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LEARNING TO NOTICE AND USE STUDENT THINKING IN UNDERGRADUATE MATHEMATICS COURSES

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Abstract

The goal of this study was to evaluate the outcomes of an intervention focused on developing mathematics graduate teaching assistants' skills of noticing and effectively responding to instances of student mathematical thinking that have significant potential to further students' learning. Four mathematics graduate teaching assistants participated in a semester-long intervention in which video of undergraduate mathematics lessons was individually analyzed and then discussed collectively with the researcher on a weekly basis. The MOST Analytic Framework (Leatham, Peterson, Stockero, & Van Zoest, 2015; Stockero, Peterson, Leatham, & Van Zoest, 2014) was introduced as a tool to aid in the analysis and discussion of the instances the graduate teaching assistants tagged using video analysis software. A pre- and post-interview was also conducted in which the graduate teaching assistants analyzed a video in real time and proposed responses to instances in the video they deemed important. In addition, the graduate teaching assistants completed an assessment of their common content knowledge. The instances tagged by the graduate teaching assistants were categorized by the researcher and then analyzed to track changes in noticing skills throughout the intervention. Results indicate that the intervention was successful in improving the graduate teaching assistants' noticing skills in a variety of ways and in their ability to propose student-centered responses to instances they identified in video. Assessment scores showed no evidence of common content knowledge contributing to differences observed in the graduate teaching assistants' noticing skills.

Chapter 1: Introduction

Research has shown that learner- or student-centered instruction leads to more effective learning for people of all ages (National Research Council [NRC], 2005). Higher education has been slow or unsuccessful in implementing student-centered instructional methods (Barr & Tagg, 1995; Felder & Brent, 1996), however, with transmissive instruction by way of lecture still prominent worldwide (Ramsden, 2003; Svinicki & McKeachie, 2014). Possible challenges for adopting student-centered instruction include student resistance, comfort level of instructors, and the time needed to see results (Felder & Brent, 1996; Seymour, 2002). Some researchers suggest that a way to promote changes in teaching methods in higher education is through graduate teaching assistant (GTA) training (e.g., Cano, Jones & Chism, 1991). Since effectiveness of GTAs typically affects undergraduate students in their first and second years of university study, GTA training is also important in retention of undergraduate students (Cano et al., 1991; Speer, Gutmann, & Murphy, 2005). With a high need for workers in the STEM fields (President's Council of Advisors on Science and Technology [PCAST], 2012), retention of students is especially important in university science, technology, engineering and mathematics (STEM) departments (Seymour & Hewitt, 1997; Suchman, 2014).

At the K-12 level, researchers have recognized a teacher's ability to notice particular aspects of instruction as it unfolds as an important area of expertise in the implementation of student-centered instruction. In particular to mathematics classrooms, *professional noticing of [students'] mathematical thinking* has been defined to include attending to, interpreting, and deciding how to respond to students' strategies and understanding (Jacobs, Lamb, & Philipp, 2010). Many studies have shown that the expertise of teacher noticing can be learned and improved through teacher education and professional development (Jacobs et al., 2010; McDuffie et al., 2014; Santagata, 2011; Schack et al., 2013; Sherin & van Es, 2009; Stockero, Rupnow, & Pascoe, 2015; van Es, 2011). Although training for undergraduate instructors and GTAs that targets teacher noticing is not widely practiced, the gains made with preservice and inservice mathematics teachers at the K-12 level suggest that similar results may be possible in higher education.

Also foundational to effective mathematics teaching are the domains of mathematical knowledge for teaching proposed by Ball, Thames, and Phelps (2008): common content knowledge, specialized content knowledge, horizon content knowledge, knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. Of critical importance is common content knowledge, the mathematical understanding and proficiency used in diverse contexts, not exclusive to teaching. Without common content knowledge, a teacher could not adequately guide students in their building of such knowledge or hope to develop the other domains of mathematical knowledge for teaching. In addition, mathematics teachers would not likely have the ability to determine which instances of students' mathematical thinking are important to notice without a strong command of common mathematics content knowledge. Because both the ability to notice and common content knowledge are factors that contribute to effective mathematics teaching, it would be of interest to investigate if and how these factors are related.

This work examines the outcomes of a professional development intervention focused on analyzing undergraduate mathematics lesson videos with a teacher noticing framework as a means to train mathematics GTAs to use student thinking more effectively, and thus enact student-centered instruction in their classrooms. Of particular interest is measuring the effectiveness of the intervention in improving GTAs' noticing of *mathematically significant pedagogical opportunities to build on student thinking* (MOSTs) (Leatham, Peterson, Stockero, & Van Zoest, 2015; Stockero, Peterson, Leatham, & Van Zoest, 2014) and in supporting their ability to propose student-centered responses to such instances. In implementing and documenting the effects of a professional development intervention that includes analyzing classroom video with the MOST framework, this work seeks to answer to the following research questions: (a) How effective is the intervention in improving GTAs' noticing of mathematically significant pedagogical opportunities to build on student thinking (MOSTs)?; (b) How effective is the intervention in supporting the GTAs' ability to propose in-the-moment student-centered responses to instances they identified in video?; (c) How valuable was the intervention from the GTAs' perspectives?; and (d) What is the relationship between the GTAs' common mathematical content knowledge and the development of their noticing skills during the intervention?

Chapter 2: Literature Review

Transitioning to Student-centered Instruction

Since the 1990s, there has been a gradual shift in education away from transmissive, teacher-centered instruction toward student-centered or learner-centered instruction. Transmissive instruction, which remains a widespread practice, assumes that students will learn simply by teachers relaying verbally what it is that the teacher knows (Jonassen & Land, 2012). A central notion of this method of instruction is that improved student learning is accomplished by improving the communication skills of the teacher. The shift to student-centered instruction has stemmed from the belief that learning is not synonymous with the transmission of ideas, but rather, learning takes place through making sense of ideas (Jonassen & Land, 2012). Student-centered instruction attempts to address this change in learning theory by advocating that students should not only be exposed to concepts, but should also be engaged with sense-making and opportunities to struggle with concepts in order to achieve significant learning (Coppola, 2014; National Council of Teachers of Mathematics [NCTM], 2014). At the core of such instruction is the understanding that "[effective] instruction must begin with close attention to students' ideas, knowledge, skills, and attitudes, which provide the foundation on which new learning builds" (NRC, 2005, p. 14).

At the college level, Barr and Tagg (1995) describe this shift as from an *Instructional Paradigm*, in which teaching consists mainly of lecturing and transmission of knowledge from faculty to students, to a *Learning Paradigm*, in which the goal is to provide opportunities for students to be engaged in building their own knowledge in order to improve student learning. While these shifts have been in motion since the

1990s, even now in the 2010s teaching at the university level is still widely entrenched in the transmissive theory of student learning, with lecturing being the most popular form of instruction internationally (Ramsden, 2003; Svinicki & McKeachie, 2014). The learning intervention in the current study attempts to promote a shift from an instructional paradigm to a learning paradigm by attuning GTAs to the ideas that students bring to the classroom.

Obstacles to Implementing Student-centered Instruction

While it is typically recognized that a shift away from traditional lecture is necessary for effective student learning, adoption and implementation of student-centered instruction has often been slow or unsuccessful (Barr & Tagg, 1995; Felder & Brent, 1996). According to Felder and Brent (1996), there are many challenges in implementing student-centered instruction. Instructors may be concerned about the amount of time this type of instruction takes and the potential loss of control compared to lecturing. Improvements in student learning using student-centered instruction are not instantaneous upon implementation. Students are likely to initially resist a shift from being passive observers in the classroom to being held more responsible for and involved in their own learning. It is factors such as these that could lead to instructors giving up on student-centered instruction and reverting to the comfort of lecturing.

A clinical observation study of college science and mathematics laboratory courses by Winter, Lemons, Bookman and Hoese (2012) found that novice instructors—a majority being graduate student instructors with little to no teaching experience—face difficulties with student-centered instruction. After follow-up interviews with the instructors, the researchers attributed these difficulties to a lack of instructor knowledge of and practice with student-centered instructional techniques, lack of instructor knowledge about how students learn, and underlying assumptions of instructors and students that all information must be presented during class time and that it is the responsibility of the instructor to present this information.

These difficulties do not only exist for novices, however. According to the NRC (2003), few STEM faculty have completed organized training on effective pedagogical methods before beginning their positions as undergraduate educators; once established in their positions, there are few opportunities for them to engage in structured training to continually develop more effective teaching skills, as such training is not widely available in the university setting. Other research suggests that faculty, especially STEM faculty, do not have enough incentive or support from their departments to pursue the professional development necessary to foster instructional improvement, especially the use of student-centered instructional techniques (Fairweather, 2008; Seymour, 2002; Suchman, 2014; Walczyk & Ramsey, 2003; Walczyk, Ramsey, & Zha, 2007). With research and publications being paramount in the earning of tenure, raises and promotions at the institutional level, STEM faculty are pressured to devote more time to research and less time to developing their teaching practices (Braxton, Luckey & Helland, 2002; Fairweather, 2008; Suchman, 2014; Walczyk et al., 2007). It has been suggested that institutions redesign the reward structure so that instructional practices receive more weight in decisions for tenure, raises and promotions (Fairweather, 2008; Seymour, 2002; Suchman, 2014; Walczyk & Ramsey, 2003; Walczyk et al., 2007). Until this happens, however, the situation of little time to focus on improving teaching is unlikely to change. In summary, the research suggests that undergraduate STEM instructors, both GTAs and faculty alike, face significant challenges in practicing student-centered instruction and in accessing the professional development they need to effectively implement such instructional practices.

STEM Retention

In the mid 1990s, a major concern in science and mathematics education was that the U.S. would face a shortage of specialists in STEM fields due to an insufficient number of college students choosing and continuing to study STEM disciplines (Seymour & Hewitt, 1997). In the interest of finding out more about the retention and attrition of undergraduates in STEM disciplines, Seymour and Hewitt (1997) conducted an interview study of 335 students at seven four-year institutions. The researchers found that poor teaching by science, mathematics and engineering (SME) faculty was a concern among over 90% of those students who switched out of an SME major. Even students who did not switch majors were concerned about quality of teaching in SME courses. From these results, it is clear that teaching must be effective in undergraduate STEM courses in order to retain students majoring in STEM disciplines. Criticisms of faculty by students in this study included that faculty exhibited limited knowledge of how students learn, particularly with respect to two elements of "good teaching": promoting discussion and the sense of discovering things together. Students also claimed that faculty often did not present lessons in a comprehensible manner or check for student understanding during instruction, key components of good lessons that students reported helped them to understand and apply theoretical concepts. These elements of "good teaching" and "good lessons" are very much in line with a more learner-centered method of instruction. When making suggestions to improve SME pedagogy, students in the Seymour and Hewitt study expressed that all faculty who teach lower level classes should obtain professional pedagogical training. In many universities today, GTAs are some of the primary instructors of lower level undergraduate classes, especially in STEM departments.

GTA Training

Cano et al. (1991) argued that GTAs are in a prime position, both in their current positions as undergraduate instructors and in the future as potential faculty, to influence change in instructional methods in higher education—an argument echoed by Speer et al. (2005) in the specific case of mathematics GTAs. Helping GTAs assert this influence is particularly important because GTAs typically are assigned to teach lower-division classes, where the retention of first- and second-year undergraduate students is critical (Cano et al., 1991; Speer et al., 2005). Anderson (1991) claimed that GTAs are a somewhat untapped resource to promote student excellence in that they are still eager and largely unsullied by traditional instructional methods. It is encouraging that it has been documented that GTAs do value training (Gray & Buerket-Rothfuss, 1991), placing the most value on training when it includes both the theoretical basis (*why*) and realistic strategies (*how*) for implementation of teaching methods in their classrooms (Hardré & Burris, 2012). The NRC (1999) has suggested that a way to improve undergraduate education in the STEM disciplines is by integrating pedagogical training into degree completion for graduate and postdoctoral students.

An interview study of STEM GTAs done by Gilmore, Maher, Feldon, and Timmerman (2014) showed that out of four hypothesized variables that might affect GTAs' orientations toward teaching (mentorship, training for teaching, teaching experience/teacher development, and research experience), only mentorship was positively associated with teacher orientations becoming more student-centered over time. However, ongoing training was noted as valuable by interviewees, especially when it included discussions with peers. Supporting the need for social interaction in GTA learning activities, Park (2004)—in summarizing the literature about GTA training in North America—noted that "a thoughtfully designed GTA training programme…should include active learning strategies, constructivist learning strategies, activities that foster social interaction, and motivational strategies" (p. 15). These sources suggest that professional development for GTAs should provide opportunities to interact with peers.

To support mathematics GTA training, Friedberg (2005) orchestrated the development of case studies describing realistic, complex issues in undergraduate mathematics courses. These case studies were designed to be topics of group discussion to allow the GTAs to process difficult teaching situations so that they may better address such situations in their own classrooms. It was found that the case studies were effective in establishing dialogue focused on teaching and in providing new teaching experiences, suggesting that the use of artifacts that mimic classroom situations in GTA professional development may promote changes in the ways that GTAs think about instruction and could, therefore, potentially improve their practice.

Speer (2004) outlined two professional development activities that she used and subsequently modified as part of a semester-long course for mathematics GTAs. In order to support the GTAs in their facilitation of collaborative group learning, a form of student-centered instruction, the GTAs watched videos of their own instruction and observed a discussion section led by a peer. Initially these activities, although helpful in some ways, were not particularly useful in focusing GTAs' attention on issues with facilitating collaborative group work, such as not asking enough questions of students, not requiring reason or explanation in student responses, and providing answers to problems too quickly. In response, GTAs were given reflective questions to provide focus for future video analyses and peer observations. This adjustment proved fruitful for the GTAs in improving both their focus on issues regarding group work facilitation in the professional development activities and the likelihood of them using better facilitation practices in their classrooms. These results suggest that structuring GTA professional development activities by providing reflective prompts leads to greater learning about student-centered instructional methods and improved classroom instruction.

In sum, the research suggests it is advantageous to target GTAs through training to support a shift toward student-centered instruction in higher education, especially when such training involves strategies for implementing instructional methods, a social component, the use of classroom artifacts such as case studies and video, and an organized structure by which to use classroom artifacts. This study focuses on providing training in a social setting to support a shift in mathematics GTAs' instructional methods away from the traditional transmissive form of instruction toward student-centered instruction by using a framework to structure the analysis of classroom video and providing practice in formulating student-centered instructor responses.

Teacher Noticing: A Basis for Effective, Student-centered Instruction

The NRC (2005) suggests that effective student-centered instruction cannot be enacted without first attending to students' ideas, knowledge and skills, since these are foundational to constructing new knowledge. These indicators of student thinking are particularly important to effective mathematics instruction, especially when used to adapt instruction to enhance student learning (NCTM, 2014). This attention to students and their ideas closely relates to what is called the practice of *teacher noticing*—the act of continually attending to and adjusting to that which occurs during instruction (e.g., Sherin, Jacobs, & Philipp, 2011). This definition highlights that teacher noticing is not passive, but rather an active process. The construct of noticing focuses on what teachers attend to—including both what teachers choose to respond to and what teachers choose not to respond to—and how they make sense of events in the complexity of the classroom (Sherin, Jacobs, & Philipp, 2011). In order to advance student learning, teachers must decide which events in the classroom are worthy of their attention, time and effort, and which ones are not as important.

Characterizing teacher noticing

Researchers in the field of mathematics education characterize teacher noticing in a variety of ways. Based on Endsley's (1995) notion of *situation awareness,* Miller (2011) adopted a definition of teacher noticing as "quickly perceiving student behavior and understanding what that behavior means in terms of student understanding and engagement" (p. 61). In attempting to improve preservice teachers' observational skills in order to later promote the noticing of important classroom events, Star and Strickland (2008) studied preservice teachers' noticing in five focused observation categories: classroom environment, classroom management, tasks, mathematical content, and communication. van Es and Sherin (2002) suggested that noticing involves three key components: identifying important classroom events, forming connections between such notable events and the broader principles of teaching and learning, and reasoning about these events. Jacobs et al. (2010) characterized the expertise of *professional noticing of* *[students'] mathematical thinking* as three skills: attending to, interpreting, and deciding how to respond to students' strategies and understanding. In providing training for GTAs, it is necessary to consider what exactly the GTAs should and should not be paying attention to. This work focuses on improving teacher noticing of student mathematical thinking, and thus adopts Jacob and colleagues' definition of noticing as comprised of three skills of expertise, since this type of noticing closely aligns with student-centered instruction.

Teacher noticing professional development

While there are GTA training programs in place that aim to shift instruction from being teacher-centered to being student-centered (e.g., Gibbs & Coffey, 2000), only a couple reported programs specifically promote teacher noticing for instructors at the undergraduate level. A study by Williams and Case (2015) focused on what international teaching assistants (ITAs) notice about their teaching in a video of their own instruction. Interviewers found that watching the video allowed the ITAs to become self-aware of their classroom presence, justify their instructional choices, and evaluate their instructional practices in order to improve. While this study involved noticing, it was focused on noticing aspects of instruction rather than student thinking. Drawing on ideas from Mason's (2002) *discipline of noticing*, Breen, McCluskey, Meehan, O'Donovan and O'Shea (2011) had a group of mathematics lecturers document in writing critical incidents that occurred in their own undergraduate mathematics classrooms each week. Their written narratives were circulated among the group, and the group met twice per semester over an academic year to collectively discuss the incidents documented in the narratives. The participants reported that their in-the-moment noticing improved over time, and that they became better able to consider students' potential perspectives on the incidents. Although this study seems to have helped the participants develop a more student-centered perspective, the writings were based on recollection rather than grounded in classroom artifacts.

The literature focused on using professional development to promote teacher noticing of student thinking with preservice and inservice K-12 mathematics teachers is more widespread and shows promising results. The work of Jacobs and colleagues (e.g., Jacobs et al., 2010; Jacobs, Lamb, Philipp, & Schappelle, 2011) provided evidence that attending to and responding to student thinking in a way that keeps instruction student-centered is not a skill K-12 preservice and inservice teachers commonly possess, but can be learned with support. For example, in a study involving mathematics teachers with a variety of levels of experience, Jacobs et al. (2010) found that professional development seemed to be effective in improving all three skills of teachers' noticing of students' mathematical thinking—attending, interpreting, and deciding how to respond. Similarly, Sherin and Han (2004) found encouraging evidence that professional development centered on the noticing of student thinking translates to more noticing of student thinking during instruction. In fact, over the past several years, a growing body of research supports the notion that targeted interventions can improve noticing skills of teachers (Jacobs et al., 2011; Jacobs et al., 2010; McDuffie et al., 2014; Santagata, 2011; Schack et al., 2013; Sherin & van Es, 2009; Star & Strickland, 2008; Stockero et al., 2015; van Es, 2011), including deciding how to respond (e.g., Jacobs et al., 2010; Schack et al., 2013). Because these interventions have taken place in a variety of settings with a range of mathematics learners, it seems possible for a noticing intervention focused on student mathematical thinking to also improve the noticing skills of GTAs, potentially providing a foundation to improve their practice of student-centered instruction in undergraduate mathematics classrooms.

Use of video in noticing interventions

The tools that are used to support teachers in the development of professional noticing vary, but the use of video with both preservice and inservice K-12 mathematics teachers is quite common (e.g., Santagata, 2011; Sherin & van Es, 2009; Star & Strickland, 2008; Stockero et al., 2015; van Es, 2011). The popularity of using video is due to a number of benefits it has in teacher education, including that video can be paused and watched again, allowing the classroom events or student comments on video to be considered at length, and that teachers can view and develop new practices through the use of video (Sherin, 2004). Videos can also be used among a number of teachers, thereby allowing for the discussion of a range of reactions to a common experience of instruction (Sherin, Russ, & Colestock, 2011). When choosing classroom videos, professional developers are able to purposefully select videos with particular features that teachers may not otherwise experience in their own instruction (van Es, Tunney, Goldsmith, & Seago, 2014). Miller (2011) suggests that video from the teacher's point of view could be especially powerful when viewers are developing noticing skills.

While video can be a valuable tool in teacher education, using video is only as productive as the way it is used to support specific learning objectives (Seago, 2004). One specific objective for which researchers have successfully used video in targeted ways in professional development contexts is to understand and develop teacher noticing skills. Sherin & van Es (2009) used video clips for pre- and post-"noticing interviews" as well as for viewing and discussing classroom interactions during video club meetings in which a facilitator prompted participants to talk about what they noticed. The study found that teachers improved in their skills of noticing and attending to student mathematical thinking, both in the video club and in their classrooms. van Es (2011) found that teachers' ability to notice student thinking advanced to a higher level of reasoning over the course of professional development designed as a video club, where a group of inservice teachers met with a facilitator 10 times to discuss what they noticed in video clips of their own teaching. Stockero and colleagues (Stockero, 2014; Stockero et al., 2015; Stockero, Rupnow, & Pascoe, under review) used full-length classroom video to train preservice mathematics teachers to notice important instances of students' mathematical thinking. Participants in these studies analyzed video individually and met as a group with a mathematics teacher educator to discuss what they noticed. Results showed that even within the complexity of full-length classroom video, the preservice teachers improved in their noticing skills in a variety of ways, including becoming more focused on students, attending to specific mathematics, and becoming more analytical of student mathematical thinking. Together, this research suggests that video is a useful and effective tool in improving the noticing and reasoning skills of student mathematical thinking.

The intervention in the current study involved GTAs individually analyzing classroom video that was recorded as close to the teacher's perspective as possible, as suggested by Miller (2011), and meeting as a group with a facilitator to discuss what they noticed, much like the noticing work described in the previous paragraph (Sherin & van Es, 2009; van Es, 2011; Stockero, 2014; Stockero et al., 2015; under review). The videos analyzed were longer in length, like those used by Stockero and colleagues (Stockero, 2014; Stockero et al., 2015; under review) to more closely reflect the classroom experience; they included both videos from the GTAs' own classroom and those that were not, a distinction from the work of Sherin and van Es.

Use of video with a framework to support noticing

Research has also shown that the use of a framework to focus noticing can enhance learning and improve what is noticed in classroom video. Santagata (2011), for example, used the Lesson Analysis Framework to support inservice teachers in their analysis of full-length instructional video as part of a professional development program. With revisions to the framework and watching shorter segments of the video in succession, the Lesson Analysis Framework became more effective in supporting the teachers to address their difficulties in attending to and reasoning about student learning. Schack and colleagues (2013) used five video excerpts of interviews with children solving mathematical problems as part of a teaching methods course for preservice elementary teachers. The videos were used within the context of learning about the Stages of Early Arithmetic Learning framework (e.g., Steffe, Cobb, & von Glasersfeld, 1988) in order to develop the three skills of professional noticing of children's mathematical thinking described by Jacobs et al. (2010). By use of a pre- and post-assessment, it was found that all three skills of noticing of the preservice teachers significantly improved. McDuffie et al. (2014) framed viewing of video using four lenses (teaching, learning, task, power and participation) to focus the noticing of preservice teachers on equitable teaching practices for mathematics when engaged in a video analysis activity in a methods course. Findings showed that the preservice teachers in this study were able to discuss noticing at a higher level initially compared to a similar study done by van Es (2011), which suggests the more structured nature of the prompts focused on the four lenses better supported noticing (McDuffie et al., 2014). Stockero et al. (2015; under review) used video in the context of a field experience course for preservice secondary teachers in order to develop their noticing of student mathematical thinking. These researchers used the MOST Analytic Framework, as described by Leatham et al. (2015), to focus the preservice teachers' noticing, both when analyzing video individually and in group meetings. Results from the study suggested that the use of the MOST framework was successful in improving noticing of student mathematical thinking in a multitude of ways. The results of these studies suggest that the use of a framework in addition to video is even more effective in enhancing the noticing skills of mathematics teachers than the use of video alone because of the structured guidance it provides in directing the teachers' attention to, and reasoning about, specific features of teaching and learning mathematics.

Since the use of a framework with video has been shown to support and improve the noticing skills of mathematics teachers at the K-12 level, it seems reasonable to expect similar results using such tools in noticing-targeted professional development with GTAs. With the goal of improving GTAs' use of student mathematical thinking in undergraduate mathematics classrooms, the current study used the MOST analytic framework to characterize instances of student mathematical thinking that are important to notice and respond to. While this framework was designed with secondary mathematics classrooms in mind, it is applicable to undergraduate mathematics classrooms, since the topics GTAs teach could also appear in secondary mathematics classrooms (i.e. precalculus, calculus I & II, trigonometry) and students in these particular classes are typically only one or two years out of high school. This framework was used to support GTAs in analyzing classroom video both individually and in group meetings, as was done by Stockero et al. (2015; under review).

Mathematical Knowledge for Teaching

While learning to notice student mathematical thinking is an essential skill in enacting student-centered instruction, there are many knowledge bases that effective mathematics teachers draw from during instruction. The work of Ball and colleagues (2008) outlines six domains of mathematical knowledge for teaching: common content knowledge, specialized content knowledge, horizon content knowledge, knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. Of critical importance to helping students learn the mathematics material is a teacher's common content knowledge. Surely, without a command of the mathematical skills and knowledge that are taught in a particular course, a teacher is not likely to effectively aid in their students' development of such knowledge. In addition, a teacher cannot hope to develop the other domains of mathematical knowledge for teaching without first developing common content knowledge, since knowledge of mathematics is foundational for understanding the interaction of this mathematics with other aspects of teaching, such as students and curriculum.

Because of its critical importance for effective teaching, tools such as *The Praxis Series* tests (Educational Testing Service, 2016) have been developed in a variety of subjects to assess teachers' common content knowledge at the K-12 level. While such tests are relatively common for teachers at the K-12 level, similar tests are not available for measuring teachers' common content knowledge at the undergraduate level. However, there are tools such as the Calculus Concept Inventory (CCI) (Epstein, 2013) that aim to measure undergraduate students' content knowledge that could also be used to assess instructors' knowledge. A tool developed for the use of measuring changes in students' conceptual understanding of calculus as a result of varied teaching methodologies, the CCI is a multiple-choice test comprised of 22 questions devoted entirely to the basic concepts of differential calculus. While it is not all-inclusive of the principles that students are expected to know after taking first-semester calculus, the CCI does attempt to measure the most basic understandings of calculus.

Because both the ability to notice and common content knowledge are factors that contribute to effective mathematics teaching, part of this study aimed to investigate if and how these factors are related. Mathematics teachers would not likely have the ability to determine which instances of students' mathematical thinking are important to notice without a strong command of common content knowledge. In order to investigate the potential relationship between content knowledge and noticing skills, this study used the CCI to provide a measurement of calculus competency in order to make a comparison of content knowledge among the GTAs. While a GTA's content knowledge of calculus is not a complete measure of the depth of their common content knowledge, the CCI measures conceptual understanding within the range of mathematical knowledge needed for this study, since the videos for the intervention in this study were from courses leading into and including the beginning calculus sequence.

Chapter 3: Methodology

Theoretical Framework

With the goal of improving GTAs' use of student mathematical thinking in undergraduate mathematics classrooms, the current study used the MOST Analytic Framework (Leatham et al., 2015) to characterize instances of student mathematical thinking that are not only important to notice, but also the most fruitful to discuss during a lesson to support students' mathematical learning. MOSTs—**M**athematically Significant Pedagogical **O**pportunities to Build on **S**tudent **T**hinking—are described as "instances of student thinking that have considerable potential at a given moment to become the object of rich discussion about important mathematical ideas" (p. 90). For a moment to be a MOST, it must satisfy three identifying characteristics: student mathematical thinking, mathematically significant, and pedagogical opportunity. For a moment to satisfy the student mathematical thinking characteristic, the student mathematics must be inferable and related to a mathematical point. For a moment to be mathematically significant, the mathematical point must be appropriate to the learning level of the students and a central goal for student learning. For a moment to create a pedagogical opportunity, the student mathematics must create an intellectual need for students to understand the mathematical point, and it must be the right time to address the intellectual need in the moment the instance occurs.

An example of a MOST from the current study occurred when a student asked, "Isn't that the same thing as M/a?" in reference to the change of base property of logarithms (see Figure 1). The student mathematics of this instance can be inferred to be, "Can't the change of base property of logarithms be simplified to M/a?" A mathematical point associated with this student mathematics is that *$log_b M$ means that the base b raised to some power yields M, while $log_b a$ means that the base b raised to some (possibly different) power yields a. Since logarithms are functions, not common factors, they cannot be "cancelled" when part of a rational expression*. The mathematical point is appropriate for precalculus students to engage with, and it is a central goal since the lesson in which the instance occurred focused on properties of logarithms. The student mathematics likely creates an intellectual need for the rest of the class since it is a common misconception for students to assume that logarithms with the same base can be treated as common factors and simplified in rational expressions. This is the right time to address the intellectual need since the question was just asked. Thus, all six criteria of a MOST are satisfied for this student question.

$$\boxed{\log_a M = \frac{\log_b M}{\log_b a}}$$

Figure 1. Change of base property of logarithms.

Instances that are MOSTs are "opportunities to engage students in making sense of mathematical ideas that have originated with students—that is, opportunities to *build* on student mathematical thinking by making it the object of rich mathematical discussion" (Leatham et al., 2015, p. 90-91). In the example MOST provided, an opportunity is created for the teacher to, for example, pose the student question back to the class in order to initiate mathematical discussion: "What do we think? Is $\frac{\log_b M}{\log_b a}$ the same thing as M/a?" The practice of engaging students in learning by making student thinking the object of rich mathematical discussion supports the NRC's (2003) researchbacked seventh principle of learning that says, “Learning is enhanced through socially supported interactions” (p. 22). The NCTM (2014) also advocates that effective mathematics teaching promotes productive discussion in which the students are actively involved in building on others’ thinking for the benefit of furthering the learning of mathematics for all students. Thus, it is these MOST moments that this work deems important for GTAs to notice so they can effectively build on them in their undergraduate mathematics classrooms to become more student-centered in their instruction. The study focused on an intervention in which GTAs utilized the MOST Analytic Framework to analyze video of undergraduate mathematics lessons to improve their noticing of MOSTs and their ability to propose responses to such instances that actively involve students.

Participants

The participants in this study were four GTAs from the mathematical sciences department of a Midwestern U.S. university. Email invitations to participate in the study were sent to six GTAs. Some of the GTAs were invited because they were early in their GTA careers, and thus likely still eager to learn about and influence changes in instructional practices as they moved forward in their careers (Anderson, 1991; Cano et al., 1991; Speer et al., 2005). Others were invited because they were near the end of their GTA career, but were considering a career in teaching after graduation and therefore also had potential to influence changes in instructional practices in their future careers. Out of the original six who were invited, four GTAs committed to participating in the study—three had completed their first year of graduate study and had one to two semesters of teaching experience, and one had completed two years of graduate study with six semesters of teaching experience. In an effort to recruit one or two more participants, an additional email invitation was sent to all GTAs (about 30 students) in the mathematical sciences department. However, no other GTAs showed interest in participating.

All four of the GTA participants had previously completed training required by the mathematical sciences department: a week of GTA orientation prior to their first semester of study, a course entitled Teaching College Mathematics in which the GTAs prepared and delivered lessons three times throughout a semester with feedback and support from peers and their instructor, and a six week seminar during their first semester of teaching in which the GTAs read and met weekly with an instructor to discuss select chapters of *Teaching Tips* by Svinicki and McKeachie (2014). Their participation in the current study was voluntary.

Intervention

The GTAs engaged in a professional development intervention facilitated by the researcher. The professional development took place in the fall 2015 semester for a period of ten weeks. This length of time was chosen both because success in similar interventions was seen within this length of time (Stockero, 2014; Stockero et al., 2015; under review) and in the interest of containing the study within one semester to favor researcher and GTA schedules. The goal of the intervention was for the GTAs to improve their skills in attending to, interpreting, and responding to MOSTs in a student-centered manner.

The design of this study was adapted from Stockero and colleagues' work with prospective secondary school mathematics teachers (Stockero, 2014; Stockero et al., 2015; under review) focused on helping them learn to notice and respond to MOSTs that surface during a mathematics classroom lesson. Before the professional development began, each GTA completed a one-on-one, video-recorded pre-interview with the researcher. During the interview, the GTA watched a short video clip from an undergraduate mathematics lesson that was recorded by the researcher at the same Midwestern U.S. university in which the GTAs were enrolled. The GTA was prompted by the researcher to stop the video if they thought a *mathematically important moment that the instructor should notice (MIM)* occurred. A definition of these moments was not given to the GTA to establish baseline data. Whenever a GTA stopped the video, the researcher asked them to describe the moment they noticed, why they chose it, and what they might do if such a moment happened in their own classroom. Using Studiocode video analysis software (SportsTec, 1997-2016), the instances chosen by the GTAs were marked on a timeline associated with the classroom video for later analysis.

In each week of the intervention, the GTAs and the researcher individually analyzed a minimally edited video of a lesson from an undergraduate mathematics classroom that was recorded by the researcher at the university. This individual analysis was in preparation for a weekly group meeting held between the four GTAs and the researcher to discuss the video collectively. One of the classroom videos was recorded in one of the GTAs' classroom from a previous semester, but otherwise the videos were not of the GTAs' own classrooms. See Figure 2 for a description of each video's content area and lesson topic.

In the first three weeks of the intervention, the GTAs used the Studiocode video analysis software to tag *MIMs* and add text to describe what they noticed and why they chose each instance. Again, the definition of such moments was left open-ended to establish baseline data. The researcher, as an experienced user of the MOST Analytic

Video	Content area	Lesson topic
Pre-interview*	Precalculus	Trigonometric equations
1	Calculus 1	Related rates
2	Calculus 1	Implicit differentiation
3	Calculus 2	Geometric, power, and Taylor series
4	Precalculus	Logarithmic properties and equations
5	Calculus 1	Limits at infinity
6	Calculus 1	Limits at infinity
7	Calculus 1	Differential equations
8	Calculus 2	Integration by substitution
9**	Calculus 1	General antiderivatives
Post-interview*	Precalculus	Trigonometric equations

*Same video **GTA participant classroom video.

Figure 2. Video content area and lesson topic.

Framework, used the Studiocode software to tag and document MOSTs—the types of instances that were the goal for GTA noticing—in the same videos. The researcher reconciled any instances of uncertainty with one of two other researchers experienced in the use of the MOST Analytic Framework. Before the group meeting with the GTAs, the researcher examined the participants' tagged video timelines and associated text and compared instances chosen by the GTAs and MOSTs identified by the researcher. Instances discussed at the group meeting were chosen with care by the researcher, limiting the number of instances so that the meeting did not, on average, last more than an hour. Instances were selected for discussion for a variety of reasons. These included instances that one or more GTAs marked—both those that the researcher identified as MOSTs and that the researcher did not identify as MOSTs—as well as instances identified by the researcher as MOSTs that were not noticed by the GTAs.

In the group meetings, the facilitator/researcher pushed the GTAs to articulate how each moment fit the early prompt of *MIMs*; that is, what the instructor had to notice in each moment and why it was mathematically important. Through discussion with peers and guidance from the facilitator in these group meetings, the GTAs worked toward building a definition of *MIMs*. This early phase of the intervention that focused on constructing a definition of *MIMs* was used as an introduction to the teacher noticing construct. That is, it got the GTAs to start thinking about what might be important to notice as an instructor during a lesson and to create a need for a more formal language and criteria with which to describe such important instances (i.e., the need for a framework).

After three weeks of analyzing video and constructing a meaning for the early *MIM* prompt, the GTAs were introduced to the MOST Analytic Framework by being given a paper to read that defined MOSTs (Stockero et al., 2014). In the week that they read the paper, the GTAs reexamined two of the videos they had already analyzed and picked out instances that they believed met the characteristics of a MOST. The group meeting discussion of the reexamined videos then revolved around whether instances fit all six criteria of a MOST.

In the remaining six weeks of the intervention, the GTAs were prompted to tag MOSTs in the new classroom videos that they analyzed and to describe in text how each instance satisfied all six MOST criteria. The GTAs were provided a text template to prompt them to specifically address each MOST criterion in their written responses in the last five weeks of the intervention. In these weeks, the researcher similarly chose with care which instances to discuss at the weekly group meetings, with the important moments now having a clear definition. The group meetings revolved around how each instance discussed fit or did not fit the definition of a MOST.

In the group meeting discussion of video 8, the facilitator prompted the GTAs to propose *building* moves in response to the MOSTs that were discussed—a teacher move that engages students in collaboratively discussing the significant student mathematical thinking that is present in the classroom (Stockero et al., 2014). These building moves, if proposed and practiced effectively, would use student mathematical thinking to further the learning of all students, which aligns with student-centered instruction and effective mathematical instruction (NCTM, 2014). The GTAs were then prompted to include proposed building moves as part of the text template in subsequent video analyses.

Following the intervention, each GTA completed a post-interview in the same style as the pre-interview. It was a one-on-one interview with the researcher in which the GTAs analyzed the same short video clip as in the pre-interview in real-time for *MIMs*—whatever the definition of such moments meant to them now, post-intervention. Upon stopping the video when viewing such moments, the GTAs were asked to describe what they noticed, why they chose the moment, and how they might respond to such a moment if it had happened in their own classroom. Again, the interview was video-recorded and timelines were created using the Studiocode video analysis software to capture the moments the GTAs chose. The researcher also asked the GTAs the following questions to get an idea of the effectiveness of the study from the GTAs' perspectives: (a) How might this work affect your teaching?; (b) Was this work valuable to you?; (c) Were there particular parts of the work that were more or less valuable to you? (group discussion, watching other people's teaching, individual coding, using the framework, etc.); and (d) How did this training support your other GTA training?

In addition to the post-interview, at the conclusion of the intervention the GTAs completed the CCI (Epstein, 2013) to measure their knowledge of calculus content. The researcher also completed the CCI and reconciled the answers with a mathematics faculty member to create an answer key. The purpose of taking the CCI was to see if there is a relation between GTA performance on the CCI (a measure of their common mathematical content knowledge) and noticing of MOSTs.

Data Collection and Analysis

The data for this study included the CCI results and the video timelines that indicate instances marked and described by the GTAs in the classroom videos during the interviews and the intervention. The CCI results were scored according to the researcher-created and faculty-confirmed key. The score of each GTA was then compared to the rest of the group to see if there were any obvious differences in scores that could potentially account for differences in noticing skills.

Categorizing Instances

Each instance marked and described by the GTAs in both the interview data and in the weekly timelines was coded and analyzed by the researcher in a multitude of ways in order to examine changes in the GTAs' noticing. First, like in the work of Stockero, Rupnow, and Pascoe (2015; under review), each instance was coded according to *agent* (who or what was noticed) and *mathematical specificity* (the way in which the mathematics was discussed). Instances that had any type of student agent were also coded for *focus* (what about the student(s) was noticed). See Figure 3 for coding categories, category descriptions, codes, and code descriptions.

Figure 4 includes examples of participant descriptions of instances with accompanying agent, mathematical specificity, and focus coding. The first and third examples were coded as Student Individual since an individual student question was the object of noticing. While the first example mentions the teacher, the description is simply to provide context for noticing "the student ask[ing] the question '[I]sn't the constant just zero?'" and therefore the teacher is not included in the agent code. The third example comes from an instance that occurred later in the pre-interview video, after the GTA had become accustomed to answering the interview question about what they might do if such a moment happened in their own classroom. Thus, while the teacher is mentioned in this example, the statements about the teacher are evaluative in nature and provide clues as to how the GTA would respond if this was their classroom. The student question "Can you pull the negative out?", not the teacher, was what was noticed as part of the mathematically important moment, so the teacher was not included in the agent code. The second and fourth examples were coded as Teacher since the teacher's actions of explaining the checking of units and "ask[ing] students to do the calculation" are clearly the object of noticing in these instances. The fifth and seventh examples have both student and teacher included in the agent code. The student is emphasized a bit more than the instructor in the fifth example since the moment was described as "the student leading the instructor on." The student is the primary actor, and therefore, the Student/Teacher agent code was assigned. Similarly, the seventh example was assigned the

Teacher/Student agent code because it was "instructor has students guide him," and therefore the instructor was the primary actor in the description of the moment.

Coding Categories	Category Description	Codes	Code Description
Agent	Who or what was noticed	Teacher	The teacher is the sole object of noticing
		Teacher/ Student	Both the teacher and student(s) are noticed, with the teacher receiving more emphasis
		Student/ Teacher	Both the teacher and student(s) are noticed, with the student receiving more emphasis
		Student Group	A collection of students is the object of noticing
		Student Individual	An individual student's contribution is the object of noticing
		Math	The mathematics itself, not a person or persons, is the object of noticing
Mathematical Specificity	Whether and how the mathematics was discussed	Non-Math	The mathematics is not discussed, usually because a non-mathematical aspect of the classroom is discussed instead, like classroom management or student engagement
		General Math	The mathematics is referenced with little to no detail
		Specific Math	The mathematics is clearly stated with enough detail to recognize the mathematical topic without having to watch the video
Focus	For instances with a student agent, what about the student(s) was attended to	Affective Interaction	A non-mathematical interaction between student(s) and teacher, usually focused on something like classroom management or student engagement
		General Understanding	The nature of a student's or students' comprehension of a concept, problem, or answer
		Mathematical Interaction	An interaction between students or between students and teacher that is mathematical in nature, usually focused on the process of working together for the purpose of learning mathematics
		Noting Student Mathematics	A specific instance of student mathematical thinking is described, like a student statement or question
		Analyzing Student Mathematics	Not only is a specific instance of student mathematical thinking referenced, but an attempt to interpret is made, like reasoning why a question was asked or a solution method was proposed

Figure 3. Components of noticing coding scheme. Figure adapted by the author from Stockero et al. (2015; under review).

Example	Codes
1. Student Mathematics: Student mathematics occurs when the student asks "isn't the constant just zero?" in response to [instructor] questioning whether or not it was clear the initial velocity was c or not. Mathematical Point: Given a differential equation, we can use initial conditions to find the constants involved and find a particular solution. Appropriate Mathematics: Yes, certainly. It's just solving a differential equation, something they have done all day. Central Mathematics: Yes. If you don't know how to use initial conditions, you don't really know how to do differential equations or pretty much any applied mathematics. Opening: Yes. I think the student was able to just jump to the solution, but I am unsure if that jump is obvious to the rest of the students. They are likely confused as to how the student was just able to say that. Timing: Yes. It seemed the next logical step in [instructor's] process anyway, and the question just seems to punctuate it. (GTA 3, Video 7)	Student Individual Specific Math Analyzing Student Mathematics Consistent MOST
2. It is important that instructors convey meaning behind variables, and the units are a great way of expressing how, in doing related rates, will work out in a way that makes sense. The instructor goes on to explain that figuring out if the units make sense in the end is a good way of making sure you did the problem correctly. (GTA 1, Video 1)	Teacher Specific Math Inconsistent MOST
3. "Can you pull the negative out?" is [how] the student worded it as a question, and it sounds like the instructor is going to address why you can't do that. The fact that the students are trying is great. The fact that the instructor is taking what they're saying and figuring out if it's going to work or not. I like this question from a student because oh, you just want to change the sign on the last term, so can't you pull out a negative. I think that's important for the instructor to explain why or what effect that's going to have on your polynomial. (GTA 1, Pre-interview)	Student Individual Specific Math Analyzing Student Mathematics Consistent Non-MOST
4. [The teacher should] ask students to do the calculation. For one thing, instructor can check whether the students are following. For another, students can be more focused on the study materials. (GTA 4, Video 1)	Teacher General Math Inconsistent Non-MOST
5. The instructor sort of posed the question to the students, and the student said 'Hey, this is a quadratic here.' And then the instructor said, 'And how would we solve that quadratic?' And then the student said 'Factor it.' And then the instructor said, 'Yes. I would factor it if I can, otherwise I would use the quadratic formula.' So the specific moment there was the student leading the instructor on. (GTA 2, Pre-interview)	Student/Teacher Specific Math Noting Student Mathematics Inconsistent Non-MOST
6. I don't think he is paying attention to the class. Maybe the camera is kind of distracting to him. (GTA 4, Pre-interview)	Student Individual Non-Math Affective Interaction Inconsistent Non-MOST
7. Instructor-student interaction: instructor has students guide him in writing what will go on the board. (GTA 2, Video 1)	Teacher/Student General Math Mathematical Interaction Inconsistent Non-MOST

Figure 4. Examples of GTA instance descriptions with assigned codes.

The Specific Math code was assigned to Examples 1, 2, 3 and 5 since the mathematics in each instance is described in detail. For instance, in Example 1 the GTA wrote that the student asked a question "in response to [instructor] questioning whether or not it was clear the initial velocity was c or not" and proposed the following mathematical point: "Given a differential equation, we can use initial conditions to find the constants involved and find a particular solution." The General Math code was assigned to the fourth and seventh examples since the descriptions refer to students doing a calculation or "writing what will go on the board," but do not provide detail about the mathematics involved. Example 6 was coded as Non-Math because the GTA did not make mention of the mathematics at all.

Since the second and fourth examples do not include a student agent, those instances were not assigned a focus code. Examples 1, 3, 5, 6 and 7 included a student agent, and thus were assigned a focus code. Example 6 was assigned the focus code of Affective Interaction since the GTA was focused on student engagement and "maybe the camera is kind of distracting to [the student]." The seventh example was coded as Mathematical Interaction since the description focused on "[i]nstructor-student interaction" in working together to "[write] what will go on the board." Example 5 was assigned the focus code of Noting Student Mathematics since a description of what the student said mathematically was provided: "the student said 'Hey, this is a quadratic here,'" and "then the student said 'Factor it.'" The first and third examples were coded as Analyzing Student Mathematics because both a description and an interpretation were provided in relation to the student question. In the first example, the GTA proposed that the individual student was "able to just jump to the solution" that the constant in the problem at hand was 0 by using initial conditions, but that they were "unsure if that jump is obvious to the rest of the students." The GTA in the third example reasoned that the student asked the question “because oh, you just want to change the sign on the last term, so can’t you pull out a negative.” That is, the student asked the question in order to address a sign issue in the two binomial terms suggested by a classmate as factors of a polynomial.

MOST Coding

Second, like the work done by Stockero and colleagues (under review), each instance was coded according to whether it was a MOST and whether the reasoning provided by the GTA was consistent with the MOST criteria. The MOSTs for each timeline were determined by the researcher’s coding since she was an experienced user of the MOST Analytic Framework. A GTA instance was coded as a MOST if it occurred at around the same time in a video as a MOST. In an instance coded as a consistent MOST, the GTA identified the characteristics of the instance that qualified the instance as a MOST according to the framework (student mathematical thinking, mathematical significance, and pedagogical opportunity). Example 1 in Figure 4 is an example of a consistent MOST because it both aligned with a MOST on the video timeline and the GTA’s text identified all characteristics of the MOST Analytic Framework. In an inconsistent MOST, the GTA captured an instance related to a MOST, but was not focused on the student mathematical thinking or was focused on non-mathematical aspects of the instance, such as student participation or motivation. The second example in Figure 4 is an inconsistent MOST because it aligned with a MOST on the video timeline, but the GTA was focused on “the instructor go[ing] on to explain that figuring out if the units make sense in the end is a good way of making sure you did the problem correctly” instead of the student mathematical thinking in the instance. A GTA instance was coded as a Non-MOST if it did not correspond time-wise with a MOST in the video. A consistent Non-MOST was an instance that the GTA reasoned about in accordance to the MOST Analytic Framework, but did not meet all of the MOST criteria. In the third example in Figure 4, the GTA inferred the student mathematics from an individual student question, "Can you pull the negative out?" and reasoned about a mathematical point related to the effect of factoring out -1 when factoring a polynomial—"why or what effect that's going to have on your polynomial." The instance was coded as a consistent Non-MOST since, although the GTA reasoned about the instance in a manner consistent with the MOST Analytic Framework, the mathematics of the instance was not at an appropriate level for precalculus students because ease in the use of the distributive property with negative numbers is expected at this level. Therefore, the instance failed the mathematically significant characteristic of a MOST, and so it did not correspond to a MOST on the video timeline. Like an inconsistent MOST, an inconsistent Non-MOST was an instance in which the GTA was not focused on the student mathematical thinking or was focused on non-mathematical aspects of the instance. The fourth example in Figure 4 did not align with a MOST on the video timeline and was not focused on student mathematical thinking. Instead, the GTA is focused on the instructor "ask[ing] students to do the calculation" so that the "instructor can check whether the students are following."

Response Coding

Because one of the goals of this intervention was for GTAs to propose more student-centered responses, the pre- and post-interview instances were additionally coded according to whether the response to each instance proposed by the GTAs was student-centered or teacher-centered. A student-centered response is one in which the teacher would involve one or more students in responding to the instances, whereas a teacher-centered response would only involve the teacher. For example, in an instance where a student shared a solution step, a GTA proposed, “Maybe ask them, 'Why would you do that? What is the purpose of that? How does that help us going forward?'” This response was coded as student-centered since the student or students in the class would be the ones thinking about and addressing the mathematics of the instance rather than the teacher just explaining the significance of what the student said. In another example, a student asked a question, and the GTA proposed, “I would quickly explain.” This response was coded as teacher-centered since the teacher would be the only one involved in reacting to the student question, rather than perhaps opening up the question to the class for discussion.

Coding Analysis

After the coding was complete, the data were summarized to look for changes in GTAs’ noticing week to week throughout the intervention. The analyses focused on the components of noticing (agent, mathematical specificity, and focus) and MOSTs (consistent/inconsistent MOSTs, consistent/inconsistent Non-MOSTs). Comparisons of student-centered responses from pre- and post-interviews were also made to look for changes in how the GTAs might respond to the moments they selected.

To provide a common unit of measure among all GTA coding, percentages were calculated for each code out of each GTA’s total number of marked instances in each video. These percentages were used to track changes in the GTAs’ noticing on an individual basis. Averages for all the GTAs’ coding per video were also calculated to reflect changes of the group as a whole. The intervention data was then split into three stages—early, middle and late in the intervention—and the data were summarized for each part of the intervention. In this analysis, *baseline* refers to Videos 1, 2, and 3, the first three videos of the intervention and before the introduction of the MOST framework. *Middle* stands for Videos 4, 5, and 6, the three videos immediately following the introduction of the MOST framework, and *final* refers to Videos 7, 8, and 9, the last three videos of the intervention when the GTAs should have had the best understanding of the framework. Data from the pre- and post-interviews, including the response coding, were analyzed separately due to the difference in the nature of the interviews. Recall that in the interviews, the GTAs engaged in in-the-moment video analysis, whereas in the individual video analysis, repeated viewings and lengthy deliberation about instances were possible.

Chapter 4: Results

Components of Noticing

The goal of the intervention in this study was to improve GTAs' noticing of MOSTs and to increase the amount of student-centered responses that GTAs propose to such instances. The data is presented to understand changes in GTAs' noticing throughout the intervention, including analyses of components of noticing (agent, mathematical specificity, and focus) and MOSTs (consistent/inconsistent MOSTs, consistent/inconsistent Non-MOSTs). Comparisons of student-centered responses from pre- and post-interviews and the results of the CCI are also discussed.

Because the GTAs identified different numbers of instances in each video, to provide a common unit of measure among all GTA coding, percentages were calculated for each code out of each GTAs' total number of marked instances in each video. Averages for all the participants were also calculated to reflect changes in the noticing of the group as a whole. The intervention data was then split into three stages and was summarized according to each part of the intervention. As a reminder, *baseline* refers to Videos 1, 2, and 3, the first three videos of the intervention and before the introduction of the MOST framework. *Middle* stands for Videos 4, 5, and 6, the three videos immediately following the introduction of the MOST framework, and *final* refers to Videos 7, 8, and 9, the last three videos of the intervention. *Pre* and *post* indicate data from the pre- and post-interviews, which are presented separately due to the difference in the nature of the interviews. Recall that in the interviews, the GTAs engaged in in-the-moment video analysis, whereas in the individual weekly video analysis, repeated viewings and lengthy deliberation about instances were possible.

Agent

Because the goals of the intervention placed an emphasis on students and their mathematical thinking, changes were examined in the GTAs' noticing of instances in which students were the primary agent—that is, the sum of instances coded with Student/Teacher (e.g., Figure 4, Example 5), Student Individual (e.g., Figure 4, Example 1) and Student Group agents. An example of the latter is, "[T]he students are forced to answer the multiple choice question with an answer and then look around and find other students [who] answered differently" (GTA 3, Video 9). Table 1 provides the percentages of such instances from each stage of the intervention. Percentages were calculated for each code out of each GTA's total number of marked instances in each video and were then averaged over each three-week stage. In the table it can be seen that both individually and as a group the general trend was an increase in the GTAs' noticing that had a primary student agent from baseline to middle to final. Impressively, the majority of GTAs averaged 100% and the group averaged 94% of primary student noticing in the final data. Furthermore, 100% of the instances marked by GTAs 1 and 3 in the final data were noticing students alone (Student Individual and/or Student Group), not in conjunction with the teacher (Student/Teacher).

Table 1

Noticing of Primary Student Agent by Stage

Participant	Baseline	Middle	Final
GTA 1	41%	100%	100%
GTA 2	50%	89%	100%
GTA 3	63%	100%	100%
GTA 4	28%	67%	75%
Group	46%	89%	94%

Figure 5 gives a visual representation of changes from week to week throughout the intervention. The figure suggests that the sharpest increase in the noticing of primary student agent instances was Video 3 for GTAs 2, 3, and 4, and Video 4 for GTA 1. With the exception of GTA 4, the level of primary student agent noticing attained in Video 4 was relatively maintained throughout the remainder of the intervention. Since the peak of such noticing occurred in Video 4, the increase in primary student noticing is likely due to the introduction of the MOST Analytic Framework following Video 3.

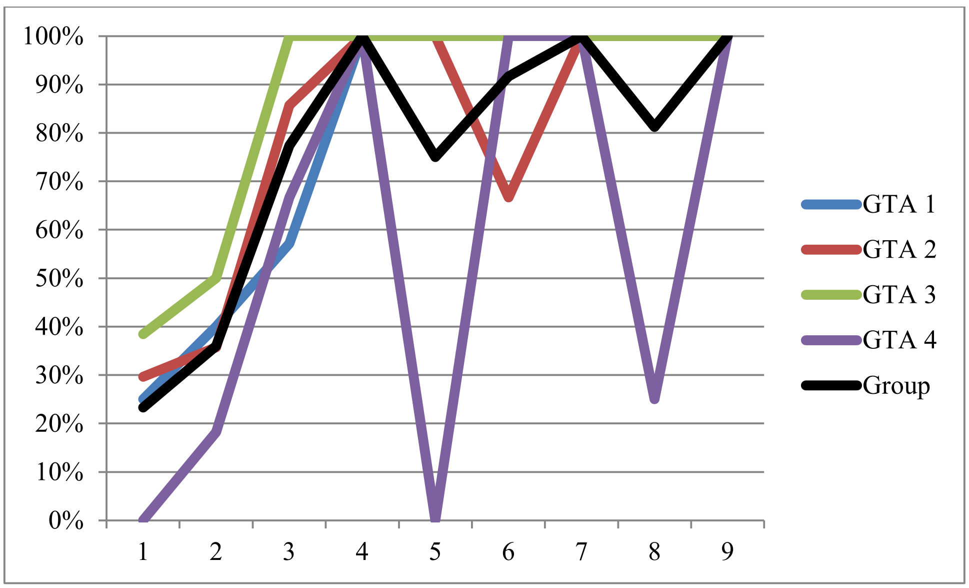

Figure 5. Noticing of primary student agent by video (in percentages).

For the pre- and post-interview analysis, Table 2 also shows improvement in the GTAs' noticing of primary student agents. While GTAs 3 and 4 displayed a high level of noticing of students in the pre-interview (75% and 80%, respectively), GTAs 1 and 2 showed the most growth from pre- to post-interview in this type of noticing. Most notably, 100% of the GTAs' noticing in the post data was primarily on students. This indicates that the GTAs developed the ability to focus their noticing on students over the teacher or the mathematics itself in their in-the-moment analysis of video.

Table 2

Noticing of Primary Student Agent by Interview

Participant	Pre	Post
GTA 1	29%	100%
GTA 2	41%	100%
GTA 3	75%	100%
GTA 4	80%	100%
Group	56%	100%

Mathematical specificity

With mathematical significance and student mathematical thinking being two of the three characteristics in identifying a MOST, a possible indicator of improvement in noticing of MOSTs is the ability to speak about the mathematics of an instance in a detailed manner. Thus, changes were examined in the GTAs' percentages of instances that were coded for mathematical specificity at the most detailed level, Specific Math (e.g., Figure 4, Example 1).

Table 3 shows that baseline data percentages for Specific Math were rather high for all GTAs with the exception of GTA 4, who only had 6% of their instances coded as such. The middle data showed an increase in mathematical specificity for all GTAs, with the most considerable increase being that of GTA 4, with a 77% increase in instances coded as Specific Math. Perhaps most important is that all of the GTAs exhibited 100% Specific Math noticing in the final data.

Table 3

Specific Math Noticing by Stage

Participant	**Baseline**	**Middle**	**Final**
GTA 1	97%	100%	100%
GTA 2	85%	100%	100%
GTA 3	81%	100%	100%
GTA 4	6%	83%	100%
Group	67%	96%	100%

Figure 6 gives a better idea of when the largest increases in instances coded as Specific Math took place. GTA 2 demonstrated their largest increase for Video 3, GTA 3 for Video 4, and GTA 4 for Video 5. Interestingly, once a GTA reached 100% Specific Math noticing, there was little to no wavering from this level for the remainder of the intervention. GTA 1's instances were 100% Specific Math from the start to the end of the intervention, with only a slight decrease to 90% in Video 2. Because of the high level of instances coded as Specific Math early on and the differences in which video the largest increases occurred among the GTAs, the improvements overall cannot be explained with any certainty. However, the MOST Analytic Framework may have been helpful in supporting the group's maintenance of 100% Specific Math noticing.

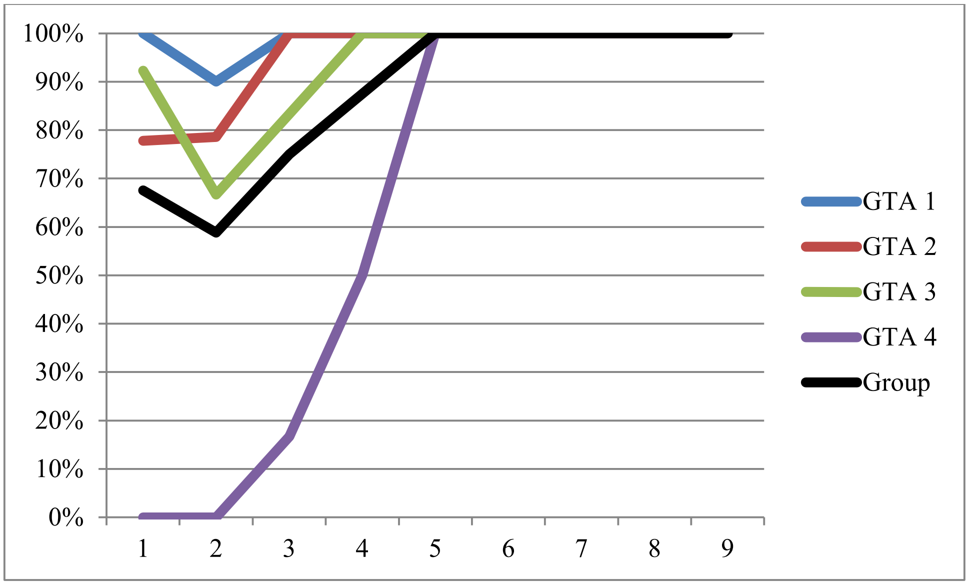

Figure 6. Specific Math noticing by video (in percentages).

Table 4 indicates improvement for all GTAs in instances coded as Specific Math from pre- to post-interview. While GTA 1 already discussed the mathematics in a high level of detail in the pre-interview, it is worth noting that this level of mathematical specificity was maintained in the post-interview. GTA 4 showed the most improvement from 40% in the pre-interview to 100% in the post-interview data. Like the final data shown in Figure 6, the post-interview data demonstrated 100% Specific Math coding for all participants. This implies that not only did the GTAs reach 100% Specific Math noticing when completing individual data analysis with more time to reflect and write about each instance, they were also able to discuss the mathematics in a high level of detail when put on-the-spot in an interview setting for their video analysis.

Table 4

Specific Math Noticing by Interview

Participant	Pre	Post
GTA 1	100%	100%
GTA 2	59%	100%
GTA 3	75%	100%
GTA 4	40%	100%
Group	69%	100%

Focus

Recall that the focus code category was only assigned to instances with a student agent and describes what about the student(s) was attended to. The two focus codes that most aligned with the goals of the intervention, due to their focus on student mathematics, are Noting Student Mathematics and Analyzing Student Mathematics. Because of their relationship to the goals, changes were explored in the GTAs' noticing with both of these focus codes. While the focus codes only apply to those instances with a student agent, the percentages calculated in the following results are out of the total of all the instances identified by the GTAs to reflect an overall, and not a narrow, sense of their noticing.

Noting and Analyzing Student Mathematics. The first focus analysis examined both Noting (e.g., Figure 4, Example 5) and Analyzing Student Mathematics (e.g., Figure 4, Example 1) in sum to capture changes in the total percentage of instances in which the GTAs were focused on student mathematics. Table 5 shows widespread improvement for all GTAs in describing and/or interpreting the student mathematical thinking in an instance. Of particular importance is that, with the exception of GTA 4, the GTAs were Noting and/or Analyzing Student Mathematics 100% of the time as soon as the middle data, immediately following the introduction of the MOST framework. While GTA 4 did not reach 100% for these noticing foci, substantial increases were still made in each stage of the intervention.

Table 5

Noting and/or Analyzing Student Mathematics by Stage

Participant	Baseline	Middle	Final
GTA 1	32%	100%	100%
GTA 2	48%	100%	100%
GTA 3	52%	100%	100%
GTA 4	9%	50%	83%
Group	35%	88%	96%

Figure 7 shows that the peak for Noting and/or Analyzing Student Mathematics of 100% was reached for all GTAs in Video 4, the first time in which the MOST Analytic Framework was applied to a new video. Therefore, it seems reasonable to attribute the increase in these foci to the introduction of the framework after Video 3. While GTAs 1, 2, and 3 largely maintained this noticing focus for subsequent videos, GTA 4 displayed sizable fluctuations in the remainder of the intervention, perhaps an indication that for GTA 4, the content of Video 5 (limits at infinity) and Video 8 (integration by substitution) was challenging, or that the framework alone was not enough to support this type of noticing. Overall, the group displayed an increasing trend in these types of noticing, which adds support to the idea that the introduction of the MOST Analytic Framework was influential in supporting these noticing improvements.

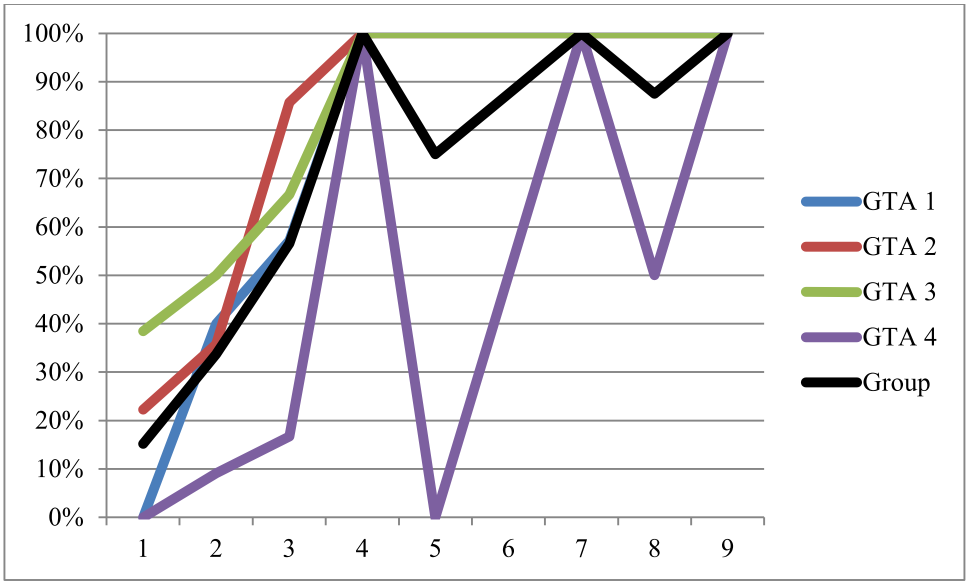

Figure 7. Noting and/or Analyzing Student Mathematics by video (in percentages).

The pre-interview and post-interview data in Table 6 exhibits increases in Noting and/or Analyzing Student Mathematics for all GTAs. Most substantially, all of the GTAs reached 100% of instances with these focus codes in the post-interview. Thus, even for in-the-moment video analysis in an interview setting, all of the GTAs developed the ability to describe and/or interpret the student mathematical thinking in their noticing of classroom instances as a result of the intervention.

Table 6

Noting and/or Analyzing Student Mathematics by Interview

Participant	**Pre**	**Post**
GTA 1	29%	100%
GTA 2	19%	100%
GTA 3	13%	100%
GTA 4	20%	100%
Group	20%	100%

Analyzing Student Mathematics. The second focus analysis investigated changes in just the Analyzing Student Mathematics focus code, where the GTAs made an attempt to interpret a specific instance of student mathematical thinking. This focus code was honed in on specifically since the MOST Analytic Framework requires not only a focus on student mathematical thinking, but also that an inference be made about the student mathematics. Therefore, changes in Analyzing Student Mathematics could be an indicator of growth in Jacobs and colleagues' (2010) second noticing skill of *interpreting [students'] understandings* and in the noticing of MOSTs.

The data in Table 7 indicates that Analyzing Student Mathematics was absent or low in the baseline videos, both individually and as a group. By the middle set of videos, substantial increases were made by all GTAs. Impressively, GTAs 1, 2, and 3 improved in their Analyzing Student Mathematics by a range of 72% to 84%, with GTA 3 reaching 100% in the final data. While improvements were made overall throughout the three stages of the intervention, there were small decreases observed for GTA 1 and GTA 2 from the middle to the final data.

Table 7

Analyzing Student Mathematics by Stage

Participant	**Baseline**	**Middle**	**Final**
GTA 1	16%	100%	94%
GTA 2	0%	72%	67%
GTA 3	24%	96%	100%
GTA 4	0%	33%	47%
Group	10%	75%	77%

In examining the Analyzing Student Mathematics data by video, Figure 8 shows that the largest increase for this type of noticing varied slightly among the GTAs. All

GTAs demonstrated the sharpest of their increases for this focus for Video 4, with two GTAs showing a consistent increase in other surrounding videos as well—Video 5 for GTA 2 and Video 3 for GTA 3. While the percentages of Noting and/or Analyzing Student Mathematics noticing held steady for most GTAs after Video 4 (see Figure 7), the Analyzing Student Mathematics data alone does not demonstrate the same behavior. GTAs 2 and 4 displayed a wide range of fluctuation in their noticing following Video 4, which prompted further investigation into particular videos that may have accounted for some of the fluctuation. Video 5, focused on limits at infinity, seemed to pose at least a slight challenge for Analyzing Student Mathematics for GTAs 3 and 4, which may point to the lesson video being a cause for this fluctuation. However, Video 5 had among the highest percentage of this noticing focus for GTAs 1 and 2, indicating that the lesson video alone was not the cause of the fluctuation. Video 9, on the other hand, seemed to challenge a majority of the GTAs (1, 2, and 4) in Analyzing Student Mathematics. This particular video was one of six in a Calculus 1 classroom, which suggests the subject matter was not the challenging factor. However, this was the only video that was recorded in one of the GTAs' own classrooms and that included a lesson strategy in which full classroom responses to multiple choice questions were displayed and used as the basis for mathematical discussion both between pairs students and the group as a whole. It is possible that these two distinguishing features of Video 9 were distractors to the GTAs and their noticing. Overall, the group's improvement in Analyzing Student Mathematics appears to coincide with the introduction and application of the MOST Analytic Framework starting with Video 4.

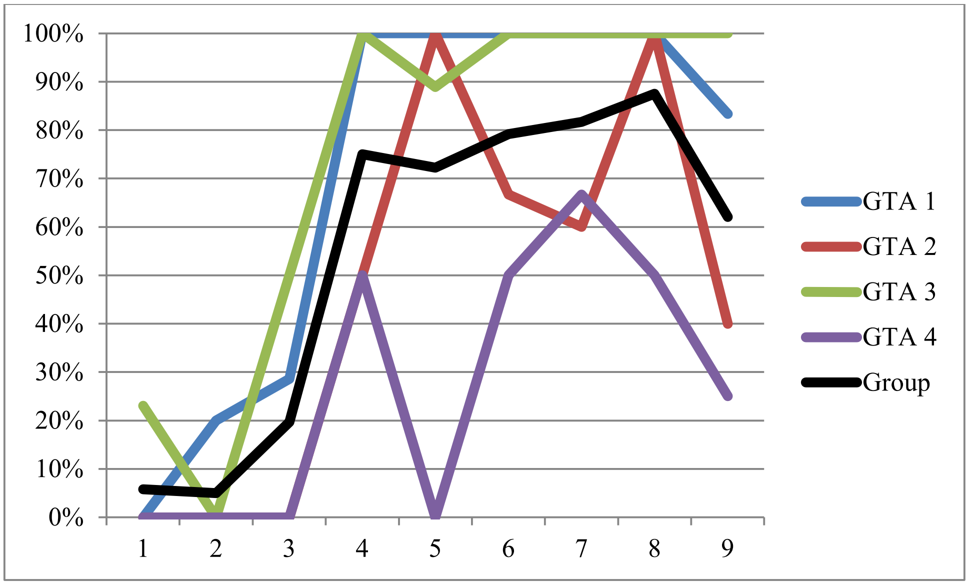

Figure 8. Analyzing Student Mathematics by video (in percentages).

The pre-interview and post-interview data shown in Table 8 exhibits remarkable growth for in-the-moment noticing that included Analyzing Student Mathematics. GTA 4 demonstrated the largest growth at 100% from pre to post, while increases for the other GTAs were also high, ranging between 75% to 87%. Interestingly, GTA 4 did not reach 70% of instances coded as Analyzing Student Mathematics for the videos they individually analyzed, but reached 100% for this focus code in the post-interview video that was analyzed in-the-moment. A possible reason for this could be that GTA 4 more completely communicated what they noticed with spoken word in the interview in comparison to written word in their text that accompanied the individually-analyzed video timelines. Another possible reason is that the shorter video used for the interview was more manageable for GTA 4 to make sense of. In general, these results suggest that the intervention was successful in developing the GTAs' ability to focus on and interpret student mathematical thinking in an in-the-moment context.

Table 8

Analyzing Student Mathematics by Interview

Participant	Pre	Post
GTA 1	14%	100%
GTA 2	0%	75%
GTA 3	13%	100%
GTA 4	0%	100%
Group	7%	94%

MOST Analysis

While the data related to the noticing components of primary student agent, Specific Math, and Noting and/or Analyzing Student Mathematics point to improvements in noticing aligned with the goals of the intervention, part of the main goal was to improve the GTAs' noticing of MOSTs. Naturally, changes were examined in the GTAs' noticing of MOSTs and Non-MOSTs. Like the other analyses, the percentages presented were calculated out of the total set of instances marked by the GTAs.

Inconsistent and Consistent MOSTs

First, the GTAs' noticing of MOSTs, both inconsistent (e.g., Figure 4, Example 2) and consistent (e.g., Figure 4, Example 1), was investigated. Recall that a GTA-marked instance was coded a MOST if it aligned with a MOST on the video timeline. Table 9 shows that all GTAs improved in their noticing of MOSTs during the three stages of the intervention. That is, the percentage of the instances marked by the GTAs that aligned with MOSTs increased from stage to stage, with the group's average percentages increasing from a baseline of 19%, to 33% in the middle data, and finally to 73% in the final data. This result indicates that the intervention improved the GTAs' noticing of moments with significant potential to improve student mathematical learning, even if perhaps the GTAs were not focused on the student mathematics in those instances.

Table 9

Noticing of Inconsistent and Consistent MOSTs by Stage

Participant	Baseline	Middle	Final
GTA 1	32%	61%	87%
GTA 2	19%	36%	76%
GTA 3	25%	57%	78%
GTA 4	19%	33%	53%
Group	24%	47%	73%

Consistent with the stage data, Figure 9 displays a general upward trend in the percentage of the GTAs' instances aligning with MOSTs in each video of the intervention. The largest increase in GTA-identified instances aligning with MOSTs occurred in Video 6 for GTAs 1 and 3 and in Video 9 for GTA 4. GTA 2's largest increases occurred about as equally for Videos 5, 6, and 9. It is important to note that 100% of the GTAs' instances aligned with MOSTs in Video 9, the last video of the intervention. Because the largest increases for the GTAs' instances aligning with MOSTs occurred in videos from different stages of the intervention, it seems that the introduction of the MOST Analytic Framework did not immediately lead to an improvement in the GTAs' noticing of MOSTs, but that extended practice with the framework supported the GTAs' ability to notice instances that aligned with MOSTs.

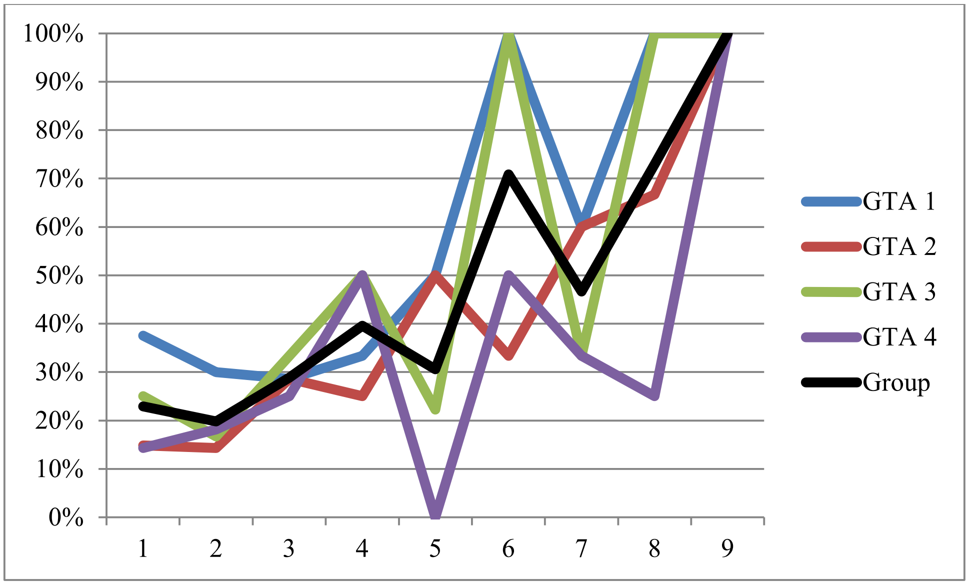

Figure 9. Noticing of Inconsistent and Consistent MOSTs by video (in percentages).

The pre- and post-interview data (Table 10) showed that all of the GTAs improved in their in-the-moment noticing of MOSTs, with increases ranging from 29% to 60%. It is worth recalling that the prompt for both the pre- and post-interview was to identify *MIMs*, which may explain why there was not a higher percentage of instances that were MOSTs among those identified by the GTAs in the post-interview. An idea underlying the intervention was that the MOST Analytic Framework would provide a way in which to characterize mathematically important moments that the instructor should notice, but perhaps the connection between *MIMs* and MOSTs was not seen by the GTAs, or was not internalized for in-the-moment analysis. Still, it is encouraging that improvements were made in the noticing of important student thinking as a result of the intervention.

Table 10

Noticing of Inconsistent and Consistent MOSTs by Interview

Participant	Pre	Post
GTA 1	29%	60%
GTA 2	19%	50%
GTA 3	38%	67%
GTA 4	40%	100%
Group	31%	69%

Consistent MOSTs

Second, the subset of MOSTs that the GTAs identified that were consistent MOSTs were examined; that is, those instances that both aligned with MOSTs time-wise in the video and were correctly characterized according to the MOST Analytic Framework. Table 11 shows improvement in the noticing of consistent MOSTs from stage to stage of the intervention. Of particular interest is that in comparison to the inconsistent and consistent MOST data in Table 9, the consistent MOST data has lower baseline data, but otherwise the middle and final data match exactly in these two analyses. This signifies that all of the GTA-identified instances that aligned with MOSTs in the middle and final data were also instances that were correctly characterized according to the MOST Analytic Framework, which alludes to the framework being an important tool in not only recognizing the right moments (align with MOSTs) but also for the right reasons (consistent MOSTs).

Table 11

Noticing of Consistent MOSTs by Stage

Participant	Baseline	Middle	Final
GTA 1	24%	61%	87%
GTA 2	19%	36%	76%
GTA 3	25%	57%	78%
GTA 4	8%	33%	53%
Group	19%	47%	73%

The data presented in Figure 10 very closely resembles that of Figure 9, with the exception of the first two weeks being lower in the consistent MOST data. This again points to the success of the intervention in developing the GTAs' skills in identifying MOSTs for reasons consistent with the MOST Analytic Framework.

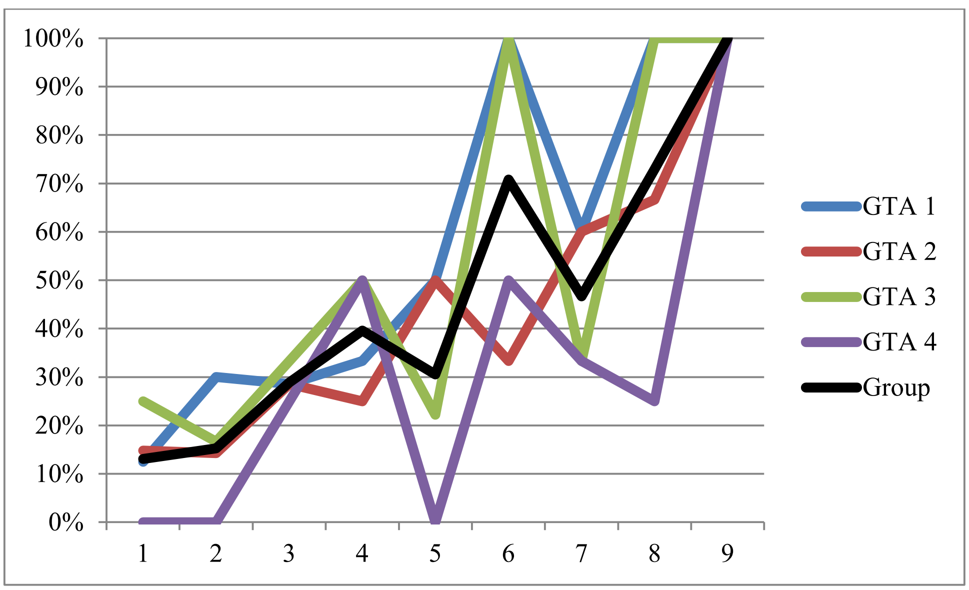

Figure 10. Noticing of Consistent MOSTs by video (in percentages).

The pre-interview and post-interview data presented in Table 12 shows considerable increases in the GTAs' in-the-moment noticing of consistent MOSTs. Consistent with the trends seen in the intervention data, the interview percentages for consistent MOSTs were lower in comparison to the combined inconsistent and consistent MOST data for the pre-interview, but were very similar in the post-interview data (see Table 10 and Table 12). In fact, with the exception of GTA 1, the post-interview percentages for consistent MOSTs matched those of the combined inconsistent and consistent MOSTs. Like the intervention data, the interview data suggests that the intervention was successful in improving the GTAs' ability to notice MOSTs and reason about them in accordance with the MOST Analytic Framework, even in an in-the-moment noticing context.

Table 12

Noticing of Consistent MOSTs by Interview

Participant	Pre	Post
GTA 1	14%	40%
GTA 2	11%	50%
GTA 3	13%	67%
GTA 4	20%	100%
Group	14%	64%

Consistent MOSTS and Non-MOSTs

Because of the interest in the GTAs' skills in using the MOST Analytic Framework to reason about instances of student mathematical thinking, changes in the GTAs' noticing of consistent MOSTs (e.g., Figure 4, Example 1) and consistent Non-MOSTs (e.g., Figure 4, Example 3) were also investigated. Recall that a consistent Non-MOST was a GTA-identified instance that did not align with a MOST on the video timeline and that was reasoned about in accordance to the MOST Analytic Framework, but did not meet all of the MOST criteria. Table 13 demonstrates that consistent reasoning was used most often by the GTAs in the middle and final intervention stages, in those videos analyzed by the GTAs after the introduction of the MOST Analytic Framework. Most impressively, with the exception of GTA 4, the GTAs were able to reason about instances of student mathematical thinking in a manner consistent with the MOST Analytic Framework in 100% of the instances they identified after the introduction of the framework. Even though GTA 4 did not reach 100% in consistent reasoning, improvements were made from stage to stage throughout the intervention. These results suggest that the framework was instrumental in improving the GTAs' ability to analyze instances of student mathematical thinking in a productive way.

Table 13

Noticing of Consistent MOSTs and Non-MOSTs by Stage

Participant	**Baseline**	**Middle**	**Final**
GTA 1	49%	100%	100%
GTA 2	46%	100%	100%
GTA 3	53%	100%	100%
GTA 4	22%	67%	83%
Group	43%	92%	96%

Figure 11 shows that the peak of 100% consistent reasoning first occurred in Video 4 for all GTAs, the video immediately following the introduction of the MOST Analytic Framework. With the exception of GTA 4, the 100% consistent reasoning was maintained for the remainder of the intervention. While this is not surprising considering the stage data in Table 13, it is interesting to note the sharp decreases in consistent reasoning exhibited by GTA 4 in Videos 5 and 8, the same videos with sharp decreases in GTA 4's video data for Noting and/or Analyzing Student Mathematics and primary student agent. Again, these results suggest that either the content of these two videos was a distractor or that the framework was not sufficient to support GTA 4's noticing and consistent reasoning about instances.

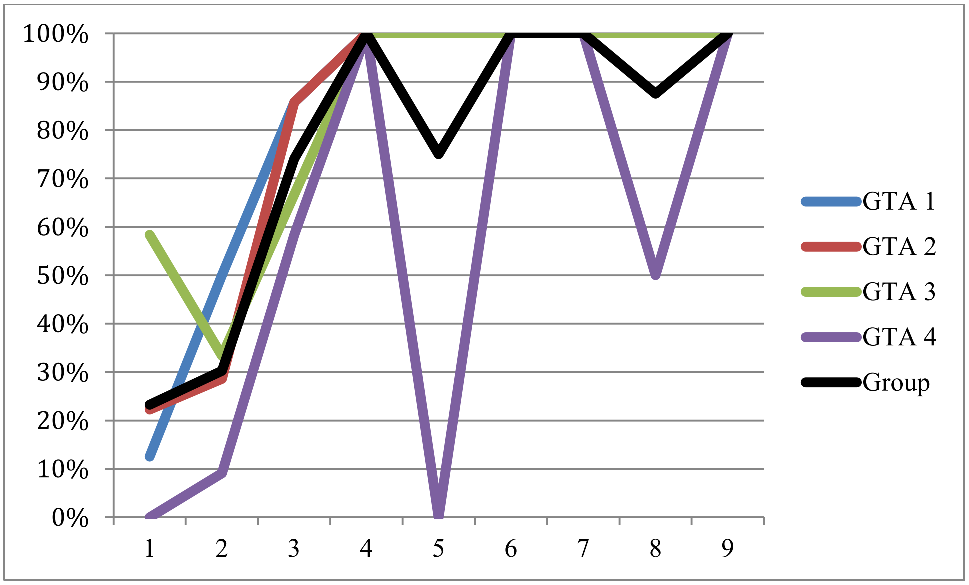

Figure 11. Noticing of Consistent MOSTs and Non-MOSTs by video (in percentages).

The interview data in Table 14 reveals sizeable increases in consistent reasoning from before to after the intervention. This suggests that, in addition to the GTAs improving in their ability to reason consistently in the context of individual video analysis, the intervention was also successful in improving GTAs' skills in productively reasoning about instances of student mathematical thinking in an in-the-moment context.

Table 14

Noticing of Consistent MOSTs and Non-MOSTs by Interview

Participant	**Pre**	**Post**
GTA 1	29%	80%
GTA 2	19%	100%
GTA 3	63%	100%
GTA 4	20%	100%
Group	32%	95%

Responses

The other main goal of the intervention was to increase the amount of student-centered responses that were proposed by the GTAs to instances that they identified in the video. Table 15 displays the interview data for the instances in which the GTAs identified as *MIM*s and provided a student-centered response to explain what they would do if such an instance occurred in their classroom. It can be seen that substantial increases in the percentage of such responses were made from pre- to post-intervention. In fact, with the exception of GTA 4, 100% of the responses provided by the GTAs were student-centered in the post data. While GTA 4 did not reach 100%, the percentage of student-centered responses that were proposed still increased from pre- to post-interview. These results suggest that the intervention was successful in improving the GTAs' skills in proposing student-centered responses in an in-the-moment context.

Table 15

Student-centered Responses by Interview

Participant	**Pre**	**Post**
GTA 1	0%	100%
GTA 2	57%	100%
GTA 3	83%	100%
GTA 4	33%	50%
Group	43%	88%

Post-interview Questions

As part of the post-interview, the researcher asked the GTAs the following questions to get an idea of the effectiveness of the study from the GTAs' perspectives: (a) How might this work affect your teaching?; (b) Was this work valuable to you?; (c) Were there particular parts of the work that were more or less valuable to you? (group discussion, watching other people's teaching, individual coding, using the framework, etc.); and (d) How did this training support your other GTA training?

In response to the question about how this work might affect their teaching, GTAs 1, 2, and 4 indicated that they would enact the practice of engaging the students more during instruction and, in the case of GTAs 2 and 4, change their teaching style in order to implement that practice. GTA 4 added that teaching styles are different in their country and, therefore, had learned a lot about the mathematics material and the teaching of that material from the other GTAs throughout the intervention. GTA 3 stated, "[A]t the very least it's going to make me more aware of these types of moments. ...I'm not always going to necessarily act on them, but I'm going to see more of them." These statements imply that from the GTAs' points of view, this work might improve their noticing and shift their teaching styles to increase student involvement during instruction.

All of the GTAs said that the work was valuable to them. GTAs 1 and 3 especially expressed their appreciation for the depth and detail of this work in regard to student interaction. In response to the question about parts of the work that were more or less valuable to them, all of the GTAs indicated that the group discussions at the weekly meetings and watching videos from a variety of instructors were helpful. These parts of the work were useful in that the GTAs learned from being engaged with diverse points of view and from viewing different teaching styles. None of the GTAs reported that there were parts of the work that they did not find valuable. Both GTAs 1 and 3 expressed their frustration with the early prompt of finding *MIMs* since the definition was left open-ended, but said that they did see the value of "a build up idea" (GTA 1) and "go[ing] through the thought process [themselves] before you can understand what's going on"

(GTA 3).

The responses were a bit more varied with regard to how this work supported the GTAs' other training. GTA 1 stated:

> ...having [the Teaching College Mathematics] class, it was very let's learn how to teach this lesson. ...This was more, oh you have your lesson set. Now let's put you in the classroom. What are you going to do with that lesson? What if this question arises? What can you do with it? And how can you help students get the most out of the lesson? So I think this was more broadly focused on how to help the students rather than let's teach this as well as we can. (GTA 1)

In a similar sense, GTA 3 said:

> I felt like with [the previous training], it was less about helping our teaching and more about teaching us how to teach, just getting started and correcting things, correcting misconceptions. This was less about correcting things and more about, at least to me, changing the way we would approach something. ...To me, [the previous training instructors] were more about creating these types of moments. I feel like part of your class was less about that and more about acting on these moments, about recognizing and being able to take advantage. (GTA 3)

These responses from GTAs 1 and 3 suggest that, from their points of view, part of their previous training was more focused on a teacher's preparation of lessons and creating opportunities for MOSTs to occur, whereas this intervention was more focused on helping students and effectively responding to MOSTs noticed during instruction.

GTA 4, whose viewpoint was a bit different from GTAs 1 and 3, said:

> Originally I ha[d] no experience. I don't know anything about the math. Maybe I

> already forg[ot] about the concept and the terminology. And the initial training from [previous training instructors], so that one was to just refresh my memory about the knowledge that is more focused on the math part. But this part, because I have the knowledge now...I want to know how I can approach the teaching part. The teaching part is more important than the pure math knowledge. (GTA 4)

This statement indicates that GTA 4 was of the perspective that their previous training was more focused on the mathematics content, whereas this work was more focused on teaching.

GTA 2 saw their work during the intervention as an enhancement of part of their previous training:

> I think it really enhanced what [seminar instructor] talked about because I could see in person how valuable going through having the students work together really strengthened what they learned. …For me personally, sometimes it was a little hard to try and implement what he suggested. That was probably mostly a product of the course that I taught...and I was also still learning too, and sort[ing] through everything being a TA. This was really nice because I could figure out which things that I should or should not focus on as an instructor, and also it was nice because I could see which styles would work for implementing the group work and all that. Also it was nice to see how to put things in action on a more personal level, or see it implemented in person through another classroom. That gave me some ideas that I probably would not have come up with on my own. (GTA 2)

Thus, the GTAs either thought that their previous training had a slightly different focus than this work (GTAs 1, 2, and 4), or saw this work as an enhancement of their previous training (GTA 2).

CCI scores

After the intervention, the GTAs took the CCI assessment to determine whether their mathematical content knowledge may have had an effect on their noticing. Table 16 shows the GTAs' performances, both as a raw score and as a percentage. Overall, the performances of the GTAs on the CCI were quite similar. This suggests that all of the GTAs in the study had about the same aptitude for calculus concepts, and therefore, there was no evidence that differences in mathematical content knowledge accounted for differences observed in their noticing.

Table 16

CCI Scores

Participant	**Score out of 22**	**Percentage**
GTA 1	17	77%
GTA 2	17	77%
GTA 3	19	86%
GTA 4	19	86%
Group	18	82%

Chapter 5: Discussion

This study, focused on a professional development intervention that included analyzing classroom video with the MOST framework, sought to answer the following research questions: (a) How effective is the intervention in improving GTAs' noticing of mathematically significant pedagogical opportunities to build on student thinking (MOSTs)?; (b) How effective is the intervention in supporting the GTAs' ability to propose in-the-moment student-centered responses to instances they identified in video?; (c) How valuable was the intervention from the GTAs' perspectives?; and (d) What is the relationship between the GTAs' common mathematical content knowledge and the development of their noticing skills during the intervention?

Results showed that the intervention was successful in improving all of the GTAs' noticing in a number of ways and in two different video analysis contexts. In analyzing video both individually and in an in-the-moment interview context, the GTAs greatly increased in their noticing of instances primarily focused on students, the percentage of instances in which they discussed the mathematics of an instance in a specific manner, their focus on describing (Noting) and/or interpreting (Analyzing) the student mathematics of an instance, their noticing of consistent MOSTs, and their ability to reason about instances in a manner consistent with the MOST Analytic Framework. Because of the timing in the intervention of the largest improvements in particular noticing skills, the MOST Analytic Framework appeared to directly contribute to the increases in the GTAs' noticing of primarily students, in Noting and/or Analyzing Student Mathematics, and in reasoning about instances of student mathematical thinking in a productive way (consistent reasoning). These results add support to the successes already seen in the mathematics education field of professional development that uses video and a defined framework to improve the noticing skills of preservice and inservice mathematics teachers at the K-12 level (Santagata, 2011; Schack et al., 2013; McDuffie et al., 2014; Stockero et al., 2015; under review) and suggest that such interventions can be successful at the undergraduate level as well.

The intervention was also successful in improving the GTAs' skills in proposing student-centered responses to instances they identified in video, the *deciding how to respond* skill of noticing (Jacobs et al., 2010). Specifically, these gains were documented from pre-interview to post-interview in an in-the-moment context. This finding builds upon those of existing studies (Jacobs et al., 2011; Jacobs et al., 2010; Schack et al., 2013) that suggest that professional development structured around noticing students' mathematical thinking in video and/or classroom artifacts can develop teachers' abilities in not only the noticing skills of *attending to and interpreting [students'] strategies and understandings*, but also the skill of *deciding how to respond* (Jacobs et al., 2010).

While most of the GTAs showed similar improvements in their noticing skills and in the proposal of student-centered responses to instances in video, GTA 4's improvements were not as consistent. This raises questions as to why. One explanation would be a difference in common content knowledge, as measured in this study by the CCI. The CCI scores were very similar among the GTAs, however, providing no evidence to support the idea that differences in mathematical knowledge was related to differences in noticing skills. Thus, the difference in GTA 4's noticing cannot reasonably be attributed to a difference in their mathematical content knowledge. GTA 4 did seem particularly challenged by Videos 5 and 8, with sizable decreases in the noticing of primary student agent, Noting and/or Analyzing Student Mathematics, Specific Math, MOSTs (both inconsistent and consistent), and consistent reasoning (consistent MOSTs and Non-MOSTs), which may account for some of the lower percentages that were found for this participant. Recall also that GTA 4's stronger post-interview results for Analyzing Student Mathematics suggested that perhaps they were able to communicate better in spoken word than in written text, another potential explanation for the lower results in the stage data. While this participant had received the same previous departmental training as the other GTAs after admission to the university's graduate program, GTA 4 was the only international student in the study, and thus had a different cultural and educational background from the other GTAs. GTA 4 mentioned in the post-interview that teaching styles are different in their country and, therefore, had learned a lot about the mathematics material and the teaching of that material from the other GTAs throughout the intervention, in addition to learning about noticing. Perhaps, then, not having a similar educational background as the other domestic GTAs provided a challenge to GTA 4's development in their noticing skills. This suggests that international GTAs may need additional support when engaging in a noticing intervention such as the one in this study. However, the data available from this study is limited to one international GTA, and thus further study is required.

While the results of this study suggest that similar interventions could be successful in training GTAs in noticing student mathematical thinking, there are limitations to this study and further questions to investigate. This was one, isolated study with a small number of participants. The significance of the results, therefore, must not be overgeneralized. Future work could involve replicating this study with more GTAs, with GTAs of varying cultural backgrounds, at other universities, with another set of videos, and in other subject areas to see if there are similar results. To expand on this study, the following questions could be investigated: How does such an intervention affect the GTAs' classroom teaching? What role does the facilitator play in building the GTAs' noticing skills during meetings? How does the set of videos used in the intervention affect the improvement of the GTAs' noticing skills?

Limitations aside, the intervention in this study was successful in improving the noticing skills of mathematics GTAs and in the proposal of student-centered responses, both steps in the right direction for advancing student-centered instruction in undergraduate mathematics courses. The development and improvement of these skills, while achieved in this study in the structure of a professional development setting, have the potential to improve the GTAs' classroom instruction. Indeed, at least from the perspective of GTAs 1, 2, and 4, the intervention might affect their instruction by enacting the practice of engaging the students more and, according to GTAs 2 and 4, changing their teaching style in order to implement that practice. Thus, an intervention such as this that targets mathematics GTAs could possibly influence the instructional methods used in higher education and improve the retention of first- and second-year undergraduate students (Cano et al., 1991; Speer et al., 2005). Students in Seymour and Hewitt's study (1997) suggested that all faculty teaching lower level classes in SME fields should obtain professional pedagogical training. Completing professional development focused on noticing important student ideas could not only improve SME pedagogy in general, but provide opportunities to practice student-centered instructional methods, which are essential for effective teaching (NRC, 2005; NCTM, 2014).

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