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Drizzle rates versus cloud depths for marine stratocumuli

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Abstract
Marine stratocumuli make a major contribution to Earth's radiation budget. Drizzle in such clouds can greatly affect their albedo, lifetime and fractional coverage, so drizzle rate prediction is important. Here we examine a question: does a drizzle rate (R) depend on cloud depth (H) and/or drop number concentration n in a simple way? This question was raised empirically in several recent publications and an approximate H^3/n dependence was observed. Here we suggest a simple explanation for H^3 scaling from viewing the drizzle rate as a sedimenting volume fraction (f) of water drops (radius r) in air, i.e. $R = fu(r)$, where u is the fall speed of droplets at the cloud base. Both R and u have units of speed. In our picture, drizzle drops begin from condensation growth on the way up and continue with accretion on the way down. The ascent contributes H ($f \propto H$) and the descent H^2 ($u \propto r \propto fH$) to the drizzle rate. A more precise scaling formula is also derived and may serve as a guide for parameterization in global climate models. The number concentration dependence is also discussed and a plausibility argument is given for the observed n^{-1} dependence of the drizzle rate. Our results suggest that deeper stratocumuli have shorter washout times.

Keywords: marine stratocumulus, drizzle rate, cloud depth

Sheets of marine stratocumulus clouds cover vast expanses of the world's oceans and cause large scale cooling. Their representation in large scale models of the atmosphere is one of the unresolved issues in climate modeling. Drizzle is an important climatological feature, e.g., the rate of 1 mm day^{-1} is roughly equivalent to a heat flux of 30 W m^{-2} and comparable to net longwave radiative flux divergence at the cloud top [1]. By limiting cloud lifetime, drizzle can decrease cloud fraction and, thereby, albedo [2, 3]. Decreased drizzle leads to deeper boundary layer and thicker clouds. Recent field campaigns devoted to marine stratocumulus [4] produced a somewhat surprising and, to the best of our knowledge, still unexplained observation that drizzle rates scale with cloud depth as approximately H^3/n, with all other parameters equal [5, 1, 6, 7]. Here H denotes cloud thickness (depth) and n is the number concentration of droplets. The purpose of this letter is to suggest a simple derivation for the H^3 scaling and examine the $n^{-\gamma}$-type scaling.

Drizzle rate is given by $R \equiv \int_{r_{\min}}^{r_{\max}} (4/3)\pi r^3 n(r) u(r) \mathrm{d}r$ where r is the droplet radius, $n(r)$ is the droplet size distribution (concentration as a function of droplet size with unit of m^{-4}), and $u(r)$ is the settling speed as a function of droplet size. Observe that the units of R are those of speed, e.g., the drizzle rate is measured in mm day^{-1} [5, 1, 6, 7]. This observation suggests a connection to the fall speed of drizzle drops. For the sake of clarity, we assume a narrow droplet size distribution so that $u(r)$ can be taken outside the integral, thereby reducing the expression for drizzle rate to

$$R = fu(r) \tag{1}$$

where f is the fractional volume of liquid water, falling through air. For illustration, typical $f \approx 10^{-6}$ and $u \approx$ 1 cm s^{-1} yields $R \approx 1$ mm day^{-1}. (The fall speed of 1 cm s^{-1} corresponds to drop radius of about 10 μm.) Note that mass density information while conventional when discussing liquid water content is irrelevant here and will be discarded in favor of f, a purely geometric quantity.

The simple equation (1) is our point of departure. Both the drizzle fraction volume and the speed scale with cloud depth H and the latter does so through the size dependence $r = r(H)$ as the larger drops fall faster. We first obtain scaling for the volume fraction $f \propto H$ by strictly thermodynamic means (condensation growth) and then obtain scaling for the terminal speed of drizzle drops at the cloud base by mechanical means (growth by collection). As a preview, this results in the following chain of scaling arguments:

$$f \propto H; \qquad u(r) \propto r_{\mathrm{cb}}^{\alpha} \propto (fH)^{\alpha} \propto H^{2\alpha}$$
$$\Longrightarrow R = fu(r) \propto H^{1+2\alpha} \qquad (2)$$

where cb is a shorthand for cloud base and α is the exponent in terminal speed u scaling with size. Since $\alpha \approx 1$ for droplet sizes in the drizzle category ($r > 40\ \mu$m) [8, 9], the above result delivers the desired H^3 dependence and a larger exponent for small drizzle ($r < 40\ \mu$m). We shall now describe the steps, in turn.

We can use the energy balance to relate the droplet volume fraction f_{ct} ('ct' for cloud top) to the cloud depth H as follows. Let air, with temperature initially at dew point, cool by ΔT. Water vapor condenses on growing droplets and the latent heat $Q_{\mathrm{in}} = mL$ (product of mass and specific latent heat), liberated in the process is spent on warming the droplets (negligible) and the volume of air cell $V = n^{-1}$ around each droplet back to the dew point so that no other droplet can form within the cell of volume V. This results in

$$Q_{\mathrm{in}} = v\rho_{\mathrm{w}}L = Q_{\mathrm{out}} \approx \rho_{\mathrm{a}}Vc_{\mathrm{a}}\Delta T \qquad (3)$$

where v is the droplet volume, ρ and c denote density and heat capacity, respectively, and subscripts a and w, refer to air and water, respectively. Substituting numerical values for density and heat capacity and solving for the volume fraction $f = v/V$ gives the fractional volume $f = v/V \approx 0.4 \times 10^{-6}\Delta T$ or, in conventional units, one gram of liquid water per kilogram of air for $\Delta T = 2.5C$. Temperature lapse rate $\Gamma \equiv \mathrm{d}T/\mathrm{d}Z$, ΔT and the cloud depth are related via $\Delta T \approx \Gamma H$, yielding

$$f \approx (0.4 \times 10^{-6}\Gamma)H. \qquad (4)$$

This represents a volume fraction analogy of the adiabatic liquid water content [10]. Thus, cloud top droplet volume fraction f_{ct} scales linearly with cloud depth. This conclusion is independent of droplet concentration and size and depends only on thermodynamics.

However, drop fall speed at the cloud base also contributes to drizzle rate and it does depend on drop size. Furthermore, at the cloud base, drizzle drop terminal speed $u(r_{\mathrm{cb}})$ depends approximately linearly on droplet size ($r > 40\ \mu$m), but the size depends quadratically on the cloud depth. So how does the fall speed scale with cloud depth? To that end, we employ simple continuous collection.

Consider a 'collector' drop at the cloud top, accreting the droplets on its way. This may be a tortuous path through a cloud when turbulent speed fluctuations are comparable to fall speed. The volume fraction swept by the drizzle drop along such a meandering trajectory, is well represented by spatially averaged f throughout the cloud. In the continuous collection version, the drop's increase in volume $4\pi r^2\,\mathrm{d}r = \pi r^2\,\mathrm{d}zf$ (the combined volume of the accreted droplets). Hence, $\mathrm{d}r \propto f\,\mathrm{d}z$ [11]. The scaling is obtained by integration over $\Delta z = H$ and neglecting the initial size

$$r_{\mathrm{cb}} \propto fH \propto H^2. \qquad (5)$$

Here we interpret f as the average volume fraction gained by drizzle drops (which typically constitutes a few per cent of the total water volume fraction). The various assumptions involved such as neglect of coalescence efficiency, employment of the continuous regime, etc, while changing numerical coefficients such as 4π, do not affect the exponent in $r_{\mathrm{cb}} \sim H^2$. The scaling $r_{\mathrm{cb}} \propto fH$ is also appealing from dimensional considerations: two lengths, r_{cb} and H are scaled by a unitless volume fraction f. As an illustration, a cloud thickness of 1 km and average $f \sim 10^{-6}$ yield $r_{\mathrm{cb}} \sim 0.25$ mm.

Combining equation (1) with equations (4) and (5) produces the sought $R \propto H^3$ dependence. Consequently and somewhat surprisingly, the cloud 'washout' time, ratio of liquid water path ($fH \propto H^2$) to drizzle rate, is inversely proportional to cloud depth so that deeper stratocumuli appear to 'empty' more quickly. This may help to explain the observations of persistent open cells or 'rifts' in cloud fields [12].

More precisely, the terminal fall speed of a drop depends on its radius as $u(r) = r^{\alpha}$ with $\alpha = 2, 1, 0.5$ in laminar, intermediate, and turbulent regime, respectively (e.g. see p 126 of [8]). This refinement yields $R = fu(r) \propto H^{1+2\alpha}$. For example, if the cloud-base drizzle radius is below 40 μm or so, α exceeds unity and $1 + 2\alpha$ can approach 4, explaining observations of $\alpha = 3.75$ [7]. On the other hand, rain rate is expected to scale only quadratically ($\alpha = 0.5$ in turbulent regime) with cloud depth and then saturate as drops become large enough to experience instability and subsequent fragmentation. The simple $R \propto H^{1+2\alpha}$ law may find applications in climate models where the detailed calculations are not feasible.

We now turn to the number concentration dependence of the drizzle rates. Since the volume fraction f is determined thermodynamically, the only dependence on n in the drizzle rate expression, $R = fu(r_{\mathrm{cb}})$, must enter via the cloud-base drop radius as $u(r_{\mathrm{cb}}) \propto r_{\mathrm{cb}}^{\alpha}$. Intuitively, the more finely dispersed cloud with smaller droplets produces lower drizzle rates (at the same water content or volume fraction) because decreased droplet sizes have lower terminal speed. At a fixed f, $nr^3 = \mathrm{const}$, and drop radius $r \propto n^{-1/3}$. For example, at constant $f \sim nr^3$, $\alpha = 1$ yields $n^{-1/3}$ scaling while $\alpha = 2$ produces $n^{-2/3}$ dependence of the drizzle rate. Yet, the drizzle rate has been observed to scale approximately as n^{-1} [5, 7]. Evidently, a more subtle argument is necessary. The number concentration dependence of the collision–coalescence process as the drops descend to the cloud base and grow by coalescence, must be addressed. The continuous collection perspective is not adequate here because the collector drop's growth depends on volume fraction f only, regardless of how the water is dispersed. Coalescence growth must, therefore, be treated as a discrete process.

Recall that these stratocumuli are a few hundred meters thick while the mean free path between collisions is on the order of 10 m, and on the order of a 100 m between coalescence events. As the droplets meander but descend to the cloud base, their volumes grow as $v_{\mathrm{cb}} = v_{\mathrm{ct}} \times (T/t)$, where T is the cloud traversal time and t is the average inter-coalescence time. Thus, the ratio of the two times is the number of coalescence events and $r_{cb}^3 \propto r_{ct}^3 \times (T/t)$. The cloud traversal time $T \propto H/u(r) \propto H/r^{\alpha} \propto r^{-\alpha}$ while $u(r_{cb}) \propto r_{cb}^{\alpha}$ so that the two nearly cancel and t^{-1} determines the net dependence on concentration.

The inter-coalescence time scales as [13] $t \propto (n\sigma Eu)^{-1}$ where $\sigma \propto r^2$ is the cross-section, and E is the coalescence efficiency. For drops between 10 and 50 μm in radius, σ, u and E each scale approximately as r^2, yielding the combined r^6 dependence for the collision rate (e.g. see p 618 of [9]). Therefore, $t \propto n$ because $\sigma Eu \propto r^{-6} \sim n^{-2}$. Thus, $t^{-1} \propto n^{-1}$ so that the drizzle rate $R = fu(r_{cb}) \propto n^{-1}$ as well in this scenario. As the drop size increases, the collection kernel (inverse inter-coalescence time) dependence on size weakens (p 618 of [9]) and so would the drizzle rate scaling with concentration. On the other hand, this might be offset by the cloud top radius dependence on concentration ($\sim n^{-1/3}$), if data are indeed compared at approximately constant volume fraction. These conclusions are less secure than the cloud depth scaling but drizzle rate scaling may offer insight into collection kernel behavior.

Intuitively, one expects the optical depth $\tau \equiv n\sigma H$ and the 'collisional depth' $\tau' \equiv n\sigma EH'$ to be related (H' total length of the meandering drizzle drop path) and, indeed, drizzling clouds typically appear dark. However, the ratio $\tau'/\tau = E(H'/H)$ does not equal unity. In fact, at a fixed optical depth τ, H' can still increase with increasing turbulence (e.g. from shear or convective activity) and, thereby, increase drizzle rate. This is yet another reason to supplement optical remote sensing observations with wind measurements.

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