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Improving capital budgeting decisions with real options

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The process of evaluating the desirability of long-term investment proposals is referred to as “capital budgeting.” Making optimum capital budgeting decisions (e.g., whether to accept or reject a proposed project), often requires recognizing and correctly accounting for flexibilities associated with the project. Such flexibilities are more formally termed real options. From a valuation standpoint, these options are valuable because they allow decision makers to react to favorable or unfavorable new situations by dynamically adjusting the capital budgeting decision process. Unfortunately, the value of real options is not explicitly considered in conventional procedures (such as discounted cash flow (DCF) models) used to evaluate long-term investment proposals. In some sense, therefore, real options can be viewed as an extension of DCF that incorporates a simple model of strategic learning.

In this article we present a short tutorial regarding real options—what they are and how they can be formally incorporated into the capital budgeting process. We also illustrate how to price a capital investment project containing real options. To explain these concepts to a wide audience in accounting, we take more of an intuitive approach and therefore abstract from more technical treatments of the topic, such as those explicitly linked to the Black-Scholes pricing model for financial options. To illustrate basic concepts regarding real options, we use a straightforward example that relates to a rental car company that is considering whether to purchase a conventional gasoline-powered car or a hybrid car as an addition to its rental fleet, a decision complicated by uncertainties regard-
ing tax incentives for commercial purchases of hybrid vehicles.

**Income-Tax Incentives Regarding the Purchase of Hybrid Vehicles**

In 2002, Congress instituted a $2,000 income-tax credit for the purchase of a new hybrid vehicle, but, according to the proposal by Congress in late 2003, that tax incentive was to have been phased out entirely by 2007. Specifically, the $2,000 tax credit would be cut by 25% in 2004, 50% in 2005, and 75% in 2006. In late 2004, however, Congress reenacted the $2,000 tax credit for hybrid cars purchased in 2004 and 2005. In 2006, Congress passed a new bill that allowed a maximum tax credit of more than $3,000 for hybrid car purchases, but the determination of which hybrid vehicles qualified for the credit and the amount of credit to be taken was based on a complicated set of rules.

We use the preceding historical review to illustrate one type of cash-flow uncertainty associated with long-term investments in depreciable assets: income-tax consequences. As illustrated, tax effects on such investments are subject to the whim of Congress. Suppose that, at the present (say the beginning of 2008), Congress is debating a bill that would give commercial owners a tax break for purchasing (and using) hybrid vehicles. Assume, however, that passage of the bill is uncertain and that our rental car company is currently considering whether to add a hybrid vehicle or a gas-powered vehicle to its rental fleet. Uncertainties regarding income-tax provisions in the new energy bill complicate the analysis of this capital budgeting decision. One of the advantages of real options is the ability to deal explicitly with these and other uncertainties.

For simplicity, we make a few additional assumptions regarding this investment decision. First, we assume that the company plans to keep either car for four years, after which time the car will be sold for an estimated salvage value. Second, if the new energy bill is passed, we assume that the income-tax credit for operating commercial hybrid vehicles will start one year hence, that is, in January 2009. Management believes there is a 40% chance that the bill will be passed and that the final fate of the bill will be known by January 2009. Third, if the bill is passed, the annual after-tax cash flow from operating the hybrid car is estimated to be $10,000; assume, too, a net-of-tax salvage value of $5,000 for the hybrid car at the end of year four. If the bill is not passed, the annual after-tax cash flow for the hybrid car drops to $4,000, and its net-of-tax salvage value at the end of year four would be $3,000.

For the gas-powered vehicle, assume that the annual after-tax cash flow is $6,000, regardless of whether the new bill passes; further, assume that the net salvage value for this asset would be $3,000 with and $4,000 without the bill. This difference reflects the fact that the passage of the energy bill would make gas-powered cars less attractive than hybrids.

Finally, assume that the (annual) risk-free rate of interest (estimated, for example, by the current yield on Treasury Bonds) is 6% and that for the business under consideration the appropriate risk-adjusted, after-tax discount rate (i.e., weighted-average cost of capital) is 10%.  

**Conventional Analysis: Hybrid or Gas-Powered Vehicle?**

First, note that the decision alternatives in this example (whether to purchase a hybrid or a gas-powered car) are mutually exclusive. According to a conventional DCF analysis, we would calculate the expected net present value (NPV) for each decision alternative and then choose the one with the higher (and positive) NPV. In our example, as shown in the top portion of Figure 1, the expected NPV for buying the hybrid car is negative, as follows:

\[
NPV_{\text{Hybrid}} = PV \left( \text{after-tax cash flows} \right) - PV \left( \text{investment} \right)
\]

\[
= \left[ \sum_{t=1}^{4} \frac{\$10,000}{(1 + 0.1)^t} + \frac{\$5,000}{(1 + 0.1)^4} \right] \times 0.4 + \left[ \sum_{t=1}^{4} \frac{\$4,000}{(1 + 0.1)^t} + \frac{\$3,000}{(1 + 0.1)^4} \right] \times 0.6 - \$25,000
\]

\[= \$2,117 \quad (1)\]

The negative expected NPV suggests that we should not buy the hybrid car now, i.e., at \( T = 0 \).
The company buys new cars and keeps them in business for four years. At the end of year four, the car is sold for the indicated salvage value. Suppose the company has to make the investment decision now (i.e., January 2008, or \( T = 0 \)) regarding the purchase of a gas-powered or a hybrid car. The new energy bill, currently under debate in Congress, offers a hefty income-tax credit on the purchase of commercially operated hybrid cars. The proposed tax credit can alter the annual after-tax cash flows and salvage values of both types of cars. With the tax credit, the hybrid car offers both higher annual after-tax cash flows ($10,000) and a higher salvage value ($5,000) for hybrid cars, while the salvage value for gas-powered cars drops from $4,000 to $3,000. These potential cash-flow changes are strictly contingent on whether the new energy bill is passed in January 2009. At time \( T = 0 \), management estimates a 40% chance that the new energy bill will be passed. Figure 1 shows that the optimal choice is to choose the gas-powered car since this alternative has an NPV of $1,478, while the NPV of the hybrid car is $-2,117. The shaded bottom portion of the figure indicates the optimal decision: **Invest in a gas-powered car now.**
As shown in the bottom portion of Figure 1, the expected NPV for the gas-powered car is positive, as follows:

\[
NPV_{Gas} = PV(\text{after-tax cash flows}) - PV(\text{investment}) \\
= \left( \sum_{t=0}^{\infty} \frac{\$6,000}{(1+0.1)^t} + \frac{\$3,000}{(1+0.1)^t} \right) \times 0.4 + \left( \sum_{t=0}^{\infty} \frac{\$6,000}{(1+0.1)^t} + \frac{\$4,000}{(1+0.1)^t} \right) \times 0.6 - \$20,000 \\
= \$1,478
\] (2)

Because the \(NPV_{Gas}\) alternative is positive and greater than \(NPV_{Hybrid}\), a conventional DCF analysis indicates that the company should invest in the gas-powered car. The corresponding cash flows associated with Figure 1 are presented in Table 1.

### The Potential Value of Real Options

To this point, we have presented the decision as a capital budgeting problem analyzed using a conventional DCF analysis. Yet in this decision problem we have already seen, but not dealt with, an embedded real option: the flexibility of our choice between the two types of cars—what we might call an “asset flexibility” option. While seemingly of little importance, this example of a real option carries an important message that is true for all forms of real options:

\[
NPV(\text{w/options}) \geq NPV(\text{w/o options}). \quad (3)
\]

As such, real options will be exercised if and only if they increase the value of a capital investment project. Consider, for example, each investment choice in a “go” vs. “no-go” decision (i.e., accept or reject). This would be the situation if we did not have the flexibility to choose between the two types of cars. When the gas-powered car is the only candidate, the expected NPV would be \$1,478, and the project would be accepted; if the hybrid car were the only option being considered, we would reject this investment because its NPV is negative. (Equivalently, failure to invest results in an NPV of \$0.) As stated by statement (3), when we introduce asset choice (i.e., when we embed asset flexibility into the decision), the expected NPV is the higher of the two—in this case, \$1,478. For this situation, we would not exercise the option of asset flexibility—it adds no expected value to the investment decision.

### An Investment-Timing Option: Buy Now or Buy Later?

The next question management might ask is: “Should we purchase a car now, or should we delay the investment for one year—until January 2009?” The flexibility offered in the timing of the capital budgeting decision is called an investment-timing option. Figure 2 expands the original decision problem (Figure 1) to address the

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**Table 1:** Comparative After-Tax Cash Flow Data—Two Decision Alternatives, Purchase Made at \(T = 0\) (i.e., Today)

<table>
<thead>
<tr>
<th>A: Purchase a Hybrid Car</th>
<th>0 (now)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>New energy bill is passed (40% probability)</td>
<td>-$25,000</td>
<td>$10,000</td>
<td>$10,000</td>
<td>$10,000</td>
<td>$10,000 + $5,000 salvage value</td>
</tr>
<tr>
<td>New energy bill is NOT passed (60% probability)</td>
<td>-$25,000</td>
<td>$4,000</td>
<td>$4,000</td>
<td>$4,000</td>
<td>$4,000 + $3,000 salvage value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B: Purchase a Gas-Powered Car</th>
<th>0 (now)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>New energy bill is passed (40% probability)</td>
<td>-$20,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$6,000 + $3,000 salvage value</td>
</tr>
<tr>
<td>New energy bill is NOT passed (60% probability)</td>
<td>-$20,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$6,000 + $4,000 salvage value</td>
</tr>
</tbody>
</table>
investment-timing question: invest now \((T = 0)\) or delay the investment one year (i.e., invest at \(T = 1)\)?)

The benefit of delaying the investment decision is that we will have more accurate (or complete) information in January 2009 when the fate of the new energy bill will be known. This new information may lead to a decision that differs from the one made based on the more limited information set available today. Intuitively speaking, if we can “wait and see” for one year, we will not get stuck for four years with a car that, based on updated information, turns out to be a suboptimal choice. The benefit of the “wait and see” option, however, comes at a cost: the after-tax cash flows that the business forgoes during the coming year. Because money has a time value, delaying the investment reduces the expected NPV of the project. If this “hidden cost” outweighs the benefit, then the embedded investment-timing option does not add value, and the investment should be made now. Thus, the optimal decision depends on the trade-off between these two considerations.

Now suppose we do defer the investment decision to January 2009. If the new energy bill is passed, the expected NPV for the hybrid alternative is

\[
NPV_{\text{Hybrid}} = PV(\text{after-tax cash flows}) - PV(\text{investment})
\]

Note that in this case all cash flows are pushed forward by one year compared to the situation depicted in Figure 1. Also, the investment outlay of $25,000 to be made one year from now is discounted by the risk-free interest rate (6%) because we assume that this amount is known with certainty.

We now find that the expected NPV for the gas-powered alternative is $285.5 which is much less than $8,337. Therefore, after incorporating the potential value of the “wait and see” option, we find that purchasing the hybrid car is the optimal choice (based on maximizing expected NPV). This is shown in the shaded upper area of Figure 3. The corresponding after-tax cash flows are presented in Table 2.

### Figure 2: Investment-Timing Option

An “investment-timing option,” one form of real option, is created if the rental car company has the flexibility to invest today \((T = 0)\) or to defer the investment for at least one year \((T = 1)\). At \(T = 0\), management estimates that the new energy bill has a 40% chance of being passed. At \(T = 1\), it will be known for certainty whether the new bill has passed. The benefit of deferring the investment decision for one year is that the company can avoid the less profitable (or money-losing) choice. The drawback of the delay decision is that the expected future after-tax cash flows are deferred. The trade-off of these two effects tells us whether the company should defer the investment to January 2009 (i.e., make the decision at \(T = 1\)).
If the company has the flexibility to wait a year to make the investment, an investment-timing option exists. Whether the company should invest now (i.e., go back to the situation depicted in Figure 1) or wait (in a sense, exercise the option to wait) depends on the expected NPV of the delayed investment. It makes sense to defer the investment only when two conditions are satisfied: (1) the expected NPV of the delayed investment is positive, and (2) this NPV is higher than the expected NPV without deferral. In our case, the shaded bottom portion in Figure 1 indicates the optimal path for the company. By waiting one year, the company can take full advantage of the knowledge that becomes available at $T = 1$ but that is unavailable at $T = 0$. Estimated at $T = 0$, and as shown in Figure 3 above, the overall expected NPV under the delay option is $3,878, which exceeds the $1,478 optimal result when the decision is made now (see Figure 1).
If the new energy bill is not passed, hybrid cars become less attractive from a financial standpoint. The expected NPV for a gas-powered car purchase becomes

\[ NPV_{\text{Gas}} = PV(\text{after-tax cash flows}) - PV(\text{investment}) \]

By using a similar process, we estimate that if the bill is not passed, the NPV for a hybrid purchase would be \(-$10,195\), which is a significant loss. The sensible choice in this situation, of course, is to purchase the gas-powered car.

The shaded portions of Figure 3 indicate the optimal choices if we do wait until January 2009 to make the investment decision. In other words, from today’s point of view we know for certain the appropriate investment decision that should be made contingent on the information available in January 2009 (i.e., \(T = 1\)), but we do not possess this information now. At \(T = 0\), all we know is that there is a 40% chance we will choose the hybrid (with \(NPV_{\text{Hybrid}} = $8,337\)) and a 60% chance we will choose the gas-powered car (\(NPV_{\text{Gas}} = $906\)). Thus, the expected NPV is $3,878 (= [8,337 × 0.4] + [906 × 0.6]). (See the box to which the two big arrows in Figure 3 point.)

Because deferring the investment decision results in a higher NPV ($3,878) compared to investing now ($1,478, as shown in Figure 1), the investment-timing option adds $2,400 of value. As such, the company should not invest in a new car now, even though that decision has a positive expected NPV.

To summarize, valuing projects with embedded options requires three major steps:

- **One:** Find the expected NPV for the project as if one particular option were taken. For example, we calculated a separate NPV for deferring and another NPV for not deferring the investment.
- **Two:** Make the decision based on the new information to be revealed at a specified future point in time. In our case, we determined the type of car that should be chosen if the bill is passed and the type of car that should be purchased if the bill is not passed. The crucial information about whether the bill passes or not is available only at \(T = 1\) (i.e., at January 2009).
- **Three:** Compare these expected NPVs. The higher NPV indicates the optimal decision. In our example, deferring the investment offers a larger benefit by $2,400 (in present-value terms).

### Table 2: Comparative After-Tax Cash Flow Data—Two Decision Alternatives, Purchase Made One Year from Today (i.e., at \(T = 1\))

<table>
<thead>
<tr>
<th>A: Purchase a Hybrid Car</th>
<th>Year</th>
<th>0 (now)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>New energy bill passed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40% probability</td>
<td></td>
<td>-$25,000</td>
<td>$10,000</td>
<td>$10,000</td>
<td>$10,000</td>
<td>$10,000</td>
<td>+ $5,000 salvage value</td>
</tr>
<tr>
<td>New energy bill NOT</td>
<td></td>
<td>-$25,000</td>
<td>$4,000</td>
<td>$4,000</td>
<td>$4,000</td>
<td>$4,000</td>
<td>+ $3,000 salvage value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B: Purchase a Gas-Powered Car</th>
<th>Year</th>
<th>0 (now)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>New energy bill passed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40% probability</td>
<td></td>
<td>-$20,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$6,000</td>
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</tr>
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<td></td>
<td>-$20,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>+ $4,000 salvage value</td>
</tr>
</tbody>
</table>
**Other Types of Real Options**

Thus far, we have looked at two specific types of real options: *asset options* and *investment-timing options*. The concepts and valuation techniques we have discussed to this point also can be extended to other types of real options. Here we briefly explain a few additional types of real options. All of these examples fall under the umbrella of providing decision makers with increased investment flexibility.

*Growth options* typically refer to the flexibility to increase the scale of an investment. Companies tend to build facilities with a certain amount of slack (or “cushion”) because the slack provides a valuable growth option. For example, a computer with a few more empty slots costs more than a comparable machine without such slots. Michigan Tech University offers its undergraduate engineering students the option to earn a “shortcut MBA” as an add-on to their undergraduate program by having them take required business courses and allowing these students to apply credits earned in their engineering program (such as engineering management, statistics, etc.) toward the MBA degree. So we can say that Michigan Tech has an embedded (academic) growth option for which it may charge a higher tuition to its undergraduate engineering students.

How do we value growth options? We can slightly modify the framework presented in the earlier example to deal with this question. For instance, we can ask whether operating a gas-powered rental car without the possibility to grow is more valuable than operating a hybrid car with the flexibility to grow in the future, say at point $T = 1$. In this case, we would compare the NPV for a gas-powered car (viz., $1,478$) with that for a hybrid. Given our assumed data, the expected NPV for the “hybrid-car-with-growth option” is

$$\text{NPV}_{\text{Hybrid-Growth}} = \text{NPV}_{\text{Operating hybrid now}} + \text{NPV}_{\text{Option to buy hybrid at } T = 1}$$

$$= \left[ -2,117 + (8,337 \times 0.4) \right] + [0 \times 0.6]$$

$$= 1,218$$

(6)

where $0 \times 0.6$ means that the growth option to buy a hybrid car at $T = 1$ is given up (hence the cash flows are zero) if the new energy bill is not passed. This situation has a 60% probability of occurrence. The example indicates that operating hybrid cars with the flexibility to grow is still less attractive than operating gas-powered cars without the possibility to grow because the expected NPV of the former is $260$ less than that of the latter.

*Expansion options* are similar to growth options. For example, a traditional phone company may choose to expand into the wireless communications business. A company may subsidize an existing product line (e.g., traditional phone) because it allows the company to quickly expand in another line (e.g., wireless) when the opportunity is deemed favorable. Ford has not abandoned its pickup truck production even though profits on pickup sales have been negative for a couple of years. A rental car company more likely may hire technicians (at the same salary level) with expertise on both gas-powered and hybrid cars, even if it is currently operating gas-powered cars only, because the flexibility in its labor force increases the value of the option to expand into operating hybrid cars at some point in the future. All these behaviors would be hard to justify without recognizing the embedded expansion options. Valuing long-term investments that offer the flexibility to expand would essentially follow the three-step procedure described above for growth options.

*Abandonment options* represent yet another type of real option. A product is more salable if the buyer is given an option to return it for, say, 70% of the original price if the buyer is not satisfied. A house can sell at a higher price if it has a higher resale value. Perhaps a company prefers to outsource its research and development (R&D) function to an external party rather than keep its own in-house scientists. Abandonment options allow the company to avoid getting stuck with a money-losing business when things are not going right. A higher abandonment (or salvage) value increases the attractiveness of the product (or project). In fact, we have actually addressed abandonment options in our rental-car example when we assumed different salvage values depending on the outcome of the new energy bill and the type of car purchased.

To value a project with an embedded abandonment option, we focus on the salvage values. The three steps and the techniques explained above are once again applicable. To illustrate, suppose there exists an option to sell (i.e., abandon) the gas-powered car in one year
for $18,000 (salvage or abandonment value). Now we can ask whether operating a gas-powered car with such an abandonment option would be more valuable than operating a gas-powered car without this option. Intuitively, this option to abandon is valuable because it allows the company to react dynamically to the uncertain result of the new energy bill debate. If the bill is passed in one year, the company can sell the gas-powered car and switch to the more profitable hybrid. The question is how much value this abandonment option would add to operating the gas-powered car. Equation (7) shows how this estimation is done.

\[
NPV_{Gas} = PV(\text{after-tax cash flows}) - PV(\text{investment})
\]

\[
= \left[ \frac{\$6,000 + \$18,000}{(1 + 0.1)^3} \right] 
\times 0.4 + \left[ \sum_{i=1}^{3} \frac{\$6,000}{(1 + 0.1)^{(i+1)}} \right] 
\times 0.6 - \$20,000
= \$1,778
\]

(7)

The $18,000 abandonment value appears in year one with an associated probability of 40%. Now we see that this abandonment option makes the gas-powered car more attractive by $300; i.e., it increases the expected NPV for a gas-powered car from $1,478 (without abandonment option, see Figure 1) to $1,778 (with abandonment option).

**Benefits and Costs of Considering Real Options**

The biggest benefit of considering real options in the capital budgeting process is that they help decision makers reach optimal investment decisions. In this regard, real options complement or extend, not replace, traditional DCF decision models. As shown in our example, some embedded real options may lead to completely different investment decisions compared to those based solely on a traditional DCF analysis. Put another way, a less attractive investment proposal may be worth significantly more once we recognize its hidden treasures—investment flexibility based on the existence of real options.

In principle, because they can help optimize the capital budgeting process, real options should always be considered when making long-term investment decisions. Once managers grasp the concepts and are familiar with the basic framework for valuing projects embedded with real options, we would expect practice to change to the point where such options are routinely considered in the analysis of capital budgeting projects. Not long ago, DCF models were new to many managers who typically relied more on simple decision models, such as payback or accounting rate of return (ARR), for making capital budgeting decisions. Today, NPV has become a common financial management tool. Because of their role as management advisors, management accountants now need to become knowledgeable about what real options are and how they can extend DCF models in a meaningful way.

Real options have a flipside, too. The major cost of incorporating real options is that the decision process can quickly become quite complex. In our previous example, we assumed that the only factor that affected the choice of vehicles was the passage of a new energy bill within one year. Other

**Further Reading**

For those who would like to know more about real options analysis, here are some additional sources.


sources of risk can be associated with this investment decision. For instance, we might consider possible fluctuations in the price of gas or innovations in the automobile industry. The more factors we consider, the more complex the analysis becomes. When we attempt to incorporate more factors into the capital budgeting valuation framework, the more “noise” we introduce, making the results of our analysis potentially less accurate.

Second, incorporating real options into the analysis typically requires an array of probability estimates, one for each possible event, outcome, or scenario. For example, we assumed a 40% probability that the proposed energy bill would pass. In practical terms, this assessment may turn out to be the largest source of uncertainty.

Third, a typical capital investment project may have many embedded real options simultaneously, and it may be impractical to consider all of them. Nevertheless, real-life decisions are inherently complex; these complexities do not go away simply because we choose to ignore them in the decision models we use. Put another way, complexity of the situation only makes real options analysis a bit less reliable.

Now that we have presented an analysis of costs and benefits, we predict that real options analysis will become one of the common tools managers and accounting professionals use to evaluate long-term investment projects. Thus, management accountants need to learn as much as they can about real options so they can use them in their decision making. ■

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Endnotes
5 We leave out the calculation procedure because it is similar to Equation (4).