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## NOTES AND CORRESPONDENCE

### When is Rain Steady?

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#### ABSTRACT

By definition, steady rain should have a nearly constant rainfall rate. Thus far, however, the criteria for determining when rain is steady remain qualitative and arbitrary. The authors suggest a definition for steadiness that can be used to quantify the elusive notion of natural variability. In particular, the logical criteria for steadiness imply statistical stationarity and lack of correlation between raindrops in neighboring volumes, requirements identical to those for the drops being distributed according to a Poisson process at all scales. Hence, steady rain is Poissonian. Explicit equations for the variance  $\sigma_R^2$  of the rainfall rates are developed. They show that, in general, raindrop clustering enhances the variance beyond that for Poissonian rain ( $\sigma_P^2$ ). It is also demonstrated by using observations that this enhancement is augmented further when the rain is statistically nonstationary. Identifying steady rain is important. To be specific, because steady rain is statistically stationary, the drop size distributions have physical, deterministic meanings independent of the measurement process. Observables such as the radar reflectivity factor and the rainfall rate are then steady and linearly related also. Techniques for determining when rain is steady are discussed. The ratio  $\sigma_R^2/\sigma_P^2$  is proposed as a useful quantitative measure of the steadiness of the rain. It is also shown that an estimate of the *minimum* possible standard deviation for steady rain is  $\bar{R}/\sqrt{\bar{k}}$  where  $\bar{R}$  and  $\bar{k}$  are the mean rain rate and average number of drops per sample, respectively. Examples using video-disdrometer data are also presented.

#### 1. Introduction

One of the highlights of the 1952 movie *Singin' in the Rain* is Gene Kelly dancing with his umbrella in a downpour while singing the hit song of the same name. This was not ordinary, natural rain, however, but a Hollywood simulation produced by sprinklers that generated a shower having a uniquely steady and, somehow, unnatural rain. What is it about that studio rain that made it so apparently artificial? One answer is that the rain was simply lacking natural variability. But, then, what exactly does that mean?

Aside from demonstrating a clear need to improve the realism of rain simulation capabilities in Hollywood, this concept of steady rain is often used as a benchmark to characterize the properties of rain. For example, List (1988) highlights the importance of observations in steady rain for determining drop size distributions and radar reflectivity factor–rainfall rate ( $Z$ – $R$ ) relations in

radar meteorology. Yet, steadiness is never defined in that paper.

A review of the literature reveals that, even at present, the identification of steady rain still remains elusive. For example, Sauvageot and Koffi (2000), in a study of observations of multimodal drop size distributions, assume that all fluctuations in  $R$  in excess of  $0.2 \text{ mm h}^{-1}$  arise from deterministic causes even for  $R$  of several millimeters per hour. But do they? Sauvageot and Koffi (2000) raise a number of important questions in their work, but the choice of the cutoff between variable and constant rain rates appears to be ad hoc. Here we attempt to provide such studies a more precise approach for defining steadiness quantitatively.

The question we address in this work, then, becomes, “Is there a more systematic approach for specifying with precision when rain is steady?” We believe that there is. With the tools developed below, it should then be possible to survey rain systematically in various kinds of meteorological conditions to identify those most commonly associated with steady rain, if it exists at all. However, we emphasize at the outset that the purpose

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of this article is not to perform such a survey but, rather, to develop the tools necessary for quantifying the steadiness of rain applicable to future research. Before proceeding, however, it is necessary first to define steadiness and to understand some important statistical characteristics of rain.

By definition, steady rain should be nearly constant. But what does that mean exactly? At the very least, steadiness logically requires that the mean values (and variances) should remain fixed; that is, they should not depend upon the point of origin of observations within the total space sampled. Rain cannot be steady if, in fact, the mean rainfall rate changes while being observed. In the words of stochastic theory, therefore, the rain should be *statistically stationary*. But that alone is not a sufficient condition for steadiness. In steady rain, the drops should also be distributed spatially as uniformly as randomness allows. Why? Because if this were not so, then drops would tend to arrive in clusters interspersed with voids relatively deficient in drops so that the rain rate would fluctuate rather than remaining steady. Such fluctuations from drop clustering can occur even in statistically stationary rain as discussed further below, so that we must have some measure for detecting drop bunching or clustering when searching for steady rain. As we will see, this measure is achieved by using the pair-correlation function. The absence of clustering is characterized, then, by having no correlation between the number of drops in neighboring volumes.

These two concepts (statistical stationarity and lack of pair correlation) led Kostinski and Jameson (1999, their footnote 5) to propose that steady rain is Poissonian rain so that any excess variance of the rainfall rate above that for an equivalent Poissonian rain can be attributed to natural variability. In the more applied framework of this work, we claim here that the rainfall rate will then be as constant as statistics will allow when the rain is Poissonian. So what is Poissonian rain?

Simply put, Poissonian rain is rain in which all the drops obey Poisson statistics on all scales. Poisson statistics require three assumptions (e.g., Ochi 1990). The first is that the probability of detecting more than one drop in a given volume  $\delta V$  is vanishingly small for sufficiently small  $\delta V$ . This first point can usually be satisfied in rain. The second requirement is that drop counts in nonoverlapping volumes must be uncorrelated, statistically independent, random variables (at any length scale, i.e., no matter how far the unit sampling volumes are separated). As we show below, this is equivalent to requiring that the pair correlation be zero at all lags. The third requirement for a Poissonian process is that it be statistically stationary.

To elaborate further on the second requirement, when we say that rain is correlated, we mean, in statistically stationary conditions, that the presence of a drop enhances (or in some cases decreases) the likelihoods that there are other drops of the same or different sizes in the neighboring volume. In other words, the drop counts in

neighboring volumes are correlated, indicative of clustering of the drops. (Here it is important to remember that such correlations are also produced if the mean values change as functions of scales.) To be more precise, the pair-correlation function  $\eta(l)$  measures the correlation between the number of drops in neighboring volumes separated at lag  $l$  (time or space) and is defined by

$$\eta(l) = \frac{[n(l)n(0) - \mu^2]}{\mu^2}, \quad (1)$$

where  $n$  is the number of drops in a sample volume and  $\mu$  is the mean number of drops across the entire measurement space. As an example using real data, in Fig. 1 we illustrate  $R$  for two brief rain events using videodisdrogrometer observations described in more detail in Jameson et al. (1999, p. 83). The rainfall rates appear to be nonstationary, but for the purpose of discussion we first assume that the data are part of a much longer term statistically stationary series. Later, we present evidence suggesting the nonstationarity of these data.

For  $k$ , the total number of drops (summed over all drop sizes) in a unit sample volume, we then plot in Fig. 2 the quantity  $\eta_k$ , the pair-correlation function for  $k$ . Here we note that if the drops in each drop size category contributing to the total number obey a Poisson process (so-called Poissonian rain), then the sum of the number of drops over all the different drop sizes also obeys a Poisson process (e.g., Evans et al. 1993, p. 124). It is obvious in Fig. 2 that the  $\eta_k$  for the two rain events do not correspond to a Poisson process, although the time to decorrelation is much faster for the second, 773-s, rain event. Note too that, had the measurements been gathered using a temporal average of about 60 s during the briefer rain, the distribution of  $k$  would have looked to be Poisson even though the process is clearly not Poissonian.

As just mentioned above, correlation between the number of drops in neighboring volumes means that the drops are not dispersed uniformly but rather tend to occur in bunches. Locations rich in drops consequently are interspersed with relative "holes" that are sparser in rain so that the rain rates themselves must be clustered [for further discussion see Jameson and Kostinski (1999a), p. 3921 and p. 3931]. This distribution means that one can define a pair-correlation function for the rain rate,  $\eta_R$ , that is analogous to that for  $k$ . The results are illustrated for these two examples in Fig. 3. In clustered rain,  $\eta_R$  is zero only in passing, whereas  $\eta_R$  is precisely zero at all lags in Poissonian rain.

Thus, we see that all of the logic for rain being steady as discussed above satisfies all three statistical requirements for a Poisson process. In particular we conclude that Poissonian rain is steady rain and that for rain to be steady it must be Poissonian and, therefore, both statistically stationary and uncorrelated.

It is important to reemphasize here, however, that both of these conditions must be satisfied. In particular just

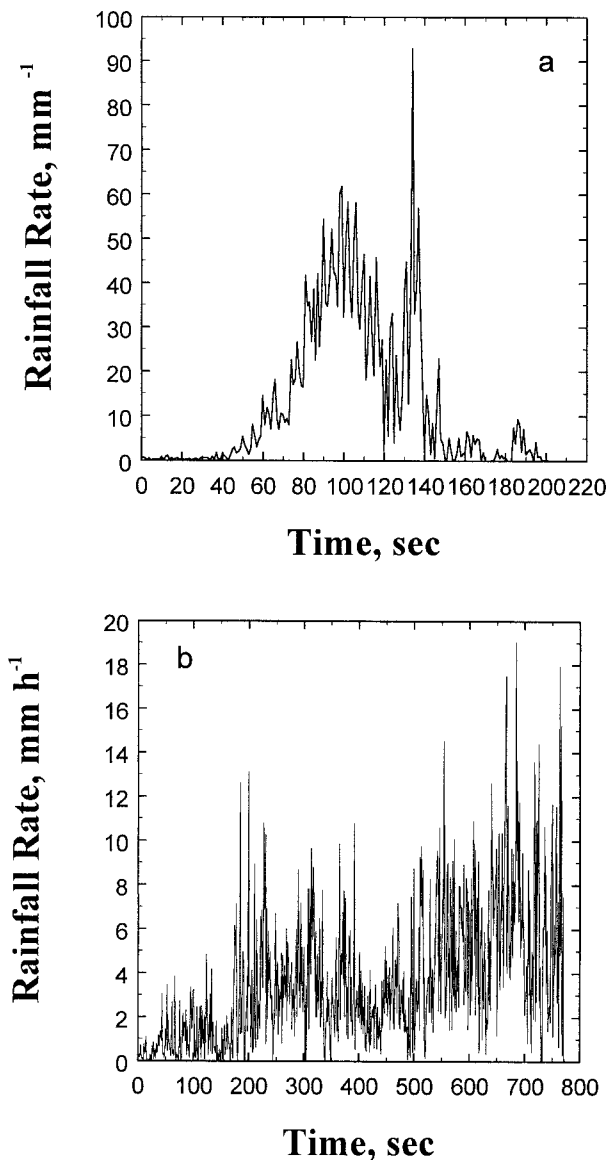


FIG. 1. The rain rates measured each second by the University of Iowa video disdrometer during (a) a brief convective shower (mean of 12.85 mm h<sup>-1</sup>, variance of 292.7 mm<sup>2</sup> h<sup>-2</sup>) and (b) a more extended, less convective rain (mean of 3.54 mm h<sup>-1</sup>, variance of 9.85 mm<sup>2</sup> h<sup>-2</sup>). From Jameson and Kostinski (1999a).

being statistically stationary alone is not sufficient. That is, *statistical stationarity* does *not* preclude significant *spatial variability* because of the presence of correlated fluctuations [see the discussion in the appendix in Jameson and Kostinski (2000)]. On the other hand, systematic changes in the random variable alone over relatively short distances (i.e., distances much less than a correlation length) should not be construed necessarily as a sign of statistical nonstationarity (i.e., data in which the mean and variance of a random variable do change throughout the set of data), because correlated fluctuations over scales of less than the correlation length can

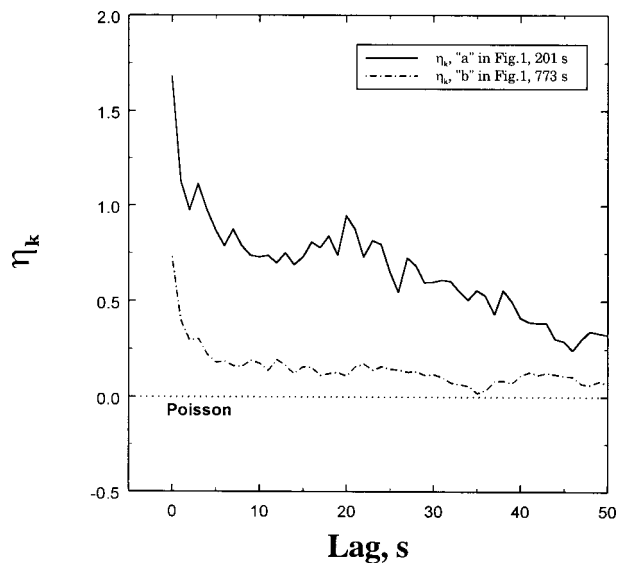


FIG. 2. The pair correlation for the total number of raindrops for the two rain events in Fig. 1. For a Poisson distribution,  $\eta_k = 0$  at all lags not equal to 0 so that the observed rains are clustered.

also produce apparent systematic changes, even in a statistically stationary process, as illustrated in Fig. 4.

The lack of correlation in Poissonian rain has important consequences, in particular with regard to the variance of  $R$ , because the variability is much larger in clustered as opposed to Poissonian rain, as Fig. 4 illustrates. A natural question, then, is whether some knowledge about the variances and/or correlations is sufficient for detecting reasonably steady rain without even having to test directly for statistical stationarity, which is a laborious process at best (see Jameson and Kostinski 2000, appendix). The answer is “yes”, as we explore in the remainder of this work, beginning with a stochastic definition of the rainfall rate described in Jameson and Kostinski (2001a).

## 2. Relative dispersions and variances of rainfall rates

The rainfall rate is defined by

$$R = \text{const} \times \sum_{i=1}^k D_i^3 V, \quad (2)$$

where  $D_i$  is the diameter of the  $i$ th drop,  $V$  is the terminal velocity corresponding to  $D_i$ ,  $k$  is the instantaneous total random number of drops,  $\text{const}$  represents some constant, and the summation is over a unit sample volume. We may then consider the quantity  $Y_i = VD_i^3$  in (2) to be the random variable resulting from the transformation of  $D_i$ , and  $R$  is the random sum described by a random total number of drops and random sizes.

It is argued extensively in Jameson and Kostinski (2001a) that  $k$  and the probability density function (pdf)

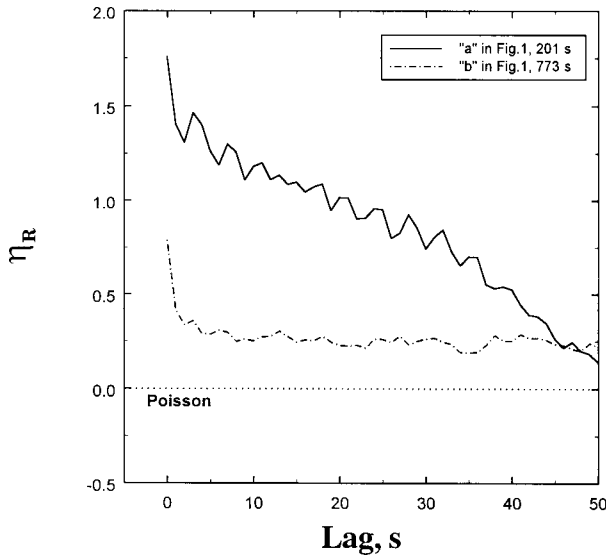


FIG. 3. The pair correlation for the rainfall rate for the two rain events in Fig. 1. For Poissonian rain,  $\eta_R = 0$  at all lags. The observed rain events are not steady by Poissonian standards.

of drop diameters,  $p(D)$ , are decoupled. They then derive the expression for the relative dispersion of  $R$  to be

$$\frac{\sigma^2(R)}{E^2(R)} = \left[ \frac{E(k)F_R\sigma^2(D^3V)}{\sigma^2(k)E^2(D^3V)} + 1 \right] \frac{\sigma^2(k)}{E^2(k)}, \quad (3)$$

where  $E$  denotes the expected value,  $\sigma^2$  is the variance, and the factor  $F_R = E_k[\sigma^2(R | k)]/[E(k)\sigma^2(D^3V)]$  is included to account for the observation that drops in such a sum do not usually occur statistically independently. Here,  $F_R$  is equal to 1 when the drops occur statistically independently, and  $F_R$  is greater than 1 when the drops are correlated. [For further discussion and explanation of the development of this expression, please refer to Jameson and Kostinski (2001a), 527–528.]

This concept can be expressed more clearly in terms of the sums of the squares of the relative dispersions of  $D^3V$  and the total number of drops  $k$  by

$$\frac{\sigma_R^2}{E^2(R)} = \frac{F_R}{E(k)} \left[ \frac{\sigma^2(D^3V)}{E^2(D^3V)} \right] + \frac{\sigma^2(k)}{E^2(k)}, \quad (4)$$

where  $\sigma_R^2$  denotes the variance of  $R$ . Thus, the relative dispersion of  $R$  is governed by two terms, one arising from  $p(D)$  and the second from the pdf of  $k$ . Notice too that the relative dispersion arising from  $p(D)$  is inversely weighted by the expected value of the total number of drops in a sample so that this term can be ignored when  $E(k)$  become very large (Jameson and Kostinski 1999a).

For the rain to be steady, it should be as statistically stationary as possible, as discussed above. It turns out that in statistically stationary rain the drop size distribution itself is steady (Jameson and Kostinski 2001a,b). As a consequence, in (4) the relative dispersion of  $(D^3V)$  is a constant in statistically stationary rain and we can,

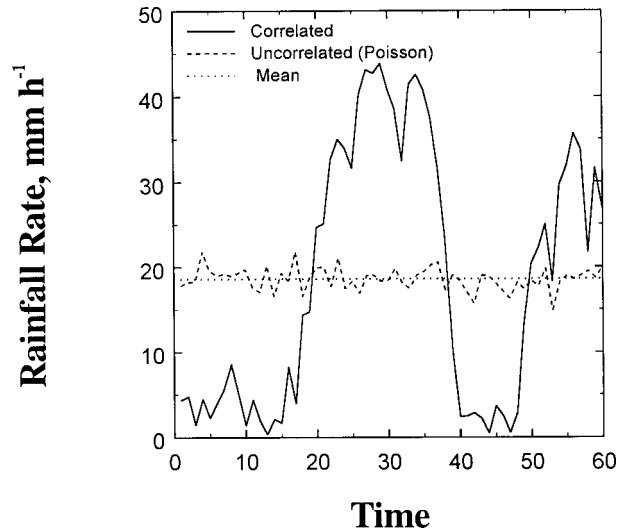


FIG. 4. Rain-rate profiles in statistically homogeneous correlated and uncorrelated (Poissonian) rain plotted as a function of unitless time. The horizontal line represents the mean for both kinds of simulated rain. Note the considerably larger fluctuations associated with the correlated rain. From Jameson and Kostinski (1999a).

instead, focus on the statistics of  $k$ . (Here it is important to remember that a statistically “constant” rainfall rate does indeed imply a constant or steady raindrop size distribution, but a steady drop size distribution does not imply that the rainfall rate is constant.)

Beginning with the most statistically uniform example, we first consider the case when the sum of the number of drops over all the different drop sizes obeys a Poisson process as discussed earlier. If so, then  $\sigma^2(k) = E(k)$ ,  $F_R = 1$ , and (4) becomes

$$\frac{\sigma_P^2}{E^2(R)} = \frac{1}{E(k)} \left[ \frac{\sigma^2(D^3V)}{E^2(D^3V)} \right] + \frac{1}{E(k)}, \quad (5)$$

where the subscript  $P$  denotes Poissonian rain. For such rain, the relative dispersion goes to 0 as the expected value of the number of drops becomes very large.

On the other hand, in clustered, correlated rain, the pdf of  $k$  sometimes appears to be well matched by the geometric distribution (e.g., Jameson and Kostinski 1998, their Fig. 8, p. 289; Jameson et al. 1999, their Fig. 2, p. 85) characterized by a long tail extending to large  $k$  and increasing probabilities as  $k$  approaches 0. Because the variance for such a distribution is given by  $E(k) + E^2(k)$ , after substitution and collection of terms (4) becomes

$$\frac{\sigma_R^2}{E^2(R)} = \frac{(F_R - 1)}{E(k)} \left[ \frac{\sigma^2(D^3V)}{E^2(D^3V)} \right] + \frac{\sigma_P^2}{E^2(R)} + 1. \quad (6)$$

As a consequence, as  $E(k)$  goes to  $\infty$ ,  $\sigma_R^2/[E^2(R)]$  approaches 1 no matter how many drops there are.

As another example and depending upon the size of the measurement volume, the distribution of drop counts at other times may be described aptly by the negative

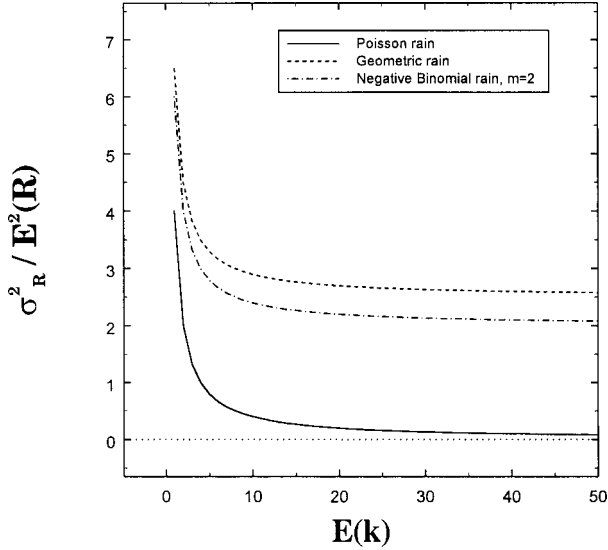


FIG. 5. The relative dispersions of the rainfall rate for three different distributions of the total number of drops  $k$ , specified in the legend, as functions of the expected total number of drops  $E(k)$ . The mean rainfall rates and drop size distributions are identical in all three cases. Here,  $F_R = 1$  for the Poissonian rain, but it is set to an arbitrary but realistic value of 1.5 in the other two examples in this figure to account for drop correlations normally associated with the appearance of such distributions of  $k$ .

binomial distribution (Kostinski and Jameson 1997, their Fig. 12, p. 2184; Jameson et al. 1999, their Fig. 8, p. 89). For such a distribution of  $k$ , the variance is given by  $E(k) + E^2(k)/m$ , where  $m$  is an integer-valued shape parameter such that, as  $m$  goes to  $\infty$ , the distribution approaches the Poisson. The relative dispersion is then given by

$$\frac{\sigma_R^2}{E^2(R)} = \frac{(F_R - 1)}{E(k)} \left[ \frac{\sigma^2(D^3V)}{E^2(D^3V)} \right] + \frac{\sigma_p^2}{E^2(R)} + \frac{1}{m}. \quad (7)$$

All of these results just discussed are illustrated in Fig. 5.

In statistically stationary rain, however, (4) can be expressed even more generally, because there is an equation that relates the variance of counts in a given volume to the pair-correlation function integrated over the same volume. This fact makes qualitative sense, because clustering acts to increase the variance (as, for example, Fig. 4 illustrates) beyond that expected when there is no correlation and Poisson statistics rule. As a consequence, the larger the average value of the correlation function in a sample domain is, the greater the clustering and, therefore, the variance are.

This insight was given precise meaning by Ornstein and Zernike (1914) who were investigating X-ray scattering by liquids near the critical density for opalescence. Their expression can be written as

$$\frac{\sigma^2(k)}{E^2(k)} - \frac{1}{E(k)} = \frac{1}{V} \int_V \eta(v') dv' = \bar{\eta}_k, \quad (8)$$

where  $\eta$  is the pair-correlation function,  $k$  is the total

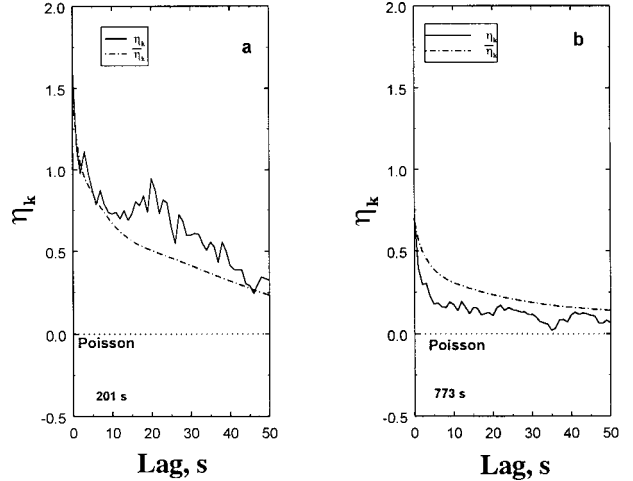


FIG. 6. The observed and average (see text) pair-correlation functions of the total number of drops for the (a) 201-s rain event and (b) 773-s rain event as a function of lag (for  $\eta_k$ ) and interval size (for  $\bar{\eta}_k$ ) as discussed in the text.

number of particles in volume  $V$ ,  $\sigma_k^2$  is the variance of  $k$ ,  $E(k)$  is the expectation value of  $k$ , and  $\bar{\eta}_k$  is the average of  $\eta$  over  $V$ . As Kostinski and Shaw (2001) point out, this result is general in so far as its derivation requires no physical mechanisms (see Landau and Lifshitz 1980). In particular, as long as the rain is statistically stationary, the distribution of drop sizes,  $p(D)$ , is steady, and the total number of drops impinging a unit surface in time interval  $t$  is the same as the total number of drops in volume  $\bar{V}_D \times t$ , where  $\bar{V}_D$  is the mean terminal drop velocity averaged over  $p(D)$ . Thus, transforming variables, we may write

$$\frac{\sigma^2(k)}{E^2(k)} - \frac{1}{E(k)} = \frac{1}{t} \int_0^t \eta(t') dt' = \bar{\eta}_k, \quad (9)$$

where  $k$  is the total number of drops counted over  $t$ .

First, we note that  $\eta$  can be negative but bounded at  $-1$  when there is perfect anticorrelation (Kostinski and Shaw 2001). Moreover, in the limiting case of no correlation,  $\eta(t') = 0$  so that we recover the Poisson relation  $\sigma^2(k) = E(k)$ . Second, the integral on the right-hand side implies that the variance on the left-hand side “remembers” the pair correlations (clustering) at all scales less than the interval size. Last,  $\bar{\eta}_k$  obviously depends on  $t$ . As a consequence, so does  $\sigma^2(k)/E^2(k)$ . That is, the observed variance will depend upon the sample resolution. This dependence is discussed further below.

As an example, in Fig. 6 we compare  $\bar{\eta}_k$  to  $\eta_k$ , corresponding to Fig. 2. From (9), it is obvious that the variance of  $k$  at smaller scales is carried over into measurements at larger scales via  $\bar{\eta}_k$ , thereby affecting the variance of  $R$ . That is, rearranging and inserting (9) into (4) and combining terms, we have simply that

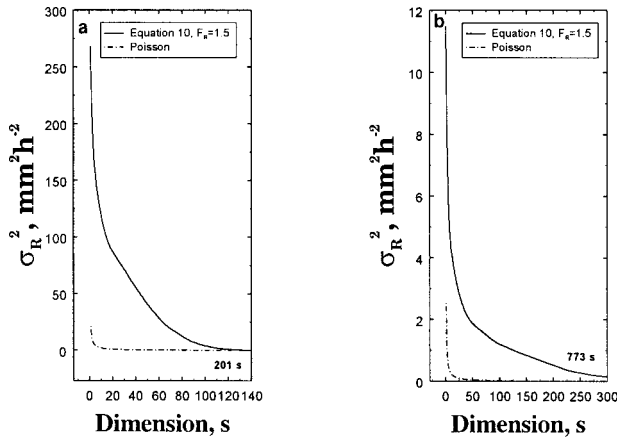


FIG. 7. The observed variances of the rainfall rate as functions of the size of the observation interval for the (a) 201-s rain event and (b) 773-s event calculated using (10) and the observed  $\overline{\eta}$  and for an equivalent Poissonian rain (see text). While the variances for the Poissonian rain quickly become negligible, the observed variances in clustered rain are strong functions of the interval resolution.

$$\sigma_R^2 = \sigma_p^2 + E^2(R) \left\{ \overline{\eta}_k + \frac{(F_R - 1)}{E(k)} \left[ \frac{\sigma^2(D^3V)}{E^2(D^3V)} \right] \right\}. \quad (10)$$

If the rain were Poissonian, then the pair correlation is zero at all scales,  $F_R = 1$ , and  $\sigma_R^2$  goes to  $\sigma_p^2$ . On the other hand, if the rain is correlated (clustered), then the variance is enhanced by the bracketed term in (10)  $\times E^2(R)$ . Data show, however, that the  $D^3V$  term is usually a very small fraction of the total variance, particularly as  $E(k)$  becomes substantial. Therefore, simplifying (10), we have

$$\sigma_R^2 \cong \sigma_p^2 + E^2(R) \overline{\eta}_k \quad (11)$$

to a very good approximation.

As (9) indicates, however,  $\overline{\eta}_k$  does depend upon the size of the measurement interval. As an example, we again return to the time-series video-disdrometer data presented in Fig. 1. Assuming that the data were statistically stationary (more on that below), we use (10) to compute  $\sigma_R^2$  as a function of  $t$ . Using (5), we also compute  $\sigma_p^2$  as if the rain were Poissonian, that is, for the same  $E(R)$  and  $p(D)$  but with  $\eta_k = 0$ . The results are shown in Fig. 7. It is obvious that  $\sigma_R^2$  decreases for both Poissonian rain and the observed rain as  $t$  increases. However, because of drop correlations, the decrease with increasing  $t$  is much slower in the actual rain than in the Poissonian rain. Nevertheless, in both cases and at sufficiently large values of  $t$ , even clustered, correlated rain may appear to have Poissonian variance or even variances smaller than Poissonian simply because  $t$  is too large to “see” the clustering anymore. It consequently is important to remember that the determination of whether the rain rate is steady will also depend, in part, upon the resolution of the observations.

Here, it is also appropriate to recall that rain, of course, need not be statistically stationary. Statistical nonstation-

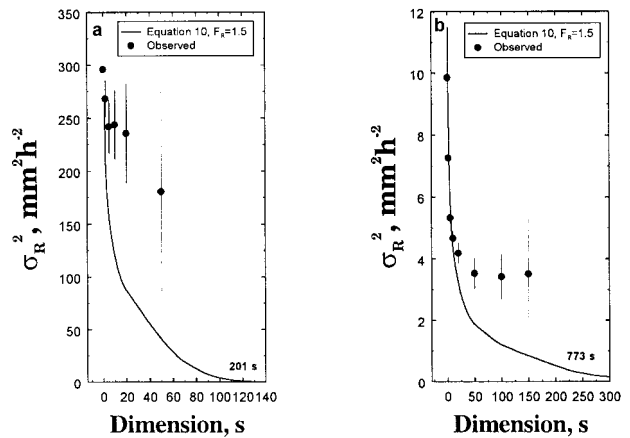


FIG. 8. The variances calculated for the two rain events using (10) based upon the assumption that the rain is statistically homogeneous compared with the variances actually observed as functions of the size of the observation interval. The enhancement of the observed variances can be attributed to statistical nonstationarity of the data, particularly at larger scales. The bars denote  $\pm 1$  standard error of the observed variances.

arity, perhaps through some randomness of the mean values, apparently acts to increase the variances of  $k$ , of  $D^3V$ , and, therefore, of  $R$ . This effect is illustrated in Fig. 8 in which the variances actually observed at the indicated resolutions [rather than the values estimated using (10) as in Fig. 7] are generally greater than those anticipated had the rain been statistically stationary.

### 3. Conclusions

Because both drop clustering, as (11) indicates, and statistical nonstationarity act to increase the variance above that for Poissonian rain, it is, therefore, reasonable to use Poissonian rain as the standard for steady rain as originally suggested in Kostinski and Jameson (1999). Although pure Poissonian rain is likely to be rare, if indeed it exists at all, the *concept* of Poissonian rain is still very useful because it provides us with a means of precisely quantifying the steadiness of any particular rain event. One can then simply compare the variance of  $R$  actually observed with that anticipated had the rain been Poissonian. This comparison is useful because, when the rainfall rate is as constant as statistics allows, there is also an accompanying constant drop size distribution. However, how does one identify a constant rainfall rate in practice?

The answer depends, in part, on just what measurements are available. If one has access only to a time or spatial series of measurements of  $R$ , then the behavior of  $\eta_R$  can be used as a discriminator because in Poissonian rain  $\eta_R$  is 0 at all lags. If, on the other hand, one has access to times-series disdrometer observations, one can compute  $\eta_k$  instead. However, one has to be careful because, if  $k$  is observed using too coarse a resolution, clustering may still exist but simply may not appear in

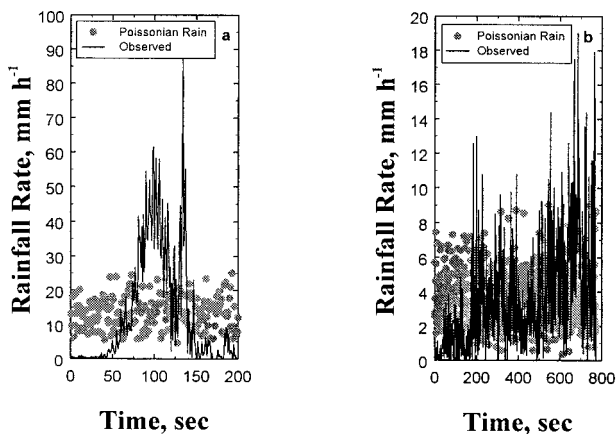


FIG. 9. The observed rainfall rates for the (a) 201-s rain event (b) 773-s event compared with those expected for Monte Carlo simulations of Poissonian rain having the same mean rainfall rate as discussed in the text. Although the second rain is much steadier, neither event strictly satisfies the Poissonian criterion for steady rain given that  $\sigma_R^2/\sigma_P^2 = 13.9$  and  $4.1$  for (a) and (b), respectively.

$\eta_k$ , as just discussed and as illustrated in Fig. 8 and in the appendix of Kostinski and Jameson (2000). (The same comment also applies to  $\eta_R$ .) One other difficulty with these two approaches is that they hedge the question of what one does with weakly correlated rain that appears to be almost but not quite Poissonian. How nearly Poissonian are they? Can a quantitative value be assigned?

The answer is yes. One approach is to use a more direct method by noting that, in Poissonian rain, the distribution of the total number of drops is also Poisson distributed at all scales. Then,  $\sigma^2(k) = E(k)$ . As a consequence, whenever  $\sigma^2(k) \gg E(k)$ , the rain is not likely to be steady. Deviations of the ratio  $\sigma^2(k)/E(k)$  from unity then can be used as quantitative measures of steadiness.

A more satisfying approach, however, is to use the rainfall rate itself. If the expected number of drops at each size category is measured, one can simply use a Poisson random number generator to create time series having the mean values appropriate to each size bin (making certain that the generator is initialized differently for each drop size to eliminate correlations). These time series then can be combined to create the Monte Carlo simulation of the rainfall rate that automatically includes the observed distribution of drop sizes. Then, not only can the variance of the simulated time series be compared with the variance actually observed, but the time series can be compared visually with the observations. For example, the observed  $\sigma_R^2$  are  $292.7$  and  $9.85 \text{ mm}^2 \text{ h}^{-2}$  for the 201- and 773-s video-disdrometer observations, respectively. The corresponding Poissonian values are  $21$  and  $2.38 \text{ mm}^2 \text{ h}^{-2}$ , respectively. Thus, the ratios of the observed variances to the Poisson variances are approximately  $14$  and  $4$ , respectively, so that neither could be classified as really steady rain. This

result is apparent when the Poissonian simulated and actual time series of the rainfall rates are compared, as illustrated in Fig. 9. Although one certainly could argue that the longer time series is more than 3 times steadier than the first, neither appears to be truly steady.

Even in the absence of any direct measurements of  $R$ , it may, at times, even be possible to use remote sensing devices such as radar, for example, at least to exclude those regions where the rain is not steady. In such locations, as the radar scans, readily detectable non-Rayleigh signal statistics of the radar reflectivity factor will appear, depending upon the scan rate and radar beam dimension [for further discussion see Jameson and Kostinski (1996, 1999b)]. Those regions in which the signal statistics of  $Z$  remain Rayleigh even though the antenna is moving are at least reasonable candidate locations of steady rain worthy of further attention.

To return to Sauvageot and Koffi (2000), it is now apparent that the limit of  $0.2 \text{ mm h}^{-1}$  offered as an upper limit for statistical fluctuations is likely too restrictive. To be specific, (5) implies that the absolute minimum  $\sigma_P^2$  occurs when the distribution of the drops is monodisperse. In that case  $\sigma_{P|\min} = E(R)/\sqrt{E(k)}$ . Based upon the information provided in Sauvageot and Koffi (2000), it appears that more realistic values of  $\sigma_{P|\min}$  range from  $0.24$  to  $3.91 \text{ mm h}^{-1}$ , particularly given that their distributions are actually considerably broader than monodisperse.

But what is important here, however, is that there is now a means of quantifying steadiness precisely. This fact is important because steady rain has some unique and useful properties. To be specific, because steady rain is statistically stationary, the associated drop size distributions have physical, deterministic meanings independent of the measurement process (Jameson and Kostinski 2001a,b). Furthermore, because such rain is Poissonian, convergence to the drop size distribution is rapid. Moreover, the relations between the rainfall rate and the radar reflectivity factor, for example, are *physical* and linear (Jameson and Kostinski 2001a). Each  $Z$  consequently is uniquely related to each  $R$  without the statistical uncertainty associated with  $Z$ - $R$  correlation power laws that apply in statistically nonstationary rain (Jameson and Kostinski 2001a).

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