged over several different size distributions. Moreover, when T is between the longest and shortest τ_M, part of the observed distribution will represent the distribution of interacting drops and the other part will represent a mixture of drops from different distributions.

Finally, this work suggests using multiple time series analysis techniques for estimating mean drop concentrations in order to use all the available information and to help reduce drop size distribution mixing associated with the conventional analysis based on fixed time intervals.

1. Introduction

The observation by Marshall and Palmer (1948) that the mean concentrations of drops of different sizes obey a simple mathematical formula (the drop size distribution) was fundamental in establishing the potential quantitative application of radar and other remote sensing devices to the measurement of rain. However, rather than just being a formula, drop size distributions are often considered to be physical entities. Studies over the last 40 years (e.g., see Pruppacher and Klett 1997) indicate that given enough time, a collection of interacting drops will naturally evolve toward an ''equilibrium'' distribution through the processes of collision, coalescence, and breakup. The physics and resulting mathematical descriptions of drop size distributions, in turn, imply that there should be statistical correlations among the different drop sizes (Bayewitz et al. 1974). However, it is not clear that equilibrium conditions ever prevail in the real atmosphere (Srivastava 1971; Valdez and Young 1985; Hu and Srivastava 1995). Hence, the first of three questions is, Are drops of different sizes correlated in the real atmosphere?

The second question concerns the measurement of drop size distributions using drop counts assigned to data bins characterized by some mean size. Because of the stochastic arrival of raindrops at a counter, some averaging is always used to improve estimates of the ''means'' and to suppress statistical fluctuations. Our second question, then, is, How much averaging is appropriate?

There are divergent responses to this question. For example, Marshall and Palmer deduced a family of drop

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size distributions by combining measurements gathered in many clouds during an entire summer. Similarly, Joss and Gori (1978) believe that ''stable'' drop size distributions appear only after combining several 1-min distributions. On the other hand, others (e.g., Sheppard and Joe 1994) describe the detailed features of 1-min distributions. Our third question, then, is, Are all of these ''true'' drop size distributions in the sense of physical entities exhibiting correlations among the different drop sizes? This and the first question are considered in the next section.

2. Measurements and interdrop correlations

Before proceeding, it is worth noting that the mathematical description of a drop size distribution depends upon accurately estimating the arithmetic mean concentration of drops in each of the different size bins. However, when estimating any temporal arithmetic mean, it is usually assumed that

1) the process is stationary (specifically, the local mean does not vary) and
2) the samples are statistically independent.

If these conditions are satisfied, then the distribution of the arithmetic sample mean approaches a Gaussian probability distribution while the variance of the sample mean decreases inversely with the number of samples, in accordance with the Central Limit Theorem, regardless of the underlying probability density functions. Often, however, condition (2) may only be partially satisfied in so far as sequential ''samples'' of the mean are correlated during the interval it takes to reach the ''time to independence.'' As an example from a different field, estimates of the mean radar reflectivity factor, Z, are computed assuming that condition 1 is satisfied during the sampling period using a certain number of ''correlated'' samples having a time to independence defined by the autocorrelation function (e.g., Atlas 1964). Here it is shown that raindrop counts are also similarly correlated in time.

In this study, the fundamental data are disdrometer counts of the number of drops of a given size for many different size categories. The data are from a Joss–Waldvogel disdrometer (Joss and Waldvogel 1967) located at Wallop's Island, Virginia, at the National Aeronautics and Space Administration (NASA) Flight Facility. This instrument determines the size of drops as they impact a surface, places them in a bin identified by a mean drop size, and counts the number of drops in every bin over a sample period, typically 1 min. However, drop impacts produce a ''ringing'' in the detector that not only causes a ''dead time'' in the instrument but also induces a decorrelation among size bins and within each bin as a function of time. For each bin, this dead time depends upon the flux of similar and larger sized drops. Thus, smaller drops will be most strongly affected, so that throughout this work only drops larger than about 1.11-mm diameter are considered. While the counts can be adjusted (see Sheppard and Joe 1994), this correction scheme introduces correlations among the various drop sizes as we shall show below. Consequently, in a statistical study such as this, we, like others (e.g., Tokay and Short 1996), adopt the more conservative approach of not correcting for this dead time.

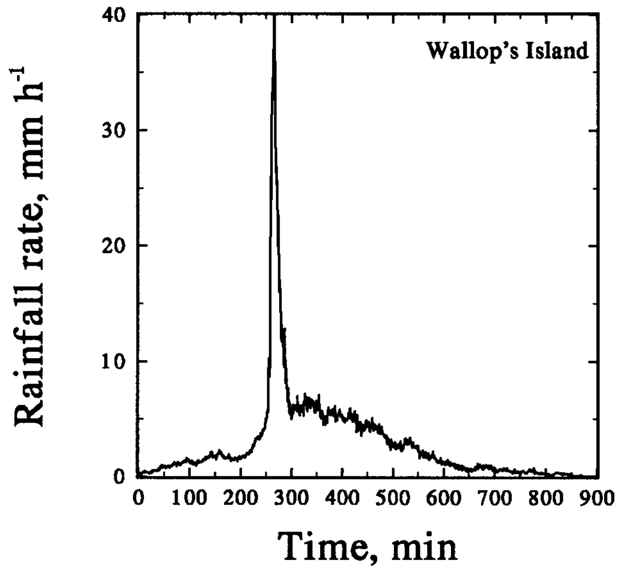


FIG. 1. The 1-min rainfall rates measured during the passage of a squall line at Wallop's Island, Virginia.

Specifically, in this work data collected during the passage of a squall line over Wallop's Island, Virginia (Fig. 1, also see Fig. 4 in Kostinski and Jameson 1997, hereafter referred to as Part I), are considered. These data consist of two parts, the first from 240 to 300 min being dominated by rain in a thunderstorm and the second (300–778 min) being characteristic of postconvective, stratiform rain.

Using these data and the concept of the excess two-point autocorrelation function for drops of one size, it is shown in Part I that the number of drops, k, occurring in interval Δt at time 0 and those occurring during Δt at a later time t are correlated. Specifically, the excess two-point autocorrelation function [a slight modification of that in Peebles (1980)] is defined by

$$\eta(t) \equiv \frac{[\overline{k(0)k(t)} - \mu^2]}{\mu^2} = \frac{\overline{k(0)k(t)}}{\mu^2} - 1, \qquad (1)$$

where μ is the long-term mean over the entire period of observations. In reality, the drop flux is always random. While the drop flux varies from moment to moment, for a statistically stationary process the average number of drops during Δt over the entire observation period T still remains constant. That is, assuming $k(0)$ and $k(t)$ are statistically independent, as

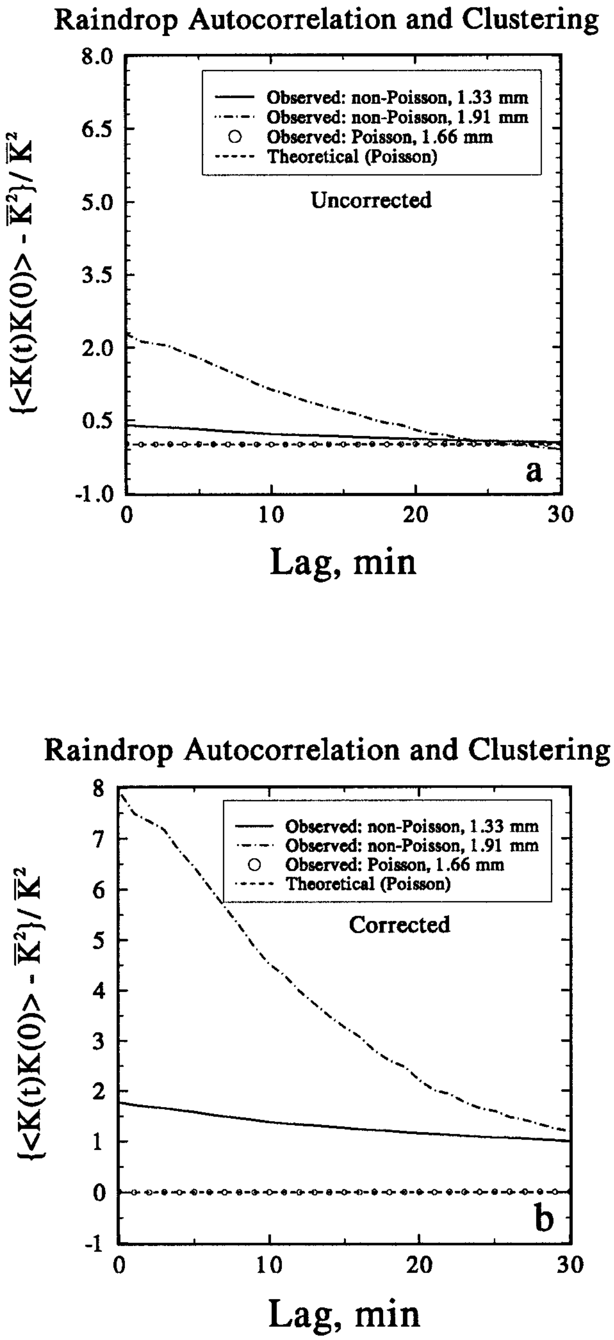


FIG. 2. The two-point excess autocorrelation function of drop counts per minute for the Wallop's Island data in Fig. 1 (a) using data not corrected for the disdrometer dead time and (b) using data corrected for dead time (after Fig. 11 in Kostinski and Jameson 1997).

they would be for a Poisson distribution, $\overline{k(0)k(t)} = \mu^2$ so that $\eta(t) = 0$. On the other hand, a positive $\eta(t)$ indicates and, indeed, defines bunching as discussed toward the end of this paper. This is what is shown in Part I and is illustrated here in Fig. 2.

In this plot ''corrected'' means that the scheme given in Sheppard and Joe (1994) was used to account for the dead time of the disdrometer. Clearly, such a correction enhances the correlations beyond those shown in Fig. 2a so that using the ''uncorrected'' measurements represents a conservative underestimate of the actual correlation. Yet both parts of Fig. 2 show significant excess correlation, albeit decreasing in time. Because of this decreasing correlation, Kostinski and Jameson (1997) define a time to independence that varies depending upon the meteorological conditions. Consequently, Kostinski and Jameson (1997) define a ''rain'' patch with respect to drops of one size. That is, in a manner exactly analogous to the radar estimation of a mean Z, stationarity applies within a ''patch,'' but the sequential samples are correlated so that the uncertainty—that is, the variance of the sample mean estimate—does not decrease as rapidly as $1/N$, where N is the number of samples of the mean concentration used in the estimate.

However, the fundamental question here goes beyond these previous results, namely: Are the counts of drops of different sizes correlated in time? If so, it can be argued that the concept of drop size distribution applies to collections of interacting drops in the atmosphere. To investigate, we modify (1) to include different size drops. Accordingly, $\eta(t)$ now becomes $\Omega(t)$ given by

$$\Omega(t) \equiv \frac{[\overline{k_1(0)k_2(t)} - \mu_1\mu_2]}{\mu_1\mu_2} = \frac{\overline{k_1(0)k_2(t)}}{\mu_1\mu_2} - 1, \qquad (2)$$

where k_1, μ_1 and k_2, μ_2 correspond to two different drop sizes. If k_1 and k_2 are statistically independent, then $\Omega(t) = 0$ whereas if $\Omega(t) \neq 0$, k_1 and k_2 are correlated. Results for the Wallop's Island data are shown in Fig. 3.

Clearly, *the counts among different drop sizes are correlated in time* implying that drop size distributions (as collections of interacting drops) are probably real. In other words, the answer to our first question is ''yes,'' drops of different sizes in the real atmosphere are correlated over some timescales as one might expect for a drop size distribution. However, in response to our third question, Fig. 3 shows that these correlations obviously decrease with time implying that there will be little if any correlation among drop sizes after the observation time, T, exceeds some coherence time, τ, for a drop size distribution. When $T \gg \tau$, the calculated drop size distribution *cannot* be viewed as a consequence of physical interactions but instead becomes a convenient formal description of the average relation among mean values of drop concentrations. Thus, seasonal drop size distributions should not be viewed the same way as a collection of physically interacting drops simulated in numerical models. Moreover, such long-term, average distributions may not exist in reality.

While Fig. 3 suggests that coherence times can be defined for drop size distributions, what do they represent and what are their values? The latter part of this question is investigated first in the next section.

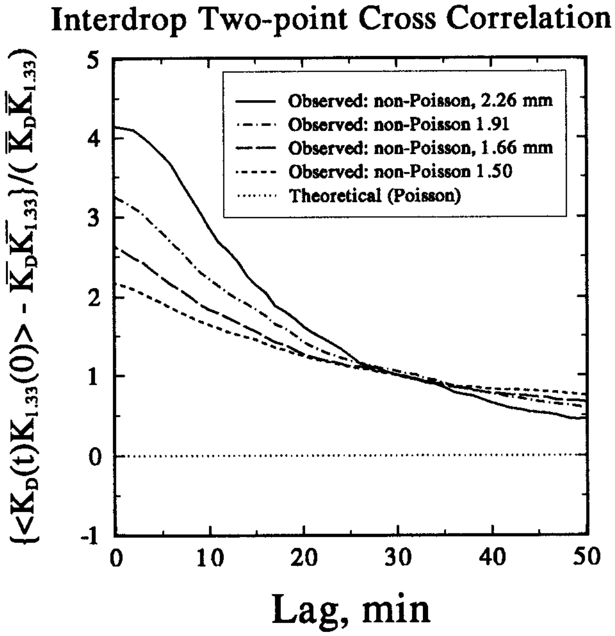


FIG. 3. The two-point excess cross correlation of raindrop counts between 1.33-mm size drops and those at the other indicated diameters.

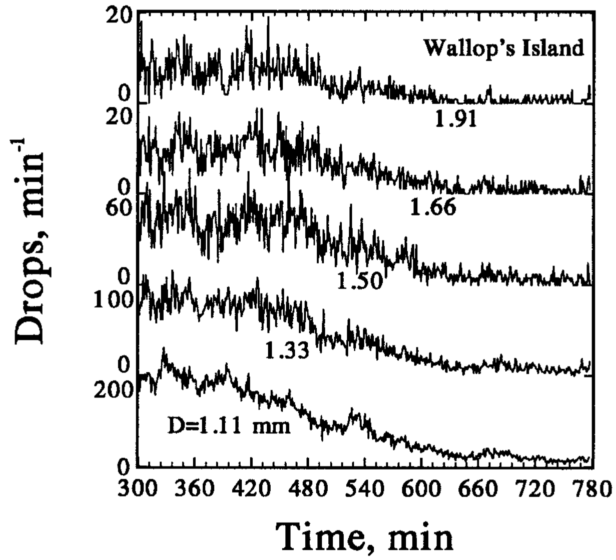


FIG. 4. A stacked time series of disdrometer counts of the number of drops per minute of the indicated mean size detected after the passage of a squall line over the NASA Flight Facility at Wallop's Island, Virginia. The scales corresponding to each drop size are indicated on the left with (0,200) at the bottom up to (0,20) at the top for the 1.11- and 1.91-mm drops, respectively.

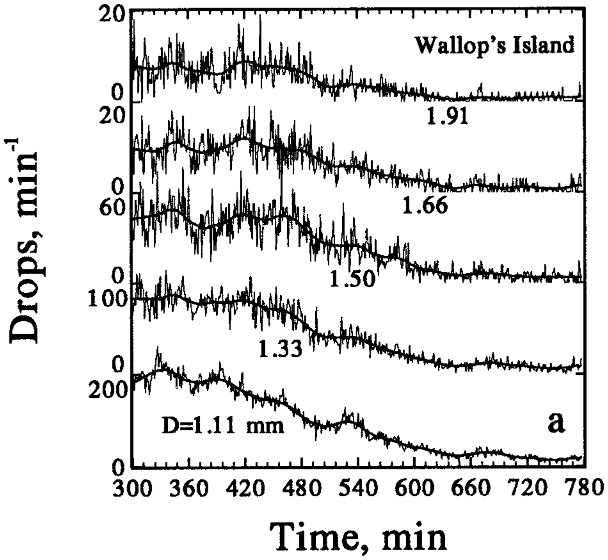
3. Coherence times for drop size distributions

Results in the previous section suggest that there appears to be a coherence time (τ) over which drop size distributions change so slowly that they may be considered nearly statistically stationary—that is, it is now reasonable to define a drop size distribution patch analogous to the rain patch defined in Part I. To estimate a coherence time for a drop size distribution, it is necessary first to estimate mean concentrations at the different drop sizes as discussed earlier. The only approach that the authors are aware of is that of calculating mean values over fixed time intervals (blocked observation times). For reasons discussed later, we instead extract estimates of mean drop concentrations using multiple time series analyses.

An example of these data is illustrated in Fig. 4 as a stacked time series of drop counts per minute during the second (stratiform) time period for sizes ranging from the smallest used in this study (1.11 mm) up to the largest (1.91 mm) occurring with significant frequency after the passage of a thunderstorm. While the fluctuations are obvious, so is the correlated behavior among neighboring times. Rather than throwing away this information, as is usually done when treating each observation time block as an isolated entity, a local weighted, least squares error algorithm (Cleveland and Devlin 1988) is used to define curves of mean number of counts per minute (fits) as functions of time. These fits are then superposed on the times series data as illustrated in Fig. 5a. Even at this stage of the analysis, there appear to be correlations among these fits at different sizes. That is, ''peaks'' and ''valley'' are present nearly simultaneously at several different drop sizes.

Subtracting these fits from the time series data then leaves time series of fluctuations. As Kostinski and Jameson (1997) found for single-sized drops, however, the mean values of these fluctuations are indeed near zero and are uncorrelated. Furthermore, it is shown here that the fluctuations among the different drop sizes are also uncorrelated. Specifically, Fig. 6 illustrates the correlation coefficients among the fluctuations and the mean counts (fits) at all different sizes for these time series data. In all cases the fluctuations are indeed uncorrelated on the 1-min timescale. This is significant for two reasons. First, these results imply that *one cannot simply take 1-min measurements of drop counts and expect to compute meaningful ''instantaneous,'' 1-min distributions because the fluctuations will likely mask any underlying drop size distribution.* Second, the very high correlation among the mean values (fits) over all of these different drop sizes suggest that at times there is indeed coherent behavior among different sizes of drops, reaffirming the results in Fig. 3. It is important to remember, however, that the results in Fig. 3 are derived entirely from the ''raw'' data and not the processed data illustrated in Fig. 5.

An analysis of fluctuations is used in order to estimate the ''coherence'' times associated with such behavior. To start, an ensemble of time series of these fluctuations is computed in which each member of the ensemble corresponds to different time blocks or observing times. That is, rather than taking 1-min data, successive counts are combined over 2 min, 3 min, etc., and a curve of



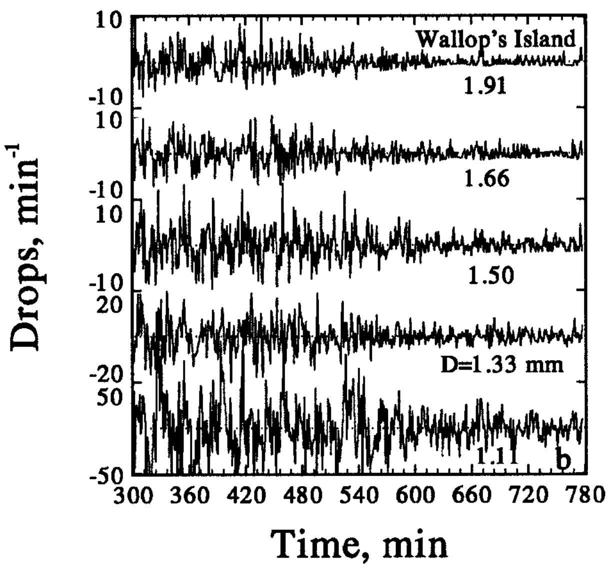


FIG. 5. Similar to Fig. 4 except that (a) is a plot of the least squares error mean curves corresponding to the different drop sizes and (b) is a plot of the residual fluctuations left after subtracting the mean curves from the raw time series data.

mean counts is fit. These are then subtracted from the reblocked data to calculate residuals. The idea is that when the time blocks begin to exceed τ, the residuals should no longer be entirely statistical fluctuations but should now begin to incorporate some of the variations of the "real" average structure. Consequently, assuming such structures should apply equally to all the drops (as

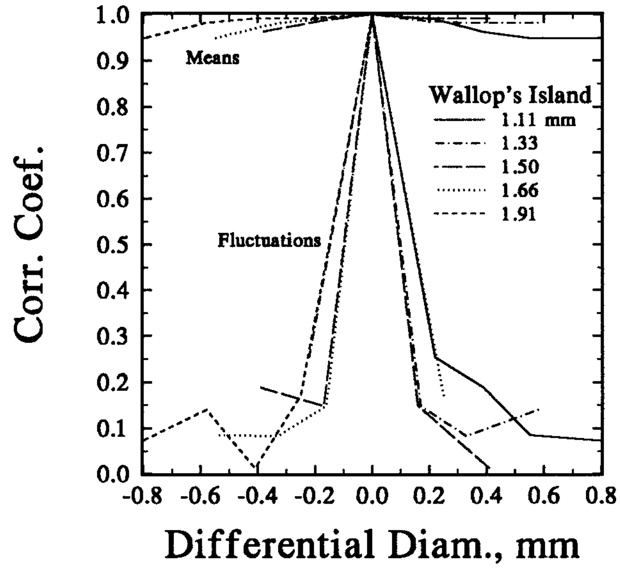


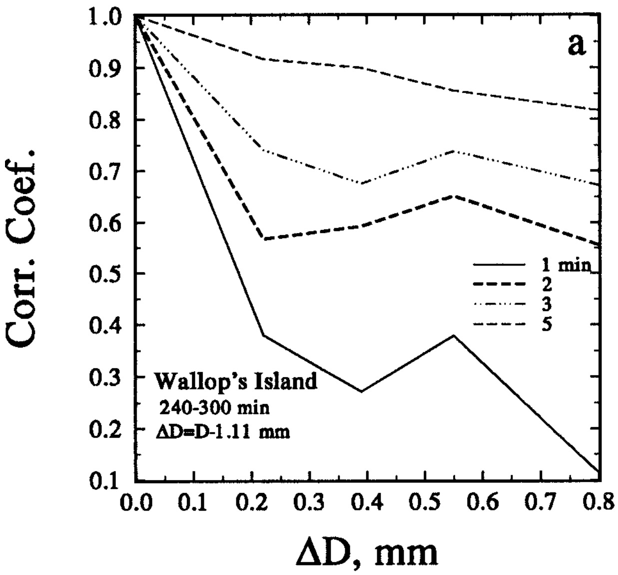
FIG. 6. The correlation coefficients among the different drop sizes for the mean values (top) and residual fluctuation components plotted in Fig. 2. The differential diameter is the difference in diameter between other drop sizes and the selected drop size listed in the legend. Note that the correlation coefficients for the fluctuations indicate that even neighboring drop sizes are uncorrelated while the means are all highly correlated.

Fig. 6 suggests), the residuals among different drop sizes should begin to correlate as the time block exceeds τ. Indeed, that appears to be the case as Fig. 7 illustrates.

The results for the thunderstorms in the squall line are shown in Fig. 7a, illustrating the correlation coefficients among drops as functions of the differential diameter ΔD. During this time, it appears that the residuals quickly decorrelate so that $\tau < 2$ min. This is not surprising since in thunderstorms one would expect to find rapidly changing mean counts over the many different scales of convection. (These short coherence times, however, have some interesting implications that are best discussed at the end of this work.) In contrast, during the stratiform rain (300–778 min), when conditions change much more gradually, Fig. 7b indicates that $10 \leq \tau \leq 15$ min. That is, the coherence times are much longer. This, in turn, suggests that the sampling times used to estimate drop size distributions must be adjusted depending upon conditions. Before exploring this in more detail, however, it is first necessary to refine our understanding of the statistical characterization of rain.

4. A statistical characterization of rain and the meaning of drop size distribution coherence times

It is widely assumed that rain drops are distributed in space as evenly as randomness permits—that is, in accordance with Poisson statistics [e.g., see the discussion in Kostinski and Jameson (1997)]. Consequently,



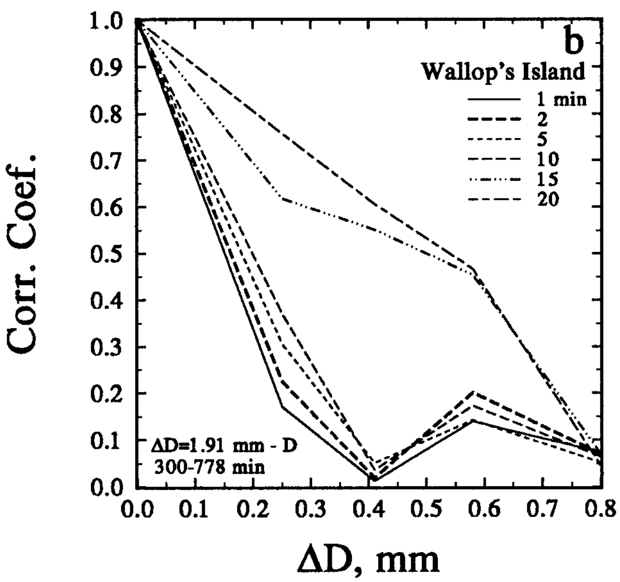


FIG. 7. The correlation coefficients of the residual fluctuations between the indicated drop size and the other drop sizes plotted as a function of the differential diameter, *ΔD,* for different sized time blocks (i.e., observation times) (a) during the passage of the squall line and (b) in the subsequent more stratiform rain. The coherence time, τ, is less than about 2 min in the squall line but increases to $10 < \tau < 15$ min in the stratiform rain.

it is assumed that the process is statistically stationary in time and/or statistically homogeneous in space and that events in nonoverlapping intervals (volumes) are statistically independent and, therefore, uncorrelated. For M drop categories, rainfall is then implicitly viewed as the superposition of M, independent Poisson distributed drop fluxes (e.g., Smith et al. 1993), and each 1-min drop size distribution is then just one realization of this random process. An ''average'' drop size distribution is then calculated from the arithmetic means of the M fluxes estimated from N samples in time (rain flux at the ground) or in space (an aircraft flying through a cloud with instruments having a known sampling volume). These assumptions are implicit, for example, in the classic work by Cornford (1967) in which the Poissonian statistical process is assumed in order to determine how many drops must be counted so that estimates of the mean concentrations and, hence, the drop size distributions fall within desired limits of accuracy.

However, it now appears that this perspective is not quite correct. Although Lovejoy and Schertzer (1990) most recently questioned the applicability of Poisson statistics to the spatial distribution of raindrops, their evidence is not convincing (see appendix). However, Kostinski and Jameson (1997) show that because the number of drops are correlated in time (e.g., see Fig. 2 in this paper) not only is convergence to the mean concentration slower, but more importantly the assumption of statistical independence of drop counts in nonoverlapping intervals underlying the Poisson distribution does not apply. Consequently, the distribution of drop counts is not Poisson. Moreover, they argue that the observed distribution of counts is the result of a *doubly stochastic Poisson process* in which the parameter of the Poisson distribution (the mean number of counts, $\bar{k}$, over interval Δt) fluctuates randomly as described by a probability density function $f(\bar{k})$. The statistics of the process is then described by the ''Poisson probability mixture'' and the net result is that the distribution of counts of drops of a single size is *not Poisson* but a much broader distribution. That is, the variance of the observed distribution of counts is greatly enhanced by the variance of $\bar{k}$. (In their studies, the distribution of counts was often well characterized by the geometric distribution corresponding to an exponential distribution of $\bar{k}$.) Hence, each of the M fluxes is described by a Poisson mixture distribution function each having its own $f(\bar{k})$ and its own coherence time τ_M.

One among several factors that contribute to the existence of $f(\bar{k})$ is drop interactions. In retrospect, it is perhaps not surprising, then, that these M fluxes are not statistically independent—that is, that the counts of drops at different sizes are correlated in time as Fig. 3 shows. To further emphasize this point, suppose we do assume M independent fluxes. What would that imply about the statistics of the count of the sum total number of drops of all sizes? Under conditions quite similar to those of the Central Limit Theorem—for example, that the different fluxes at different sizes are uncorrelated, that none dominate, etc.—statistical theory (Kovalenko et al. 1996) shows that a *sum of independent fluxes of random counts tends to a Poisson distribution regardless of the underlying probability density functions of the individual components.* Hence, if rain consisted of

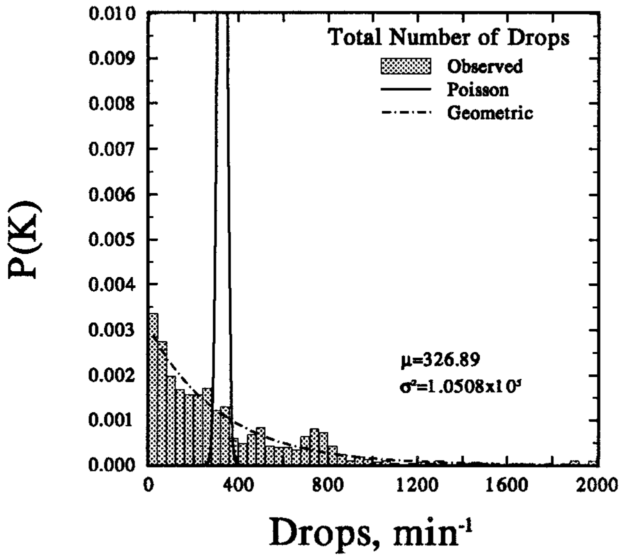


FIG. 8. Histogram of the total number of drops each minute compared to that expected for a Poisson distribution and a geometric distribution $p(K) = [1/(1 + \mu)][\mu/(1 + \mu)]^K$ for the indicated mean value, μ.

M independent fluxes, the statistics of the sum total should tend toward a Poisson distribution regardless of the probability density functions of each of the M contributing fluxes.

In Fig. 8 the histogram for the 1-min sum total counts obviously deviates significantly from a Poisson distribution. The likely reason is that the M fluxes are not independent (Fig. 3). Interestingly, this observed distribution (Fig. 8), like the results in Part I for single size drops, is well described by a geometric distribution. One explanation is that for each drop size distribution patch characterized by near stationarity, the sum total counts are nearly Poisson distributed as the above theorem suggests. However, the observed distribution includes results from many such drop size distribution patches so that the identical conclusion of Kostinski and Jameson (1997) for single size drops applies as well to the total number of drops—namely, that the observed distribution of the sum total counts is a also likely the result of a doubly stochastic Poisson process in which $f(\bar{k})$ is well approximated by an exponential distribution. These results then lead to *the statistical characterization of the rain as a doubly stochastic Poisson process (Poisson mixture) of* M-*correlated random variables (fluxes) each having its own* $f(\bar{k})$ *and its own coherence time.*

This characterization is important. For example, let the shortest coherence time associated with one of the M fluxes of a drop size distribution be τ_1 and that, for the longest coherence time be τ_2 so that $\tau_2 \gg \tau_1$. Then if the total observation time $T \gg \tau_2 \gg \tau_1$, the measured distribution will obviously be a mixture of this as well as some other drop size distributions. Thus, T obviously exceeds the coherence time of the drop size distribution. On the other hand, if $\tau_2 \gg \tau_1 \gg T$, then the observations should accurately reflect the drop size distribution. In that case T is clearly less than the coherence time of the distribution. The difficulties arise when $\tau_2 \gg T \gg \tau_1$. Then part of the observed distribution will represent a single size distribution, but the other part will be a mixture of drops from more than one distribution. Thus, the coherence time must be equivalent to the shortest coherent time of all the M fluxes measured if the purity of a physical drop size distribution is to be preserved. This has important implications with respect to the measurement of drop size distributions as discussed and illustrated below.

5. On the measurement of drop size distributions

In order to preserve the purity of a physical drop size distribution, then, it is necessary that $\Delta t \ll T \ll \tau$. But if τ then happens to correspond to a smaller drop size, the scarcer larger drops may be undersampled, for example. On the other hand, if one were to follow the logic in Cornford and increase T in order to get ‘‘good’’ estimates of counts at larger sizes, then $T \gg \tau$ and the measured distribution would represent a mixture. Moreover, in this latter case the measured, but distribution mixed, concentrations at smaller sizes would actually become a function of T (i.e., ill-defined) and less reliable even though the number of sampled drops is increased! This is illustrated below.

Figure 9a shows the effect of different blocking (averaging) times on the estimates of the variance and mean value of drops of size 1.11 mm during the passage of the thunderstorm when $\tau < 2$ min. In this case, as T increases beyond a few minutes, it quickly exceeds τ, and the variance increases rapidly while the mean value continually changes. The same holds in the postconvective rain as well (Fig. 9b). After 15–20 min, the variance increases dramatically as the mean value wanders.[1]

Moreover, these estimates lose statistical robustness because of the greatly enhanced variance. That is, unlike the normal expectations of decreasing sample variance with increasing sample size when $\Delta t \ll T \ll \tau$, the statistics of the counts corresponds to that of a Poisson mixture probability density function (Part I) when $T \gg \tau \gg \Delta t$. For such a mixture, as Fig. 9 illustrates, the variance actually increases as more drops are counted because the variance of the field of mean values (‘‘meteorological’’ variance) now contributes significantly.

However, while Fig. 9 corresponds to a single drop

[1] Note that the mean wanders even during times less than the coherence time. The reason for this is that the plotted random variable is a statistical estimate of the mean using partially correlated samples. Hence, there is significant statistical uncertainty and variability associated with such estimates. Nevertheless, the most likely ‘‘best’’ statistical estimate of the mean is that when the blocking time approaches but does not exceed the coherence time.

size, the same applies to other drop sizes as well. Thus, if one were to compute the slope of an exponential fit, say between the 1.11- and 1.91-mm size drops, Fig. 10 illustrates that increasing T beyond τ actually produces ill-defined slopes (i.e., the specific value depends upon T) and increasing uncertainty.[2]

If time-blocked data analysis is to be used, however, the observations need to be monitored, at least, in order to detect when mixing is likely to be occurring. One approach is to keep track of the variance of the data. That is, as long as the measurement interval $\Delta t \ll T \ll \tau$, then for each drop size the samples can be considered drawn from a Poisson distribution. Since a sum of independent Poisson-distributed random variables remains Poisson distributed (Ochi 1990), the variance should remain equal to the mean value. On the other hand, if $T \gg \tau$, then the mean value of the Poisson distribution corresponding to each sample during Δt can change unpredictably (randomly), and the probability density function (pdf) describing the entire observation becomes that of a Poisson mixture as discussed earlier. Consequently, when mixing occurs, the variance will exceed that for a Poisson distribution. In other words, significant deviations of the ratio of the variance to the mean (i.e., the Poisson variance) from unity can be used to detect deviations from Poisson statistics due to mixing.

This is well illustrated in Fig. 11a, and which clearly indicates mixing at 255 and 265 min, for example. For such lumpy mixtures, particularly when the sampling interval Δt approaches the coherence time as is the case for these data, it is likely to be difficult or perhaps impossible to measure a physical drop size distribution with any confidence.

Moreover, while such effects might be detected simultaneously for all drops as in Fig. 11a, this mixture signature may also appear at different times for different drop sizes (Fig. 11b). Consequently, at times, blocked data may be contaminated by mixing at some drop sizes but not others, as discussed at the conclusion of the previous section (i.e., $\tau_2 \gg T \gg \tau_1$).

As mentioned previously as well, an alternative to using fixed observation time blocks for determining mean values from a sequence of successive blocks is to consider all the drops simultaneously at all times using multiple time series analysis. Hence, in contrast to blocking data (information) at other times are not ignored but are used to provide more stable estimates of the means while minimizing fluctuations and reducing the contaminating effects of mixed data. This approach also provides an important method for achieving maximum temporal resolution using disdrometers and aircraft measurements provided $\Delta t \ll \tau$.

[2] For this purpose the uncertainty is given by the sum of the relative errors for each concentration (standard deviation/mean) divided by the difference in drop sizes.

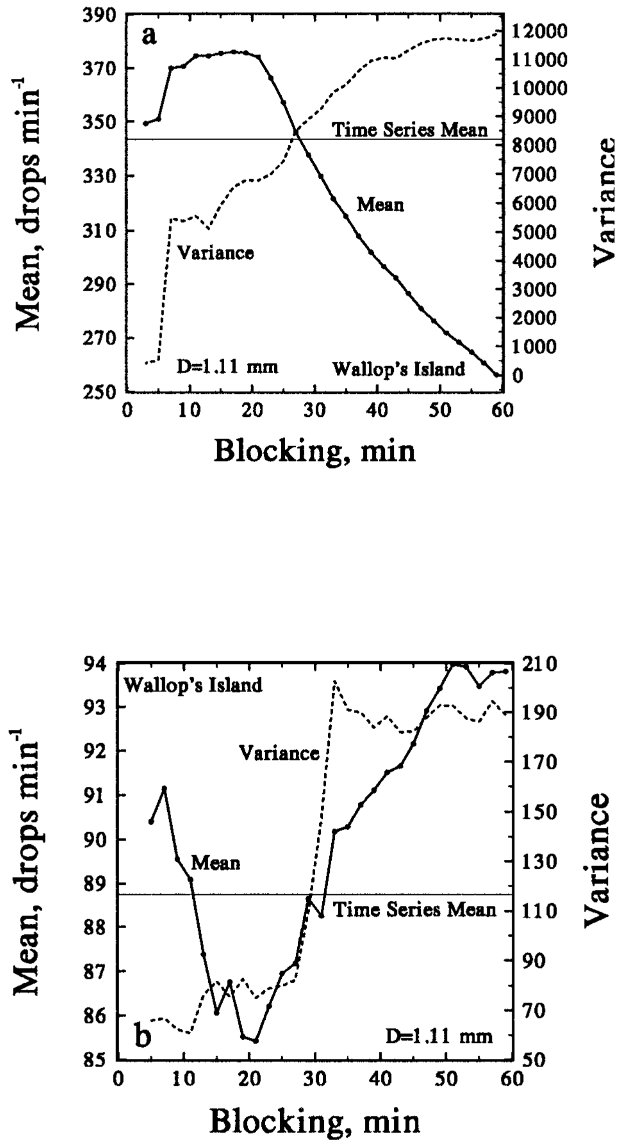


FIG. 9. Plots of the mean number of drops per minute and the variance for the 1.11-mm drop size as functions of different sized time blocks (i.e., observation times) for (a) the squall line and (b) post convection stratiform rain. Note that beyond the coherence times, the mean value is just a function of the observation time while the variance substantially increases with increasing averaging. The time series mean are those estimated from the time series analyses described in the text (e.g., see Fig. 5).

Where the mean values at different sizes are changing only slowly, it should then be possible to compute a statistically robust drop size distribution. On the other hand, where the mean values are obviously changing rapidly with respect to one another particularly on timescales approaching Δt, computed drop size distributions can be considered only formal relations among average concentrations.

As an example, if we were using a 5-min observation time during the 240–300-min period of the Wallop's

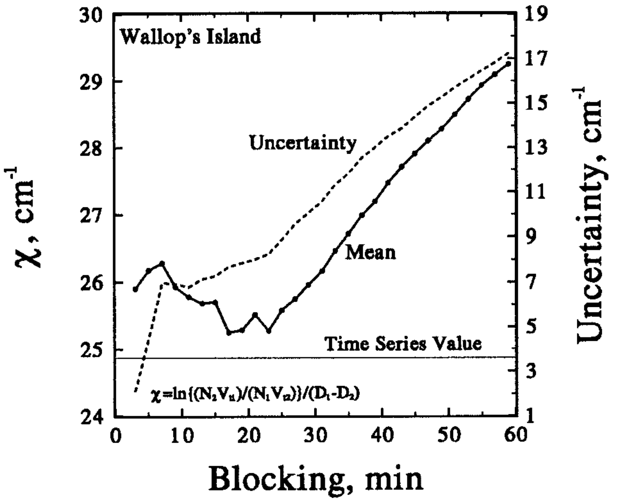
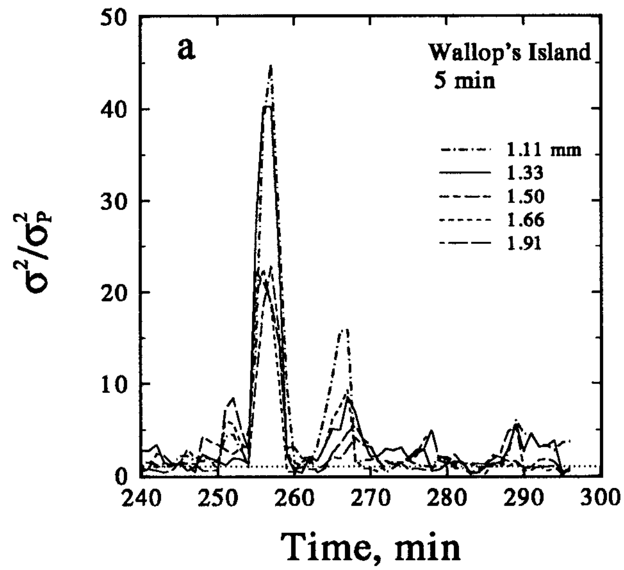


FIG. 10. The slope (χ) of an exponential fit between 1.11- and 1.91-mm drops and the associated uncertainty as a function of increasing observation time during the convective part of the squall line. Beyond the coherence time, more averaging produces increasing uncertainty and slopes that are functions the averaging time. The time series value is calculated using the time series estimates of the mean concentrations at the two sizes. Here, $N_{1,2}$, $V_{t1,2}$, and $D_{1,2}$ are the counts, terminal fall speeds, and diameters corresponding to the two different drop sizes.



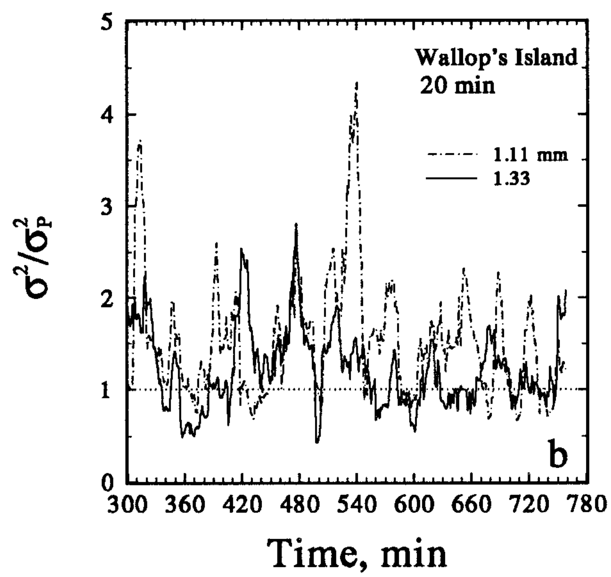


FIG. 11. The ratio of observed variance σ^2 to σ_p^2, the Poisson equivalent variance (i.e., equal to the observed mean value) (a) during the squall line and in the stratiform rain (b) for the indicated observation (block) times. Values significantly greater than unity indicate locations where statistical mixing is occurring.

Island data, Fig. 11a indicates that any calculated drop size distribution would likely be meaningless at 255, 259, or 260 min because the mean values themselves are changing on timescales much smaller than T and, in fact, are changing on the order of Δt. Yet, if we look at the time series mean values (Fig. 12) during periods between rapid changes, it is possible to pick out some intervals (such as during 260–265 min) when it might still be possible to compute a robust drop size distribution using the time series mean values because they are changing relatively slowly.

On the other hand, it is clear from Figs. 11 and 12 that there are times (e.g., at 255, 259, and 265 min) when conditions are changing so rapidly that it may not be possible to compute a statistically robust drop size distribution even using time series analyses. Here, Δt is simply too large.

6. Results and some concluding remarks

To summarize, we return to our three initial questions, namely: Are drops of different sizes correlated in the real atmosphere? How much averaging is appropriate in order to reduce statistical fluctuations? Are drop size distributions computed using measurements averaged over seasons down to minutes all true drop size distributions in the sense of physical entities exhibiting correlations among the different drop sizes? The answers are that, yes, over sufficiently short durations (time less than the coherence time of the distribution), drop counts at different sizes are correlated indicating that size distributions likely exist as physical collections of interacting drops. However, the drop size distribution is likely only coherent for the shortest coherence time of the M fluxes of the different drop sizes so that overaveraging (averaging over times longer than the coherence time) transforms the concept of the drop size distribution from one of a physical collection of interacting drops to a

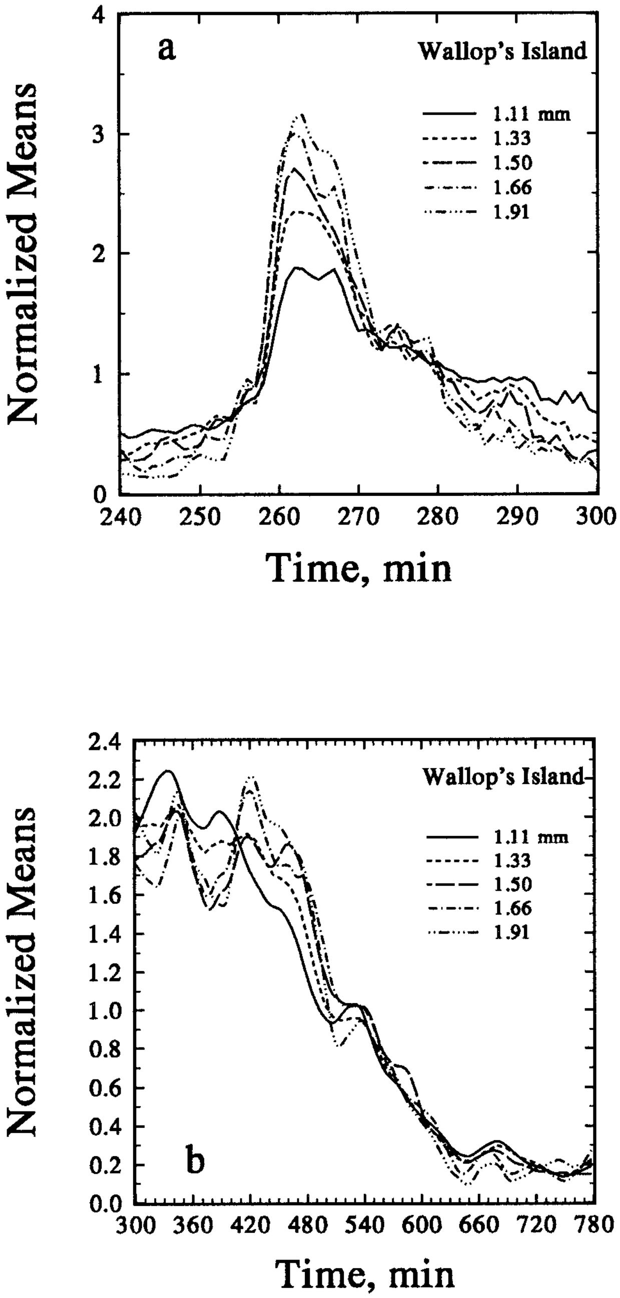


FIG. 12. The time series–derived mean number of drops per minute corresponding to the different drop sizes and normalized by each of their overall respective mean values during the indicated period for (a) the squall line and (b) the stratiform rain.

formal relation among the M-observed average drop concentrations.

While a problem for those trying to define drop size distributions everywhere, all the time, perhaps as importantly, such coherent, correlated behavior suggests that at times on many scales, there may be a physical clustering of raindrops associated with the statistical mixing of drops. This may be important to many problems, such as understanding the effects of small-scale precipitation loading on cumulus dynamics, understanding the onset of multiple scatter of microwaves passing through rain, and understanding the effects of precipitation fluctuations on the signal statistics of all scanning remote sensing devices for measuring precipitation, from lasers to radiometers (Jameson and Kostinski 1996).

So what is this clustering? This is visualized using a geometric (borrowed directly from astronomy) rather than a statistical interpretation of the two-point correlation functions illustrated in Figs. 2 and 3. Specifically, to see if raindrops are ‘‘clustered’’ one begins with the product of the number of drops in two volumes separated by a fixed distance, ζ, and then subtracts the square of the average value computed over the entire volume under study, V, for an ensemble of several such pairs in V. That is, let us consider the quantity

$$\phi(\zeta) = \langle k(0)k(\zeta)\rangle - \mu^2, \tag{3}$$

where $k(0)$ and $k(\zeta)$ are the number of raindrops in two identical volumes separated by distance ζ, μ is the mean number over V, and the brackets denote an ensemble average. [Note that (3) is essentially the numerator in (1) after substituting ζ for t.] If the number of drops in two separated volumes are distributed uniformly, then $\langle k(0)k(\zeta)\rangle = \mu^2$ so that $\phi(\zeta) = 0$. If, on the other hand, the number of drops in volumes separated by scales of ζ tend to cluster (due to intermittent turbulence, for example), $\phi(\zeta)$ would then deviate from zero, that is, $\langle k(0)k(\zeta)\rangle \neq \mu^2$. In other words, there would be a ‘‘bunching’’ of raindrops compared to the average number expected for a uniform, statistical spatial distribution over V. While spatially separated measurements are required to compute (3), Figs. 2 and 3 suggest that such clustering is occurring simultaneously (at times) over many (but not necessarily all) spatial scales and over many drop sizes.

The qualifying parenthetical expressions in the previous sentence are important. In particular, clustering need not always occur, as Fig. 10 and 11 in Part I as well as Fig. 11 in this work illustrate. Because deviations from Poisson are a prerequisite for fractal structures, such evidence of nonclustering, Poissonian structure conflicts with any ubiquitous fractal description of rain. Figure 11 suggests that the ratio of variances may then become a convenient research tool because it can be used readily to detect deviations from the Poisson distribution (clustering) over the relevant scales.

Moreover, in contrast to a formal fractal geometric description, in this study (Parts I and II) we offer an alternative probabilistic characterization of rain. In particular we

1) introduce and identify three relevant timescales and discuss them in the context of appropriate meteorological regimes;
2) identify, obtain, and analyze data most relevant to

the internal characterization of rain (i.e., drop counts);

3) offer a physical explanation for the fluctuation enhancement in the framework of a doubly stochastic Poisson process caused by turbulent mixing, as well as other factors; and
4) discuss important implications of such enhanced fluctuations for droplet growth and size distribution development.

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APPENDIX

Apparent Fractal Dimension of Poisson Distributed Raindrops

To quantify the spatial distribution of raindrops, Lovejoy and Schertzer (1990) resurrected an old technique for determining drop size distributions, namely exposing surfaces of dye coated paper briefly to rain. They then used the dye stains left by the raindrops for various analyses. In particular they analyzed one such observation having 452 drops scattered over a 128 cm × 128 cm surface and found fractal dimensions ranging from 1.93 down to 1.79 with a preferred value of 1.83.

Because the Poisson distribution is expected to yield the most uniform, random distribution of drops (due to statistical independence of nonoverlapping volumes), the fractal dimension for a Poisson distribution of drops on a surface should be close to 2.0. Hence, observations of fractal dimensions differing markedly from 2.0 are used as evidence for deviations from Poisson statistics. While the results in our present study indicate that Lovejoy and Schertzer may be correct sometimes, we shall demonstrate that the described experiment is insufficient to establish real departures from a Poisson distribution.

Specifically, as a Monte Carlo simulation, we take 453 drops and disperse them randomly over a 128 cm × 128 cm surface in accordance to a Poisson distribution as illustrated in Fig. A1. This figure is quite similar to Fig. 1 of Lovejoy and Schertzer.

We then calculate the fractal dimension of such a dispersion of drops following the approach of Lovejoy and Schertzer. That is, we first compute the number of "drops" in a circle of radius r for all the drops. We also then calculate the dimension for drops lying between logarithmically spaced annuli. These approaches are standard for calculating the dimension of "percolation fractals" (e.g., Schroeder 1991), for example. The results are shown in Fig. A2.

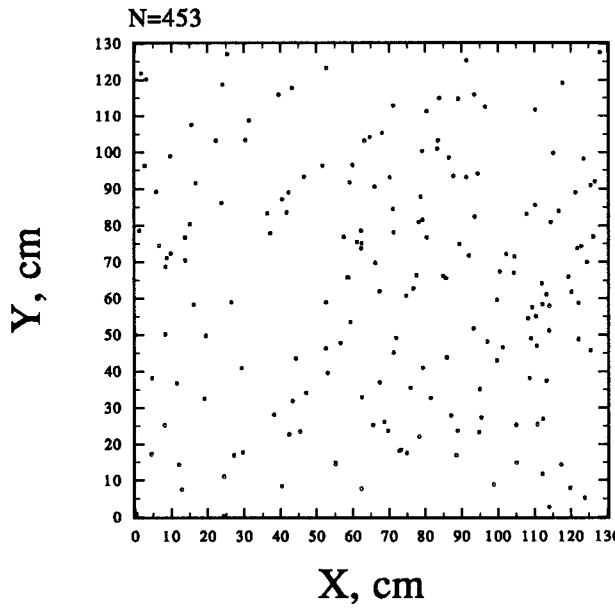

FIG. A1. Poisson Monte Carlo simulation of 453 raindrops distributed over a 1.28 m × 1.28 m area.

Amazingly, we get the same fractal dimensions as reported by Lovejoy and Schertzer. Since this simulation is consistent with the Poisson distribution, we conclude that the existence of the fractal dimension need not be

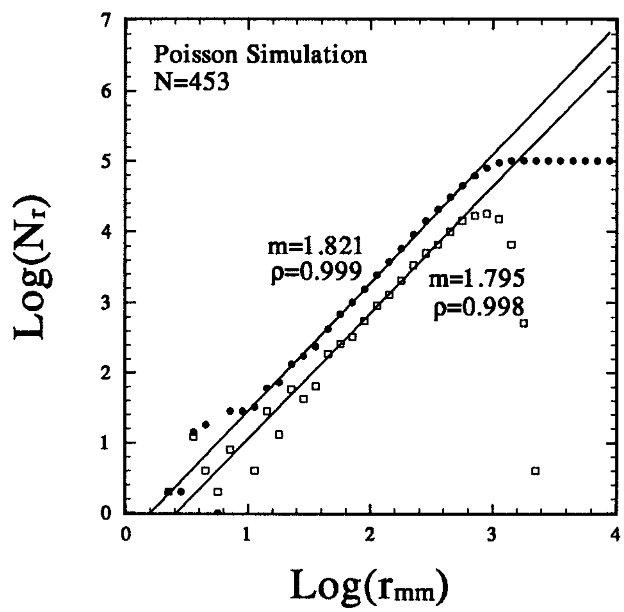

FIG. A2. Plots illustrating calculation of the fractal dimension using the simulation in Fig. A1 computed using the average number of drops in a circle of radius r (solid circles) and between logarithmically spaced annuli [open squares, after Lovejoy and Schertzer (1990)] The dimension found in this simulation agrees with that of Lovejoy and Schertzer.

the result of departures from Poisson, but may also simply be the result of an incomplete, finite sample drawn from a Poisson distribution. Thus, this experiment can no longer be used to support claims of fractal universality extending from the largest down to these small scales in rain.

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