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ABSTRACT

What is the effect of capital structure policy on credit spreads? In a widely cited paper by Huang and Huang (2003), a few representative structural models of credit spreads have been calibrated and compared. Among them are the model by Collin-Dufresne and Goldstein (2001) and the one by Leland and Toft (1996). The former assumes exogenously given stationary mean-reverting leverage, the latter optimizes its capital structure endogenously. Their study shows that all the models they calibrated perform similarly in that they all explain a very small portion of the observed spreads. We make three improvements based on that study. First, we correct a mistake in their calibration for the CG model. Second, the Leland (1994) model of perpetual bond was used for simplicity. We strictly calibrate the LT model with finite maturities. Third, we use the same set of targets, so the model comparison is more meaningful. We argue that leverage ratio cannot be an appropriate target for calibrating the Leland-Toft model. This differentiates our calibration approaches. All these three issues have not been recognized thus far and we address them in this paper.

INTRODUCTION

The effect of a firm's capital structure policy on the credit spreads on the firm's debt is controversial. In a widely cited paper by Huang and Huang (2003), hereafter HH (2003), a few representative structural models of credit spreads have been calibrated and compared. Their study shows that all the models they calibrated perform similarly in that they all explain a very small portion of the observed spreads, especially for investment-grade bonds. Their results seem to be robust despite drastically different capital structure assumptions underlying those models. Indeed, structural models have long been recognized in their inability of generating high enough spreads between corporate and Treasury bond yields. This has prompted studies to include other missing factors. For example, Liu *et. al.* (2006) incorporate personal taxes, de Jong and Driessen (2005) and Driessen (2005) investigate liquidity factor, Johnson and Qi (2006) consider personal taxes, rating transition risk and liquidity factors.

However, before we search for other possible sources that contribute to the credit spreads, it is important to have more accurate knowledge of how much spread is due to default risk, and to what extent would capital structure policies affect the credit spreads. The well known study by HH (2003) provides such benchmarks and thereafter has been cited by many academic papers. A quick Internet search shows that 186 academic papers have cited their results as of the time when this paper is being written. Thus, it is important that their approach, results and claims are clear and accurate. While applauding their ambitious work and agreeing with their study in general, we have identified three flaws that are substantial enough to warrant careful re-investigation. To our best knowledge, this is the first time and paper to point out and attempt to address these three flaws. First, their calibration of the Collin-Dufresne and Goldstein (2001) model, hereafter the CG model, is based on inconsistent leverage targets by mistake. We will calibrate the CG model using the same set of target leverage ratios consistent with all other models in their study. This is a minor yet necessary correction to their study. Second, the true Leland-Toft model, hereafter the LT model, is for bonds with finite maturity. But in HH (2003), the Leland (1994) model of perpetual bond (and inaccurately called the LT model) was used to replace the true LT model for its simplicity, and they believe that if the LT model (of finite maturity) were used, the spreads would have been even lower. Thus, all structural models perform *similarly* (poorly). We will show that this is completely true. To address this flaw, we calibrate the true LT model. Our results show that their claims have to be understood in context. In particular, consistent with their study, the majority of the observed spreads cannot be explained by neither of the two models. Nevertheless, the calibrated LT model appears to have significantly higher explanatory power than the CG model. In addition, we also calibrated the LT model for 4-year bonds which is missing from HH (2003) because of their use of perpetual bond model. Third, it appears that the calibration targets are the same for both the LT and the CG

models (and others too). We argue that the inclusion of leverage ratio as a target for the LT model is problematic because of its underlying tradeoff nature. In this paper, we will explain why leverage is an appropriate a target for calibrating the CG model but not for LT model. This differential treatment of the CG and LT model makes our calibration approach different from theirs. We believe that our three modest contributions provide necessary corrections, clarifications and extensions to the current literature regarding credit spread and its relation with the capital structure policy.

The paper proceeds as follows. Section 2 reviews the models to be studies. Section 3 explains model implementation, calibration and the choice of calibration targets. Section 4 provides the results and analysis. Section 5 concludes the paper.

BRIEF MODEL REVIEW

Both the CG (2001) model and the LT (1996) model are constructed on the first-passage time approach which models bankruptcy as a stochastic process hitting a threshold for the first time. This bankruptcy threshold can be thought of as the firm's market debt value, hence the higher the leverage ratio, the more probable this threshold being hit. Ceteris paribus, there is a monotonically increasing relationship between default probability and leverage ratio. This is true to both models. However, what differentiate them is how to treat the leverage ratio. The CG model assumes an exogenously determined target leverage ratio which follows a mean-reverting stochastic process. The LT model is based on the tradeoff between corporate tax benefits and bankruptcy costs, which endogenously sets an optimal leverage ratio. This difference turns out to be vital for correct model calibration.

The Collin-Dufresne and Goldstein Model

In the CG model, the leverage is exogenously given. The firm value V_t follows a geometric Brownian motion. The firm value V_t is specified under the risk-neutral measure by

$$\frac{dV_t}{V_t} = (r_t - \delta)dt + \sigma dz_1(t) \qquad (1)$$

where z_1 is a Brownian random variable, δ is the payout ratio and σ is the volatility. The spot rate r_t follows the Vasicek (1977) process,

$$dr_t = \beta(\theta - r_t)dt + \eta dz_2 \qquad (2)$$

where β, θ and η are constants with the coupling $dz_1 dz_2 = \varsigma\, dt$. Default occurs when the firm value hits the bankruptcy threshold K, or $l_t = \log(K/V_t) = 0$. The log-default threshold k_t is mean-reverting:

$$dk_t = \lambda[y_t - \nu - \phi(r_t - \theta) - k_t]dt \qquad (3)$$

where $y_t = \log V_t$, $(y_t - \nu)$ sets the target threshold with ν and λ adjusting the mean-reverting speed, and $\phi \geq 0$. Upon default, bondholders can recover a fraction of the face value. Denoting the loss rate by L, then the price of a risky zero-coupon bond is given by:

$$P^T(r_0, l_0) = D(r_0, T)[1 - LQ^T(r_0, l_0, T)] \qquad (4)$$

where $D(r_0, T)$ is the price of the risk-free zero-coupon bond and $Q^T(r_0, l_0, T)$ is the cumulative default probability before T under the T-forward measure, and its specific form is given in CG (2001). For defaultable coupon bonds, we assume the coupon loss rate is 100 percent, i.e., $L_{coupon} = 1$. Coupon bond is treated as a portfolio of zero-coupon bond. The yield to maturity Y satisfies the following equation:

$$P^T = e^{-YT} + \sum_{i=1}^{T} ce^{-Y \times i} \qquad (5)$$

where c is the coupon. The yield spread, $YS(T) = Y - Y^T$, is defined as the difference between Y, the yield on a corporate bond, and that on a Treasury, Y^T. Both have the same maturity T.

The Leland and Toft Model

The LT model (1996) endogenously decides a bankruptcy boundary V_B which balances the tradeoff between the benefit of corporate tax shields and the bankruptcy costs. Firm value of an unlevered firm, V, is assumed to follow the diffusion process:

$$\frac{dV}{V} = [\mu(V,t) - \delta]dt + \sigma dZ \qquad (6)$$

where $\mu(V,t)$ is the expected rate of return on the firm's asset, Z is a standard Wiener process. The firm issues debt continuously to replace the debt that is expiring, hence maintaining stationary leverage. Within a unit of time, the firm issues debt d with a continuous constant coupon flow $c(T)$, principal $p(T)$, and maturity T. Upon default, bondholders receive a fixed portion ρ of the asset value V_B. Given this debt policy, the firm would have debt outstanding with time to maturity from 0 to T. The total debt value D is then given by integrating $d(V,V_B,t)$ over the period of T:

$$D(V,V_B,T) = \int_{t=0}^{T} d(V,V_B,t)dt \qquad (7)$$

The levered firm value $W(V,V_B,T)$ equals the unlevered firm value V plus leverage benefits $h(V,V_B)$ less bankruptcy costs $B(V,V_B)$,

$$W(V,V_B,T) = V + h(V,V_B) - B(V,V_B) \qquad (8)$$

where $h(V,V_B) = \frac{\tau_C C}{r}\left[1 - \left(\frac{V_B}{V}\right)^{a+z}\right]$ and $B(V,V_B) = (1-\rho)V_B\left(\frac{V_B}{V}\right)^{a+z}$. Other parameters are the corporate income tax rate τ_C, $a = \frac{r - \delta - (\sigma^2/2)}{\sigma^2}$, $b = \ln\left(\frac{V}{V_B}\right)$, $z = \frac{[(a\sigma^2)^2 + 2r\sigma^2]^{1/2}}{\sigma^2}$. Applying the smooth-pasting condition $\left.\frac{\partial E(V,V_B,T)}{\partial V}\right|_{V=V_B} = 0$ to maximize the equity value $E = W - D$ and using the additional par-bond constraint, the closed-form solution is obtained by LT (1996) for the price of the firm's newly issued debt (per unit of time),

$$p(T) = \frac{\frac{c}{r} - e^{-rt}\frac{c}{r}[1 - F(T)] + \left[\rho(T)V_B - \frac{c}{r}\right]G(T)}{1 - e^{-rt}[1 - F(T)]}, \qquad (9)$$

where the cumulative default probability is $F(t,V,V_B) = N[h_1(t)] + \left(\frac{V_B}{V}\right)^{2a} N[h_2(t)]$ and $G(t,V,V_B) = \left(\frac{V_B}{V}\right)^{a-z} N[q_1(t)] + \left(\frac{V_B}{V}\right)^{a+z} N[q_2(t)]$, with $N(\cdot)$ denoting the cumulative standard normal distribution. The parameters $h_1(t)$, $h_2(t)$, $q_1(t)$ and $q_2(t)$ are functions of volatility σ, interest rate r, and bankruptcy boundary V_B. They can be found in LT (1996). The yield of this par bond is simply $Y = c/p$ and the yield spread is defined as the difference between Y and the riskfree rate r.

MODEL IMPLEMENTATION AND CALIBRATION

Numerical Implementation

To implement the CG model, I choose similar parameter values as in HH (2003). For example, the mean-reverting coefficient λ = 0.2; the long-term average leverage ratio ν = 38%; the coupling coefficient $\varsigma = -0.25$ for $dz_1 dz_2 = \varsigma\, dt$; coupon rate c = 8.13% and payout rate $\delta = 6\%$. For the Vasicek interest, $\beta = 0.226$, same as HH (2003). We choose interest rate volatility $\eta = 1.5\%$ as in CG (2001). Similarly, for the LT model, I also choose

constant interest rate r = 8%; bankruptcy cost $(1-\rho)=15\%$ of the firm value at default V_B; The reasons for making these choices are given in detail by HH (2003). The corporate tax rate is chosen to be $\tau_C = 35$ percent. The initial value of the unlevered firm's V = 100.

Model Calibration and the Target Choices

The essence of calibration is to tune some unobserved variables such that the model generates the default probability to match the observed historical data for the same sample time period. This is in agreement with the general spirit of HH (2003). However, what differs from theirs is that we treat the CG and LT model differently. In the CG case, leverage is exogenously given, thus the average leverage for each bond rating can be used as an important calibration target. While in the LT case, leverage is endogenously optimized based on the tradeoff between corporate tax shields and bankruptcy costs. It is well known that tradeoff theory cannot explain the observed relationship between bond rating and the average leverage ratio. It predicts that profitable firms (e.g., AAA-rated firms) should use more debt than less profitable firms (e.g., BB-rated firms).

Therefore, based on the model's structure, we make the following choices (also shown in Table 1 and 2). For the CG model's calibration targets, we choose default probability, equity risk premium, leverage ratio, and recovery ratio (of the face value). For the LT model, we choose default probability, equity risk premium, and recovery ratio (of the firm value at default V_B). Such differential treatment is essential for a meaningful calibration. For the CG model, leverage ratio is inputted, and the model will generate a credit spread for this initial leverage input. However, the LT model will endogenously generate both the optimal leverage ratio as well as the default probability simultaneously. If we include leverage ratio in the targets to calibrate the LT model, then it is impossible to simultaneously have the model agree to the target leverage ratio as well as the default probability. In this case, the model is said to be over-specified. Thus, the question boils down to which one is an appropriate target for the LT model, the default probability or the average leverage ratio? Certainly, understanding that the LT model is a tradeoff model, we should drop leverage as a target. This is in line with Hittle *et. al.* (1992) that only 11 percent of the surveyed 500 large OTC firms use optimal target capital structure.[1] Therefore, we do not expect the leverage ratio of the majority of the firms to be explained by the tradeoff theory. Then why do we still calibrate the LT model? This is because there are still firms (say, 11 percent by Hittle *et. al.*, 1992) that follow the tradeoff theory. These firms shall have similar default probability to that of other firms in the same credit rating class.

RESULTS AND ANALYSIS

Table 1 reports the credit spreads predicted by the CG model after calibration. Our results show that the credit spreads generated by the CG model given the correct leverage targets are generally significantly lower than those from HH (2003). The only exception is Aaa bond, for which our credit spread is 0.6 bps while theirs is 0 bps. We believe this exception may come from rounding errors in the numerical calculation. The overall results are not surprising because they mistakenly used much lower leverage targets as follows.[2]

	Aaa	Aa	A	Baa	Ba	B
Leverage targets chosen for CG calibration in HH (2003)	7.8	12.7	19.2	26.0	32.1	39.4
Leverage targets for CG calibration in this study	13.1	21.2	32.0	43.3	53.5	65.7

[1]We also note that Pinegar and Wilbricht (1989) surveyed the Fortune 500 firms and find 31 percent of them use target (i.e., optimal) capital structure.
[2] For example, see Column 2 of Table 6 in HH (2003), and compare leverage ratios they used to calibrate other models.

Table 2 shows the target parameters and our calibration results for the LT model. HH (2003) only calibrated for 10-year bonds because they used a perpetual bond model to approximate the finite maturity LT model. We calibrated for both since we implemented the true finite maturity LT model.[3] For Aaa-rated bonds, there is a considerable difference between spreads from the finite maturity LT model (26 bps) and HH (2003)'s perpetual version (37 bps). For other investment grade bonds, this difference is surprisingly negligible. This finding is in contrast to the statement in HH (2003) - "at first glance, the LT model seems to generate higher credit spreads for investment-grade bonds than the LS model did in our base case. This, however, is due to the fact that the LT model considered here has a perpetual bond, which, for investment-grades, should have a higher credit spread than 10-year bond." The above quoted statement is based on their intuition without quantitative estimates. As shown by our results in Column (7) and (8) of Table 2, compared to their perpetual bond approximation, the LT model (of finite maturity) generates fairly similar spreads and even higher spreads for Baa-rated bonds. However, this credit spread difference on Baa-rated bonds is only 4 bps.

The central theme in HH (2003) is that pretty much all structural models perform similarly poorly in generating credit spreads after calibration. Our results confirm this belief only to certain extent. For investment-grades, both the CG and the LT model can only explain a small portion of the observed spreads. However, the calibrated LT model has significantly greater explaining power. If we compare Table 1 and 2, it is easy to see that the calibrated LT model (of finite maturity) can explain 8 times as much spreads for Aaa-rated bonds as can the CG model (i.e., 26/3.2 = 8.1), and almost 3 times as much for Baa-rated bonds (i.e., 64/25 = 2.6). Thus, the calibrated LT model clearly can generate significantly higher spreads for investment-grades. It is inaccurate to claim that all structural models of term structure of credit spreads perform similarly poorly after calibration. There is a significant difference between the two models we studied. However, the overall explanatory power is unsurprisingly unsatisfactory because there are other factors left out of the picture. This indeed motivates studies of other factors affecting the yield spreads. For example, Liu *et. al.* (2006) incorporated personal taxes, de Jong and Driessen (2005) and Driessen (2005) investigate liquidity factor, Johnson and Qi (2006) consider personal taxes, rating transition risk and liquidity factors.

CONCLUSIONS

In this paper, we calibrated two structural models of credit spreads with different capital structure policies – the CG and LT models. This work is motivated by a few flaws we identified in a widely cited paper by Huang and Huang (2003). While our approach agrees with their general calibration spirit, we disagree with their treatment of the LT model on the leverage target. We argue that leverage ratios can only be used for calibrating the CG model but not the LT model because the latter is a tradeoff theory model. It is impossible to calibrate the LT model by having it simultaneously generate the observed leverage ratios and the default probabilities. It is well documented that tradeoff theory cannot explain the observed trend in average leverage across bond ratings.

There are mainly three contributions from our results. First, we applied the consistent leverage targets for calibrating the CG model for both 4- and 10-year bonds. This may serve as a minor correction to HH (2003) since their calibration of the CG model is based on inconsistent leverage targets by mistake. Second, the true LT model (of finite maturity) was not calibrated in HH (2003). Instead, the Leland (1994) model of perpetual bond was used for its simplicity, and it is believed there that all structural models perform similarly (poorly) since they argue that if the LT model of finite maturity were used, the generated spreads would be even lower. However, our results do not fully support these beliefs. Both models can only explain a small portion of the observed spreads for investment-grades, but the LT model has significantly more explanatory power (e.g., 8 times as much for the Aaa-rated bonds). Third, we explain why leverage should not be chosen as a target for calibrating the LT model. This discernment between how we treat the CG and LT model makes our calibration approach different from theirs. We provide a theoretical argument for why leverage cannot be chosen as one of the calibration targets. Our study contributes some clarifications to the literature.

[3] Rigorously speaking, the perpetual bond model is not the LT model. It is the Leland (1994) model.

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Table 1. The CG Model and Credit Spreads

This table lists the target parameters that we calibrate the model against. The sixth column shows the observed average yield spread. The data presented here are from HH (2003) for the period of 1973 – 1993. The original sources are Lehman bond index and Moody's. Columns (10) and (11) are HH's results for comparison.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Rating	Parameters used in the model ("trivial" target parameters)*			Target parameter		**This investigation**		From HH (2003) (Based on different target leverage ratios by mistake)	
	Recovery rate (% of face value)	Leverage ratio (%)	Equity premium (%)	Cumulative default probability (%)	Observed spread (bps)	Due to default (bps)	Due to default (%)	Due to default (bps)	Due to default (%)
4-Year Bond									
Aaa	51.31	13.1	5.38	0.04	55	**0.6**	1.0	0.0	0.1
Aa	51.32	21.2	5.60	0.23	65	**3.1**	4.9	6.3	9.7
A	51.31	32.0	5.99	0.35	96	**4.8**	5.1	9.9	10.3
Baa	51.32	43.3	6.55	1.24	158	**17.1**	11.0	31.1	19.7
Ba	51.31	53.5	7.30	8.51	320	**114.5**	37.5	168.0	52.5
B	51.32	65.7	8.76	23.32	470	**340.1**	72.4	435.3	92.6
10-Year Bond									
Aaa	51.31	13.1	5.38	0.77	63	**3.2**	5.1	11.4	18.2
Aa	51.32	21.2	5.60	0.99	91	**5.1**	5.6	14.9	16.4
A	51.31	32.0	5.99	1.55	123	**8.4**	6.8	22.5	18.3
Baa	51.32	43.3	6.55	4.39	194	**25.1**	13.0	52.3	26.9
Ba	51.31	53.5	7.30	20.63	320	**127.0**	39.7	182.7	57.1
B	51.32	65.7	8.76	43.91	470	**304.2**	64.7	371.6	79.1

*Trivial target parameters are observed values that can be directly inputted into the model. The implied asset volatilities were chosen such that the model generates exactly the same default probabilities matching those in column (5).

Table 2. The LT Model and Credit Spreads

This table lists the target parameters and calibration results. The sixth column shows the observed average yield spread. The data presented here are from HH (2003) for the period of 1973 – 1993. The original sources are Lehman bond index and Moody's. Columns (10) and (11) are HH's results for comparison (4-year bond results are not available).

(1)	(2) (3)	(4)	(5)	(6)	(7)	(8)	(10)	(11)
Rating	Parameters used in the model ("trivial" target parameters)		Target parameter		**This investigation**		From HH (2003) Perpetual bond approximation	
	**Recovery rate (% of default firm value V_B)	Equity premium (%)	Cumulative default probability (%)	Observed spread (bps)	Due to default (bps)	Due to default (%)	Due to default (bps)	Due to default (%)
4-Year Bond								
Aaa	85	5.38	0.04	55	**3**	5.3	N/A	
Aa	85	5.60	0.23	65	**12**	18.3		
A	85	5.99	0.35	96	**17**	17.4		
Baa	85	6.55	1.24	158	**48**	30.6		
Ba	85	7.30	8.51	320	**106**	33.1		
B	85	8.76	23.32	470	**232**	49.4		
10-Year Bond								
Aaa	85	5.38	0.77	63	**26**	41	36.9	58.6
Aa	85	5.60	0.99	91	**30**	33	34.5	37.9
A	85	5.99	1.55	123	**38**	31	38.5	31.3
Baa	85	6.55	4.39	194	**64**	32	59.5	30.6
Ba	85	7.30	20.63	320	**160**	50	165.7	51.8
B	85	8.76	43.91	470	**315**	67	408.4	86.9

** As in HH (2003), I also choose the lower of 85% of V_B or 51.31% of the face value as the recovered amount when default happens.