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Explaining Bond Spreads Via Default Risk, Taxes, Rating Transition and Liquidity

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Abstract

This study develops a semi-structural framework of bond pricing that incorporates default risk, taxes, and bond rating transition, whereas prior papers have primarily focused on the first (and more recently the second) factor. After capturing the three effects, the remaining spread between corporate bond rates and risk free rates can intuitively be attributed to liquidity. Models estimated without all three effects cannot intuitively dismiss the "unexplained" spread as a liquidity premium. This is confirmed by applying the framework to samples from two periods (1973-1993, and 2004-2010).

JEL: G24, G30

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1. Introduction

In this paper, we generalize the structural models for defaultable taxable bonds to incorporate rating transition estimated exogenously. The incorporation of rating transition is not an arbitrary decision. First, as we show, it improves the structural model calibration procedure which originally ruled out the possibility of rating migrations before default. Second, since ratings and the fundamental change in issuers' risks are not accurately reflected or instantly correlated,[1] it is sensible to model the investors' pricing process as the expectation of the underlying risks signaled by ratings. Third, as we demonstrate, the unexplained spread component in structural models with default and personal taxes alone displays a strong rating-related pattern. A consistent and robust finding is that default and taxes have the least explanatory power for BBB ratings in terms of percentage of the observed spreads. This is direct evidence of rating transition risk being priced. If transition risk is ruled out, one would have to accept the conclusion that junk bonds are more liquid than investment grade bonds, which is extremely counter-intuitive since many institutional investors are prohibited from investing in junk bonds.

These observations indicate that the unexplained spread component left by default and tax-related premia is very hard to attribute to liquidity risk alone. Instead, it strongly suggests a very different factor is at work – rating transitions. Accordingly, we would expect to remove or greatly reduce the bothersome rating-specific pattern in the unexplained spread, mitigate the over-shooting problem for low rate bonds and create an intuitive liquidity premium. These suppositions are shown to be true in our model.

2. Background

Most structural bond pricing models based on default risk can explain only a very small portion of the observed spreads for investment-grade bonds. On the other hand, the models appear to be able to explain a significant portion of the observed spreads for low grade bonds, and in some cases, even exceed or "overshoot" the observed values. Both are problems in structural models that deserve investigation.

Jones *et. al.* (1984) demonstrated that the Merton-type model predicts a credit spread well below the observed yield spread, especially for high-grade bonds.[2] Since then many structural models have emerged to incorporate other factors: stochastic interest rates (Longstaff and Schwartz 1995), endogenous default, (Black and Cox 1976, Leland 1994, 1998, and Leland and Toft 1996), strategic default by stockholders, (Anderson and Sundaresan 1996, Anderson, Sundaresan, and Tychon 1996, and Mella-Barral and Perraudin 1997), and stationary leverage ratios (Collin-Dufresne and Goldstein 2001).[3]

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[1] See Kliger and Sarig (2000), and Altman and Rijken (2004).
[2] See Helwege and Turner (1999) and Elton *et. al.* (2001).
[3] In addition, Duffie and Lando (2001) consider noisy accounting information. Zhou (2001) incorporates jump risk. Goldstein *et. al.* (2001) include dynamic capital structure.

Despite these efforts, there is no fundamental improvement in predicting credit spreads (see Lyden and Saraniti 2000; and Eom *et. al.* 2004). Huang and Huang (2003) investigated most representative structural models only to find that their performance is fairly stable for different parameter choices and credit risk accounts for only a very small portion (20 to 30 percent) of the spread for high-grade bonds. The performance appears to be better for low-grade bonds, but the enhanced explanatory power may be spurious because of the overshooting problem of the structural models for junk bonds (e.g., see Eom *et. al*. 2004). Therefore, the performance of these structural models is far from satisfactory and reliable for all ratings.

One factor previously ignored in term structure models is personal taxes. It has been found that personal taxes explain a significant portion of corporate bond spreads. In many cases, the tax premium exceeds the default premium (Elton *et. al.* 2001). Further, tax effects with endogenous capital structures demonstrate rich interactions with default risk and have direct influences on bond prices. The tax effects also have a strong impact on capital structure, which in turn indirectly affects bond prices (Liu *et. al*. 2006).

The primary focus to date has been on the explanatory power of these models without careful thought to the "unexplained" portion of the spread, which is typically attributed to the unobservable liquidity factor. Naturally, if the model completely explained the spread, one would not need to bother with the unexplained spread. Driessen (2005) attempted to decompose credit spreads into the default-, liquidity- and tax-driven components. As we will show here, while personal taxes improve the model's descriptive power, the resulting pattern in the "unexplained" spread is not intuitively related to liquidity.

To date, bond rating transitions have not been combined with the structural models. In reality, most firms experience downgrades before they end up in bankruptcy. Graham and Harvey (2001) found that CFOs ranked bond ratings higher than the tax advantage of interest deductibility when determining their capital structure. According to Kisgen (2006), “firms near a credit rating upgrade or downgrade issue less debt relative to equity than firms not near a change in rating.” Indeed, the concern for credit ratings is justified. Credit ratings determine whether many institutional investors are allowed to hold a firm’s bonds.

Many rating-based reduced-form models have been developed. The two most widely used are by Duffie and Singleton (1997) and Jarrow *et. al.* (1997). One may ask whether ratings simply reflect fundamental change in issuers’ risks or whether they carry some additional information value. Kliger and Sarig (2000) showed that rating information does not affect firm value, but that debt value increases (decreases) and equity value decreases (rises) when Moody’s announces better (worse) than expected ratings. This effect implies that the rating’s signaling is not accurate. Investors tend to believe that bond ratings are slow in responding to changes in corporate credit quality.[4] Similar findings are reported in the surveys by Ellis (1998) and Baker and Mansi (2002).

[4] See, e.g., Altman and Rijken (2004).

Therefore, (1) bond ratings are an important factor in determining bond prices; (2) they are dynamic in nature; and (3) they contain additional information value.

3. Model Calibration and Rating Transitions

Our calibration procedure finds its root in Huang and Huang (2003). The essence of the procedure is to choose the unobservable parameters (e.g., asset volatility) such that the model generates other variables (e.g., cumulative default probability, equity premium, etc.) that match the observed values. This procedure is used to make sure that we did not obtain spurious results. For example, a model may appear to perform very well, but we may find that the seemingly accurate model-generated spread is mainly due to an unrealistically high model-implied default risk. Therefore, the true performance of a model is the accuracy of the generated spread when the implied default risk is adjusted to match the observed values.

In this paper, we applied the models to 4- and 10-year bonds. The aggregate bond data for the period 1973-1993 from Huang and Huang (2003) is used for direct comparability. The target values of the selected parameters and the observed aggregate historical spreads for each bond rating are given in Table 1.

For tractability, we assume away tax-timing effect if the bonds are traded before maturity. The long-term capital gains tax rate is a fraction α of the regular income tax rate τ. Tax rates applied to capital gains and losses are equal and there is no limit on loss deduction. In the event of default, a portion of the principal (or residual) is paid, and the remaining loss is treated as capital loss immediately deductible from an investor's taxable income.[5]

The rating transition is accounted for by applying our endogenously obtained spreads to a rating transition matrix under the equivalent martingale measure.[6] This should capture two effects that have not been previously considered in models: First, the probability of a firm assigned one rating having its true fundamentals similar to those in other ratings, i.e., the imperfectness of rating-based information; second, the probability of a firm indeed migrating among different rating categories before the bond maturity.

[5] Interest income and short-term capital gains are taxed at the ordinary income tax rate while long-term gains are taxed at a lower capital-gains rate. The ordinary income tax rate for the highest income group is 35% for both individuals and corporations. Corporate bond income is subject to state taxes but Treasury bond income is not. Maximum state marginal income tax rates generally range from five to ten percent, per Commerce Clearing House (1997) and Elton *et. al*. (2001). The effective tax rate of corporate bonds is equal to $\tau = \tau_F + \tau_S(1 - \tau_F)$, where τ_S is the state income tax rate and τ_F is the federal income tax rate. The last term reflects that state taxes are deductible from income for the purpose of federal taxes. Amortization is an important feature of bond investments. We assumed straight-line method.

[6] The observed transition matrices are not appropriate for such operations. The existence of a unique transition matrix under the equivalent martingale measure is implied by the assumption of market completeness. We thank Manfred Frühwirth for this observation.

TABLE 1
TARGET PARAMETERS FOR MODEL CALIBRATION

This table shows the values of the target parameters used for model calibration. The last two columns include the average observed yield spreads. The target data are from Huang and Huang (2003) for the period 1973-1993. We set recovery rate for the LT model to 80% per Andrade and Kaplan (1998) and Eom *et. al*. (2004), which indicates that the cost of financial distress is in the range of 15 to 20 percent of the firm's going concern value. We set the target recovery rate to 80% for the LT model. For the CG model, recovery is defined as a percentage of bond face value regardless of debt maturity, and we set the recovery rate to 50 percent, similar to that used in Colin-Dufresne and Goldstein (2001) and Huang and Huang (2003).

	Target Parameters for Model Calibration			Observed Average Yield Spread	
Credit Rating	Equity Premium (%)	4-Year Cumulative Default Probability (%)	10-Year Cumulative Default Probability (%)	4-Year Bond (bps)	10-Year Bond (bps)
AAA	5.38	0.04	0.77	55	63
AA	5.60	0.23	0.99	65	91
A	5.99	0.35	1.55	96	123
BBB	6.55	1.24	4.39	158	194
BB	7.30	8.51	20.63	320	320
B	8.76	23.32	43.91	470	470

We followed the procedure of Wei (2000) to construct our rating transition matrix. That is, we adopted the risk neutral 1-year rating transition matrix estimated by Wei (2000). Next we derived the 4- and 10-year transition matrices by assuming the transition is purely Markovian and time-homogeneous.[7] Furthermore, we normalized the transition matrices by redistributing the probability mass for categories other than AAA through B. This was required to isolate the transition effect from default risk (i.e., probability of migrating to default), noises from not-rated categories, and the possibility of withdraw (or withdrawn ratings). These standard adjustments are normally made to smooth the transition probabilities (see for example, Wei 2000, and Carty 1997). Table 2 shows the 4- and 10-year rating transition matrices we used in this study.

[7] These two assumptions are commonly used in literature, e.g., Jarrow, Lando and Turnbull (1997), to simplify the computation, although there has been research showing that these two assumptions may not hold well due to the "momentum effect" (see Bahar and Nagal 2001).

TABLE 2
RATING TRANSITION MATRIX UNDER THE EQUIVALENT MARTINGALE MEASURE

Panel A is the accumulative four-year rating transition matrix, and panel B is the accumulative 10-year rating transition matrix. Both are derived from the one-year transition matrix under the equivalent martingale measure given in Wei (2000). Given ratings below B and other not-rated (NR) categories are not considered, the transition matrices are normalized to 100 percent for all possible migrations among AAA through B ratings (see Wei 2000, and Carty 1997).

Panel A: Four-year accumulative rating transition matrix (%)

	AAA	AA	A	BBB	BB	B	***Total***
AAA	77.17	18.67	3.38	0.58	0.15	0.05	100
AA	2.14	71.92	21.73	3.30	0.52	0.39	100
A	0.31	7.10	73.23	15.81	2.50	1.04	100
BBB	0.13	1.35	14.97	67.55	12.17	3.83	100
BB	0.12	0.42	3.16	19.46	56.52	20.33	100
B	0.10	0.92	3.43	8.44	37.77	49.32	100

Panel B: Ten-year accumulative rating transition matrix (%)

	AAA	AA	A	BBB	BB	B	***Total***
AAA	53.08	30.59	12.27	3.01	0.68	0.36	100
AA	3.60	47.29	34.98	10.47	2.34	1.32	100
A	0.81	11.71	52.95	24.86	6.43	3.24	100
BBB	0.38	3.99	24.49	46.05	16.41	8.69	100
BB	0.30	1.74	10.42	28.60	33.46	25.48	100
B	0.25	1.46	6.21	14.80	27.87	49.41	100

4. Risky Bond Pricing with Default Risk, Taxes and Rating Transitions

The tax-related premium cannot be separated from the default premium.[8] In addition, personal taxes affect the optimal leverage, which in turn affect corporate bond pricing and yield spread; therefore taxes must be incorporated under specific capital structure assumptions. We next develop models for exogenous and endogenous leverage and rating transitions.

4a. Exogenous Leverage

We start by extending the basic framework of Colin-Dufresne and Goldstein model (2001), hereafter the CG model. It assumes a stationary mean-reverting leverage ratio. The CG model is a two-factor model where the spot rate r_t follows the Vasicek (1977) process and the firm value V_t follows a geometric Brownian motion. Default occurs when the firm value hits the bankruptcy threshold K, or $l_t = \log(K / V_t) = 0$. The firm value V_t is specified under the risk-neutral measure (Π) by:

[8] For example, the tax rebate is directly related to the timing of default and the value of recovery.

$$\frac{dV_t}{V_t} = (r_t - \delta)dt + \sigma dz_1{}^{\Pi}(t) \tag{1}$$

where δ is the payout ratio and σ is the volatility parameter. The risk-free tax-free spot rate r_t obeys the following process:

$$dr_t = \beta(\theta - r_t)dt + \eta dz_2{}^{\Pi}(t) \tag{2}$$

where β, θ and η are constants with $dz_1{}^{\Pi} dz_2{}^{\Pi} = \varsigma\, dt$. The log-default threshold k_t is mean-reverting:

$$dk_t = \lambda[y_t - \nu - \phi(r_t - \theta) - k_t]dt \tag{3}$$

where $y_t = \log V_t$, $(y_t - \nu)$ sets the target threshold with ν and λ adjusting the mean-reverting speed, and $\phi \geq 0$. This stationary process reflects the fact that firms tend to issue additional debt when their leverage ratio falls below some target and are less willing to replace maturing debt when it is above that target.

If default does not occur before maturity T, the bond investor receives the principal scaled to a unit payoff at T. Otherwise, a fractional loss in principal, L, is incurred. The price of a risky tax-free bond is thus given by:

$$P^T(r_0, l_0) = D(r_0, T)[1 - LQ^T(r_0, l_0, T)] \tag{4}$$

where $D(r_0, T)$ is the price of the risk-free zero-coupon bond and $Q^T(r_0, l_0, T)$ is the cumulative probability of default before T under the T-forward measure. Using the method of the first passage time probability density and discretizing the interest rate range (with a lower bound r_L and an upper bound r_U) into n_r equal intervals by $r_i = r_L + i\Delta r = r_L + i \times (r_U - r_L)/n_r$ for $i \in (1,2,...,n_r)$ and time into n_T equal intervals with $t_j = jT/n_T = j\Delta t$ for $j \in (1, n_T)$, where $\Delta t = T/n_T$, $Q^T(r_0, l_0, T)$ at time zero is:

$$Q^T(r_0, l_0, T) = \sum_{j=1}^{n_T} \sum_{i=1}^{n_r} q(r_i, l_0, t_j) \tag{5}$$

where l_0 is the initial leverage ratio and $q(r_i, l_0, t_j)$ is the probability mass in a grid ($\Delta r \times \Delta t$) at the level of (r_i, t_j). The specific form of $q(r_i, l_0, t_j)$ is given in Colin-Dufresne and Goldstein (2001).

For defaultable coupon bonds, we assumed that the unpaid coupons have a 100% write-down, i.e., the coupon loss rate $L_{coupon} = 1$. If default does not occur, the bondholder pays a regular income tax on each coupon and a capital gains tax on the difference between the principal and purchase price. If default occurs before maturity, the bondholder receives

(1 – *L*) of face value, and a tax rebate based on the capital gains tax rate $\alpha\tau$, where τ is the ordinary income tax rate and α is equal to 1 if the loss is short term or less if it is long term. According to Qi *et. al.* (2010), the price of the risky taxable coupon bond with straight-line amortization is:[9]

$$P^T\left(r_0,l_0\right)=\frac{Z_M+\sum_{m=1}^{M}\left[\left(\left(1-\alpha\tau\right)\left(1-L\right)+\frac{\alpha\tau\times t_m}{t_M}\right)\Gamma_m\right]+\sum_{m=1}^{M}\left[\left(1-\tau\right)c-\frac{\tau}{t_M}\right]Z_m}{1-\frac{\tau}{t_M}\sum_{m=1}^{M}Z_m-\frac{\alpha\tau}{t_M}\sum_{m=1}^{M}\left[\left(t_M-t_m\right)\Gamma_m\right]} \tag{6}$$

where,

$$\Gamma_m=D\left(r_0,t_m\right)\Delta Q^T\left(r_0,l_0,t_m\right) \tag{7}$$

$$Z_m=D\left(r_0,t_m\right)\left[1-Q^T\left(r_0,l_0,t_m\right)\right] \tag{8}$$

$D(r_0,t_m)$ is the price of the risk-free tax-free zero coupon bond with maturity t_m, and $\Delta Q^T\left(r_0,l_0,t_m\right)$ is the incremental default probability over time period $\left(t_{m-1},t_m\right]$ under the *T*-forward measure using $D\left(r_0,t_m\right)$, the price of riskless tax-free bond with maturity $t=t_m$, as the numeraire.

4b. Endogenous Leverage

The Leland-Toft Model (1996), hereafter the LT model, is based on the tradeoff between the benefit of corporate tax shields and the costs associated with bankruptcy. The firm can endogenously choose a bankruptcy boundary which maximizes firm value. The optimal leverage is therefore set endogenously. Firm value of an unlevered firm, *V,* is assumed to follow the diffusion process:

$$\frac{dV}{V}=\left[\mu\left(V,t\right)-\delta\right]dt+\sigma dZ \tag{9}$$

where $\mu\left(V,t\right)$ is the expected rate of return on the firm's asset, δ is the total payout ratio,[10] Z is a standard Wiener process, and σ is the volatility parameter.[11]

The LT model assumes the firm issues debt continuously to replace the debt that is expiring, hence maintaining stationary leverage. Within a unit of time, say one year, the firm issues debt *d* that has a continuous constant coupon flow *c*(*t*), principal $p(t)$, and

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[9] We have studied cases with and without amortization. The difference in the results is small, thus we only present "with-amortization" framework. All the results are available upon request.

[10] This is a proportion of firm value paid to all security holders.

[11] A similar process was used by Merton (1974), Black and Cox (1976), and Brennan and Schwartz (1978).

maturity t. The asset value process of the levered firm continues without time limit until it hits a default boundary V_B, at which the firm defaults on its debt. Upon default, bondholders receive a fixed portion ρ of the asset value V_B, or $(1 - \rho)$ is the fraction of firm value lost due to default. Suppose the firm continuously issues new debt with maturity T years as long as the firm remains solvent. The value of all outstanding debts D is determined by integrating $d(V, V_B, t)$ over the period of T:

$$D(V, V_B, T) = \int_{t=0}^{T} d(V, V_B, t)\,dt \tag{10}$$

The levered firm value equals the unlevered firm value plus leverage benefits less bankruptcy costs. Given the unlevered asset value $V(t)$, equity value $E(V, V_B, T)$, tax benefit $h(V, V_B)$ of leverage, bankruptcy cost $B(V, V_B)$, and outstanding debt $D(V, V_B, T)$, the levered firm's value $W(V, V_B, T)$ is:

$$W(V, V_B, T) = V + h(V, V_B) - B(V, V_B) \tag{11}$$

where

$$\begin{aligned} h(V, V_B) = {} & \left(1 - \frac{(1-\tau_C)(1-\tau_E)}{1-\tau}\right)\frac{C}{r}\left[1 - \left(\frac{V_B}{V}\right)^{a+z}\right] \\ & + \tau_{EC} E(V, V_B, T)\left(\frac{V_B}{V}\right)^{a+z} \end{aligned} \tag{12}$$

$$B(V, V_B) = (1-\rho) V_B \left(\frac{V_B}{V}\right)^{a+z} \tag{13}$$

where $a = \dfrac{r - \delta - (\sigma^2/2)}{\sigma^2}$, $b = \ln\left(\dfrac{V}{V_B}\right)$, $z = \dfrac{[(a\sigma^2)^2 + 2r\sigma^2]^{1/2}}{\sigma^2}$, and τ_C is the corporate income tax rate, τ_E is the effective tax rate on equity returns, and τ_{EC} is the capital gains tax rate on equity appreciation.[12] Both dividends and capital gains (or losses) of equity are subject to taxes. Dividend income is taxed at the ordinary income tax rate τ, whereas capital gains are taxed at $\tau_{EC} = \alpha\tau$.[13] The effective tax rate on equity returns τ_E is the weighted average of dividend and capital gains tax rates. Graham (2003) suggests that the effective equity tax rate is $\tau_E = (1-\delta)\alpha\tau + \delta\tau$ where the weight depends on the payout ratio δ.[14] Given the par-bond condition assumed in the LT model, Liu *et. al.* (2006) showed the price of the firm's new debt (per unit of time) is:

[12] For the derivation of (12) and (13), see Leland and Toft (1996) and Qi *et. al*. (2010).
[13] This tax treatment holds for the sample period of the data we use to calibrate the model.
[14] See Graham (2003, p. 1095; 2000, p. 1912; 1999, p. 153).

$$p(T)=\frac{\frac{(1-\tau)c}{r}-e^{-rt}\frac{(1-\tau)c}{r}\left[1-F(T)\right]+\left[(1-\alpha\tau)\rho(T)V_B-\frac{(1-\tau)c}{r}\right]G(T)}{1-\alpha\tau\left\{e^{-rt}\left[1-F(T)+G(T)\right]\right\}-e^{-rt}(1-\alpha\tau)\left[1-F(T)\right]} \tag{14}$$

where

$$F(t,V,V_B)=N\left[h_1(t)\right]+\left(\frac{V_B}{V}\right)^{2a}N\left[h_2(t)\right]$$

$$G(t,V,V_B)=\left(\frac{V_B}{V}\right)^{a-z}N\left[q_1(t)\right]+\left(\frac{V_B}{V}\right)^{a+z}N\left[q_2(t)\right]$$

and $N(\cdot)$ denotes the cumulative standard normal distribution. The parameters $h_1(t)$, $h_2(t)$, $q_1(t)$and $q_2(t)$ are functions of personal taxes and their analytical forms can be found in Liu *et. al*. (2006).

4c. Defining Yield and Spread

For a coupon bond with maturity $T = t_M$, price $P(T)$, and tax rate τ, the yield to maturity $Y(T,\tau)$ at t_0 can be obtained by solving the following equation:

$$P^T = e^{-Y(T,\tau)T} + \sum_{i=1}^{M} ce^{-Y(T,\tau)t_i} \tag{15}$$

where t_i is the time associated with the cash flow in the ith period and c is the coupon rate. The yield spread is the difference between the yield on a corporate bond $Y^C(T, \tau)$ and that on a Treasury $Y^T(T, \tau_F)$ with the same maturity T,

$$YS(T) = Y^C(T,\tau) - Y^T(T,\tau_F) \tag{16}$$

Since corporate bonds are subject to default risk and state taxes while Treasury bonds are not, yield on corporate bonds contains a premium to compensate investors for these disadvantages.

4d. Rating Transition and Corporate Bond Spreads

So far the structural models are static in that they implicitly assume a bond would either remain in the same rating or in default before maturity.[15] It does not consider the possibilities that a bond may end up in other different ratings at maturity. The markets are

[15] This is because the asset volatility σ is treated as a constant in both the LT and the CG models over the bond's maturity and cannot be calibrated to match the default rate for multiple ratings at a time. The CG model, in addition, is also calibrated to match only one rating's leverage over the bond's life. By doing so, they all implicitly consider two possibilities – maintaining the same rating (with the same asset volatility σ and leverage), or in default.

quite different for investment-grade bonds and junk bonds, the possibility of rating migration itself (especially across this border) would introduce unique market-related risk[16] which is not considered in the original structural models. Incorporating rating transition risk accounts for the probability of a bond being downgraded and/or upgraded before its maturity as well as the probability that a bond assigned one rating has financial fundamentals or true credit risk more similar to bonds in other ratings.[17] In actual procedure, we derive the 4- and 10-year transition matrices based on the 1-year risk neutral estimate by Wei (2000). We assume the rating transition is a time-homogeneous Markov process and smoothed the transition probabilities by normalizing the total probability to one for the ratings we consider. This is a standard procedure that allows us to remove the impact of migration to/from other ratings.[18] The procedure involves two steps as follows.

First, the models with taxes are calibrated strictly based on the rating-specific data in Table 1. The spreads obtained in this manner are completely unaffected by rating transitions and we use YS_i to denote the generated spread for the *i*th rating.

Second, the transition matrices shown in Table 2 are applied to the spreads generated in the first step. This is done with the follow approximation (with a slight abuse of the summation sign),

$$\overline{YS_i} = YS_i + \sum_{j=AAA}^{B} \left(YS_j - YS_i\right) \times a_{ij} \tag{17}$$

where $\overline{YS_i}$ is the spread for the *i*th rating after we account for rating transitions, and a_{ij} is the element of the transition matrix, which represents the cumulative probability of migrating from the *i*th to the *j*th rating. Since the value of a_{ij} will be exogenously determined, (17) is a reduced-form relationship. The added terms, $\sum_{j=AAA}^{B} \left(YS_j - YS_i\right) \times a_{ij}$, are modifications for rating transition effect. Essentially, (17) can be thought of as the weighted average of all possible ratings a bond may end up with before maturity. This is

[16] Imagine a BBB-rated bond being downgraded to BB, one immediate consequence is that many institutional investors (such as pension funds and savings banks, etc.) holding this bond would have to liquidate their holdings. Simply due to such an increase in supply, bond price would drop even though the firm's fundamentals have not changed much. Furthermore, with many institutional investors bailing out, the issuer of this newly downgraded BB bond would find it much harder to raise new capital which in turn makes the existing debt riskier. Therefore, the significantly higher default rate of junk bonds may be partially attributed to this rating change.

[17] There is evidence that rating changes lag rather than lead security-price changes. Wakeman (1981) argues that rating changes merely provide "a single, easily communicated code that incorporates all the major ingredients of the bond's risk". Kliger and Sarig (2000) point out that rating changes are triggered by economic events, and therefore it is unclear how much of the price reaction to rating changes is due to the rating announcement and how much is to triggering economic event itself.

[18] This is designed to remove or reduce the biases in rating transition risk estimates due to not-rated categories, firms withdrawn from ratings, and bankruptcy states.

to overcome the shortcoming of the models that only one asset volatility σ (hence one rating) is allowed. The linear form of (17) is used because the transition matrices are under the risk neutral measure and any other form implies risk premium.

The resulting spreads $\overline{YS_i}$, where i ranges from AAA through B, are therefore generated endogenously within our structural framework and reflect the impacts from all three factors: default risk, taxes and transition risk.

5. Results and Analysis

In our simulation, the parameter values were chosen as close as possible to the original models. For the CG model, we chose $\beta = 0.1$, $\delta = 0.06$, $\sigma = 0.2$, $\eta = 0.015$, $\theta = 0.08$, $\lambda = 0.18$, $\varsigma = -0.2$, $\phi = 2.8$, $\nu = 0.6$, $L = 0.5$ and the initial short rate $r_0 = 0.08$. These parameter values resemble those used by CG. In the numerical calculations, we set $r_L = 0.001$ and $r_U = 0.18$ as the lower and upper bounds for the range of the spot rate r_t and discretize this range into $n_r = 25$ equal intervals. The coupon rate was set to 8%. For the endogenous leverage model, we employed the same interest rate (r = 8%) and payout ratio ($\delta = 6\%$) as used by LT. The bankruptcy cost rate was set to 20% of the firm value as suggested by Andrade and Kaplan (1998). The coupon rate, *c,* is a decision variable, which is optimally generated by the calibrated LT model.

5a. Yield Spreads Generated by the Models without Rating Transition Risk

Figure 1 reports the CG and LT model results for 4-year bonds with the personal tax rate equal to 0% (i.e., Default risk only) and 23% (Default and taxes). The 23% personal tax rate was chosen such that the equity return tax rate, estimated as a weighted average of personal income and capital gains tax rates, i.e., $\tau_E = (1-\delta)\alpha\tau + \delta\tau$, is equal to 12% as estimated by Graham (1999). The results for τ = 10% and τ = 30% are available and have similar properties to the results displayed here. The upper panels are expressed in basis points and the lower panel in percentage of the observed values.

As seen in Figure 1, default risk alone explains very little in absolute terms or percentage terms of the observed spread for high quality bonds. Personal taxes considerably enhanced the model performance, especially for high quality bonds. For example with τ = 23%, the CG model explained 41 basis points of the observed spread on 4-year AAA bonds compared to 0.6 basis points with no taxes. As rating decreases, the relative improvement due to taxes becomes smaller. However, even for BBB bonds, taxes still contribute at least an additional 44 basis points. The 4-year bond results of the LT model display an analogous pattern to the CG model.

The results for the 10-year bonds are found in Figure 2 and mimic the 4-year bond results. That is, default risk alone explains a small percentage of the spread while personal taxes increase the model's performance for all bond ratings, especially high quality bonds. For example, the CG model’s explanatory power increased from 3 to 45 basis points for AAA bonds by incorporating income taxes, while the LT model can still improve its predictive power by 15.8% on the BBB bonds.

In summary, personal taxes are an important factor in explaining the spreads on the risky bonds and after incorporating personal taxes the structural models do much better in explaining the spreads on risky bonds.

5b. Evidence for the Rating Transition Risk

As shown in these Figures 3 and 4, structural models with personal taxes perform significantly better than traditional models considering default risk only. Our study supports this view. However, when we express the model-generated spreads as a percentage of the observed values (shown in the lower panels), a troubling pattern emerges. The model explanatory power after accounting for default risk and personal taxes exhibited a strong rating-dependent pattern that cannot be readily explained by liquidity risk.[19] In particular, the curves showed the least explanatory power occurred at the BBB rating: a feature that held true independent of bond maturities (i.e., true for 4- and 10-year bonds), and the specific models used despite their drastic difference in the capital structure assumptions. This is a highly uncomfortable result which does not match financial intuition. That is, if only taxes and default risk are included, the remaining unexplained spread attributable to liquidity would imply that junk bonds are more liquid than most investment grade bonds.

In reality, there are regulations that prohibit certain institutional investors from investing in junk bonds. When a bond is down-graded to BBB, its investors have increased concerns that any further down-grading may result in a sudden relative increased supply (from these restricted investors) which pushes down the bond price. This concern warrants an additional premium that is not related to default risk (because BBB bond ratings are not default states); not related to taxes (because tax treatments do not change); and not strictly related to illiquidity (because BBB rating should be more liquid than junk bond ratings). Thus, in order to capture the premium due to a possible rating change, we incorporate rating transition risk into our models.

In addition to the strong evidence that liquidity is not the only missing factor, liquidity is also the most elusive factor in asset pricing research because there is no accurate quantified definition for it.[20] Accordingly, a model should first consider as many important quantifiable factors as possible and understand their behaviors. This allows a better understanding of the nature of the liquidity risk and the estimation of its magnitude. In sum, there are both theoretical and empirical reasons for incorporating rating transition risk in the model.

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[19] Previous studies, for example, de Jong *et. al*. (2005), and Liu *et. al*. (2006), have tended to implicitly assume the portion of the spread that is unexplained by taxes and default risk comes from liquidity.

[20] In most cases, asset pricing is done by assuming market completeness and no arbitrage. Liquidity is a result of limited market of demand and supply. As a result it is hard, if not impossible, to "catch" the liquidity factor in a structural arbitrage-free framework.

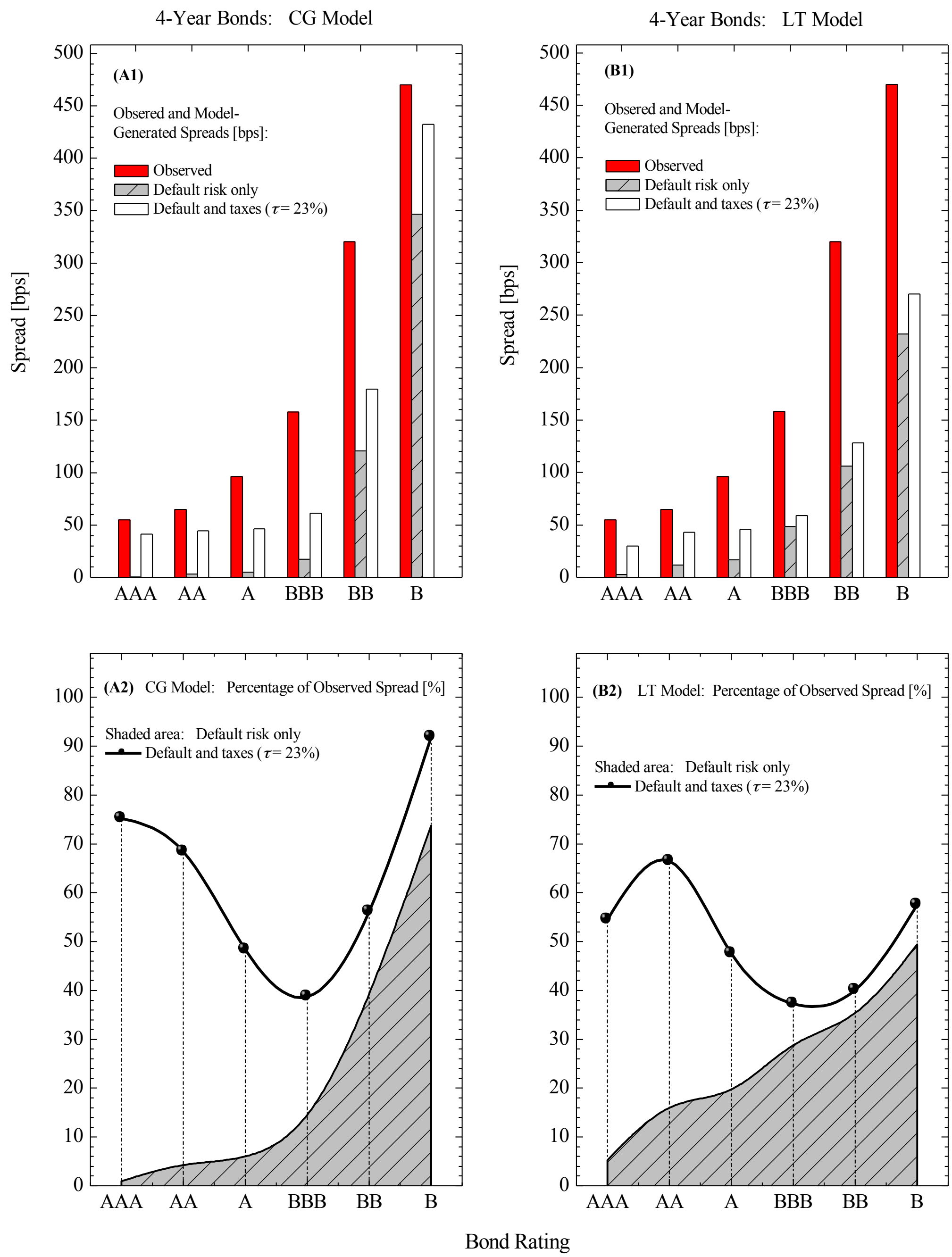

Figure 1. Default and tax premiums on 4-year bonds. The upper panels show the observed and model-generated spreads in bps. The lower panels express the model-generated spreads as a percentage of the observed values. The two left panels are for the CG model (*where leverage ratio is exogenously given*) and the two right for the LT model (*where the model endogenously chooses the optimal capital structure*). Notice that the explanatory power of both models unanimously reaches the lowest level in percentage terms at BBB.

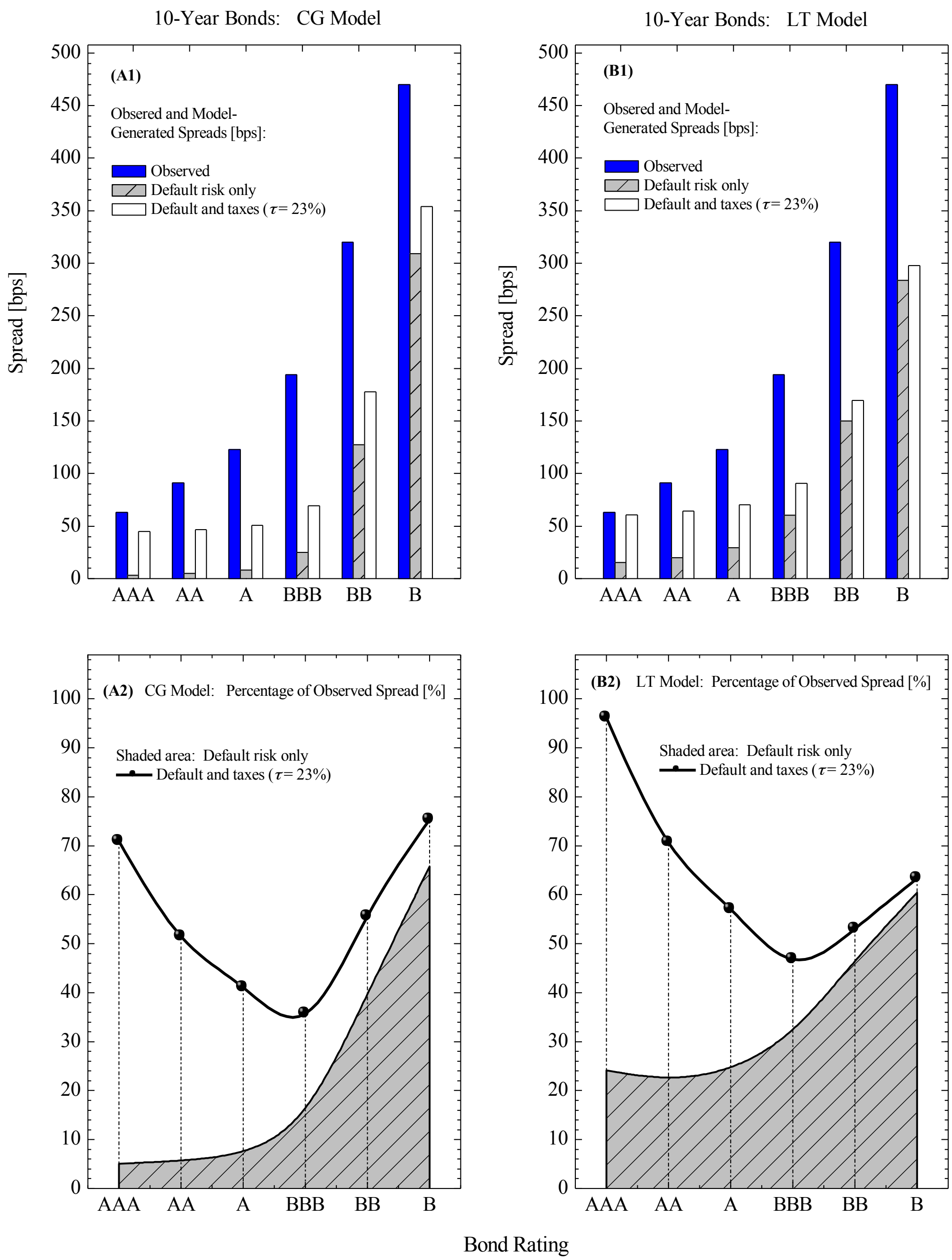

Figure 2. Default and tax premiums on 10-year bonds. The upper panels show the observed and model-generated spreads. The lower panels express the model-generated spreads as a percentage of the observed values. The two left panels are for the CG model (*where leverage ratio is exogenously given*) and the two right for the LT model (*where the model endogenously chooses the optimal capital structure*). Notice that the explanatory power of both models unanimously reaches the lowest level in percentage terms at BBB.

5c. Rating Transition

Incorporating the rating transition factor should replace the rating-dependent liquidity pattern with a new liquidity pattern that makes more intuitive sense. We also expect the model to generate higher spreads for high-grade bonds and mitigate the possible overshooting problem for low grade bonds. More specifically, the unexplained spread attributed to liquidity should be smaller for high quality bonds and larger for low quality bonds.

The overshooting problem of structural models has been documented in several studies (e.g., Eom *et. al*. 2004). It manifests in two ways: (1) for junk bonds, the model may appear to have a much larger explanatory power (even in percentage terms) for the observed spreads; (2) the model-generated spreads may exceed the observed spreads, especially for junk bonds. While the latter can be easily noticed, the former is more subtle and may lead to spurious conclusion, which could be more harmful. We argue the main reason for this overshooting problem is the neglecting of rating transition. In previous structural models, it is implicitly assumed that a bond will either default with cumulative default probability p_d or remain in the same rating with probability $1 - p_d$ before maturity. There is no other scenario for a bond. However, it is well documented that low-grade bonds have a net propensity to move up over time.[21] In other words, focusing only on the default probability (as in the previous structural models) oversimplifies the situation and the "sunnier side" is ignored for low-grade bonds. That is, the spread explained by default, taxes and rating transition will be smaller for low quality bonds than found in a model with only default and taxes. Thus, we should expect the overshooting problem to be mitigated after rating transitions is incorporated in the model.

By the same token, for high quality bonds, rating transition implies that beyond remaining unchanged or default, the bond also has a probability to be downgraded before maturity. Considering this alternative path of rating evolvement, we expect the rating transition effect to further increase the model-generated spreads. For bonds in the middle range of the rating spectrum, the overall effect of rating transition depends on which factor is dominant (i.e., the influence of being upgraded or downgraded).

Figures 3 and 4 are the main results of our paper. Figure 3 presents spreads with and without rating transitions. Figure 4 documents the implied liquidity premiums with and without rating transitions. The results are exactly as we expected.

In Figure 3, the thin curve in each panel represents default risk and taxes to serve as a comparison. The thick line shows the spreads (in percentage of the observed values) after the rating transition is incorporated. Panels A1 and A2 (the upper two panels) are for the CG model for 4-year and 10-year bonds respectively; Panels B1 and B2 (the lower two panels) are for the LT model for 4-year and 10-year bonds respectively.

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[21] For example, Carty (1997) reports that "there is a relatively greater chance for a non-defaulting B-rated issuer to enjoy a net upgrade than a non-defaulting Ba-rated issuer".

As desired, Figure 3 shows the three-factor model is able to explain a larger percentage of the spread for high quality bonds. For B bonds, the model-explained portion drops after the transition factor is incorporated. For example, the CG model's explanatory power on the B bonds drops from 92% to 61% for 4-year bonds. Stated differently, models with only default and taxes would require relatively minuscule liquidity premiums for B bonds to accurately match the observed default rates. Indeed, a reduction in the explained spread for B bonds is required to both allow a liquidity premium and mitigated the potential overshooting problem for the junk bonds.

Figure 4 documents the unexplained portion of the observed spreads, which is attributable to illiquidity. The upper three panels are expressed in basis points and lower three panels are in percentage of the total observed spreads. In Panel A1, the CG model predicts a higher illiquidity premium (in basis points) on BB bonds than on other ratings (including B bonds) for both 4- and 10-year maturity. This is the most direct evidence that a model with default and taxes cannot satisfactorily explain the spreads by assigning the unexplained portion to liquidity risk. Furthermore, when we express the results in percentage terms (in Panels A2 and B2), both the CG and LT models demonstrated the same pattern: the relative liquidity premium is highest on BBB. This serves as indirect evidence of the rating transition factor we have focused on in this study. Panels C1 and C2 present our results from both models for 4- and 10-year bonds after incorporating rating transition risk. Indeed, not only did the model-inferred liquidity premium in basis points move up as rating declines (in Panel C1), but this is also true in percentage terms (shown by Panel C2 in contrast with A2 and B2).[22]

To summarize the findings, we note that the U-shape ratings-related pattern (with the minimum at BBB) is basically removed.[23] The model-explained portion of the observed spreads goes up considerably for almost all ratings, especially for the BBB bonds. The model's relative explanatory power drops for the B bonds, which in fact helps to mitigate the potential overshooting problem of these models for junk bonds. The improvements range from a few to tens of percentage points.

[22] Finally, we checked the results for sensitivity to the risk neutral transition matrix. For example, Guan (2006) uses a different method to estimate the transition matrix. Carty (1997) and Altman et al (2004) draw attention to the volatility of agency-rating migration matrix. We altered the elements in the base-case matrix by a few to more than 10 percentage points. Again, the results are robust.

[23] Although the CG model still shows a weak U-shape for the 4-year bonds, the model performance increases by more than 10 percentage points, which is quite significant.

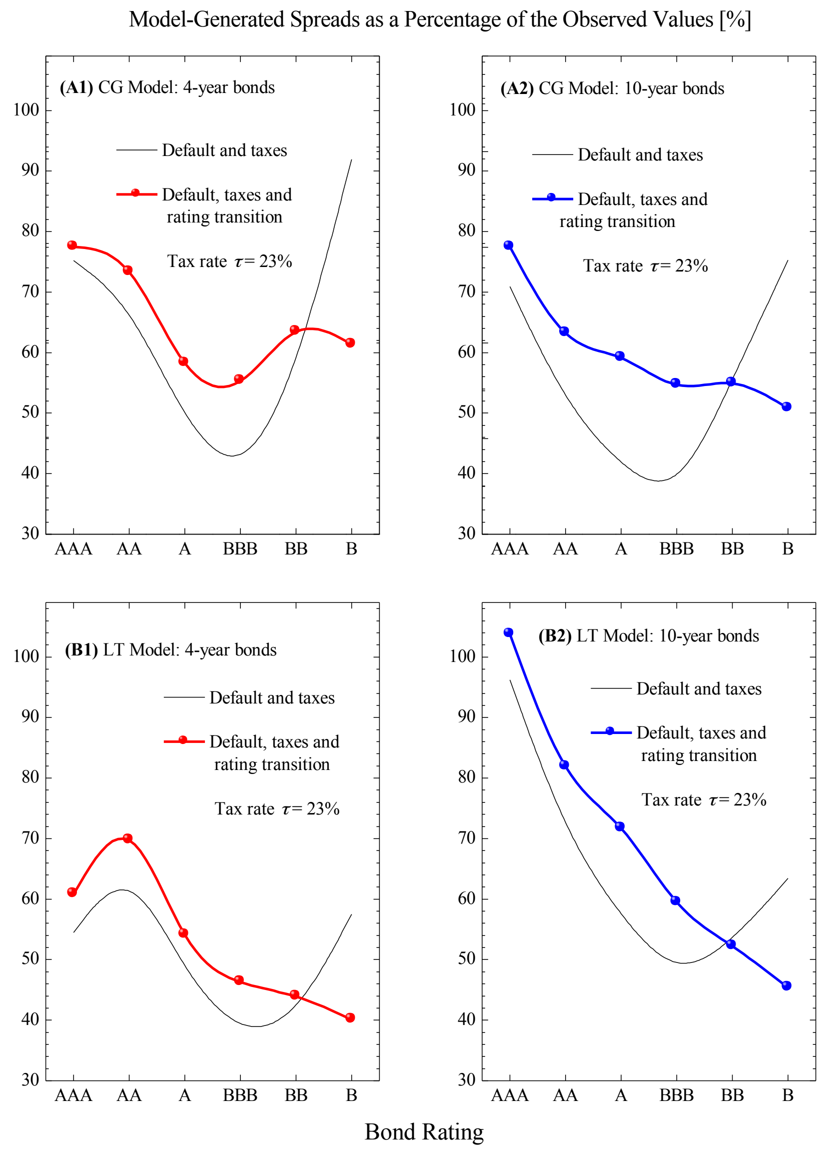

Figure 3. Defaultable bond spreads with personal taxes with and without rating transition. The upper panels are for the CG model and the lower for the LT model. The two left panels are for 4-year bonds and the two right for 10-year bonds. The personal tax rate is chosen (23%) such that the equity return tax rate, estimated as a weighted average of personal income and capital gains tax rates, i.e., $\tau_E = (1-\delta)\alpha\tau + \delta\tau$, is equal to 12 percent estimated by Graham (1999). We note that incorporating rating transition risk into the model considerably increases the explanatory power of both models for all ratings (except B bonds); especially the U-shape pattern (dipping at BBB) is removed or greatly reduced.

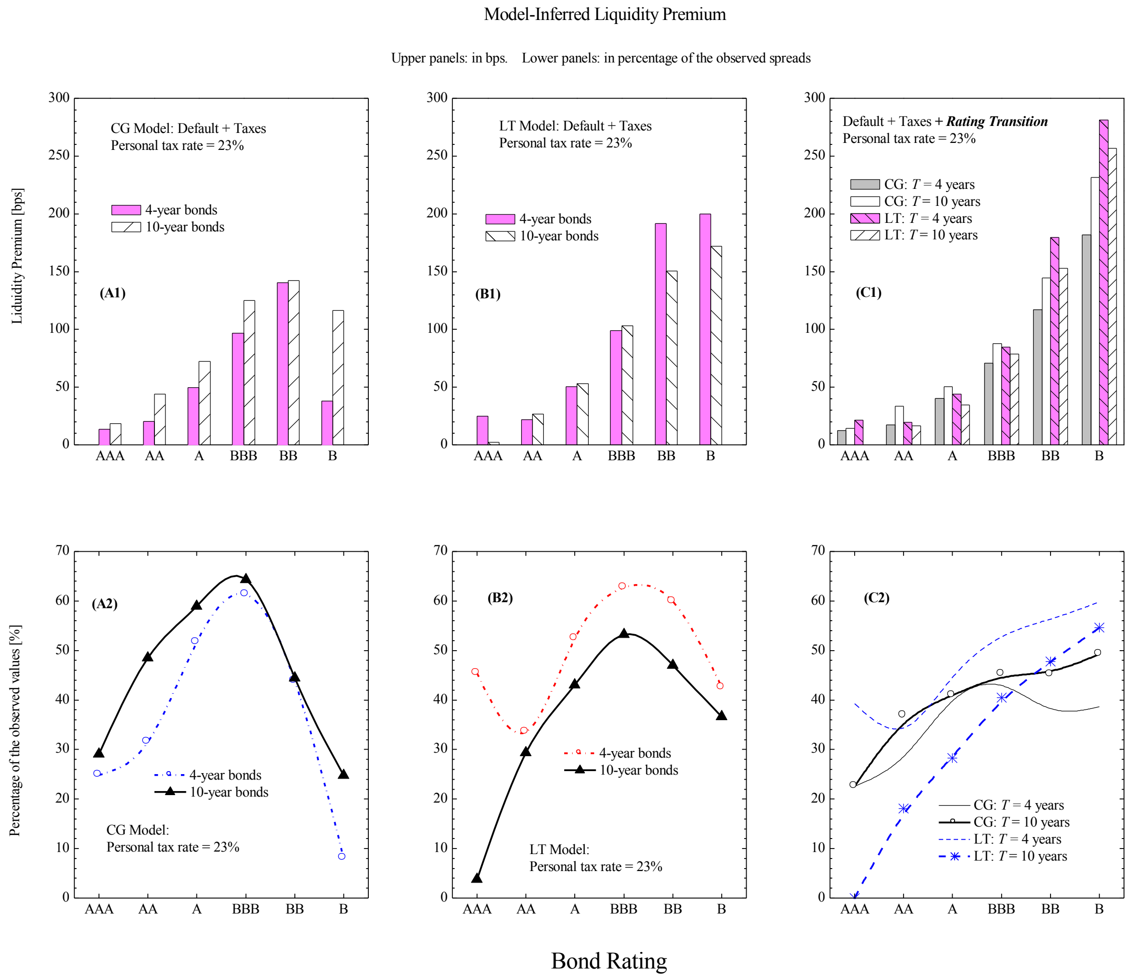

Figure 4. Model-inferred liquidity premium. The upper panels are in basis points and the lower panels are expressed in the percentage of the total observed spreads. In Panel A1, the CG model after calibrated with taxes implies that liquidity premium is lower on B bonds than on some investment-grade bonds, a quite disturbing result. Notice the hump-shapes pattern in Panels A2 and B2 is replaced with generally upward-sloping curves after the incorporation of rating transition risk (shown in Panel C2), which is more consistent with the nature of ratings-liquidity relationship.

5d. Financial Crisis and Model Performance

The recent financial turmoil, especially the subprime crisis, has posed serious challenges to financial industry and academia. To see how our model performs in light of these new developments, we focus on a new sample period of 2004-2010. The main reason to choose this sample period is data availability and a desire to challenge our model with the 2007-2009 subprime crisis. Table 3 shows the results. Panels A and B are for the CG model over short-term and medium-term horizons, and C and D are for the LT model. Following Rossi (2012), we report both short-term (one month to four years) and medium-term (four year to ten year) horizons.

TABLE 3

YIELD SPREADS AND LIQUIDIY PREMIUM: 2004-2010

This table shows the observed spreads (Rossi, 2012) for the period of 2004-2010 and the model-predicted values by considering default risk, taxes, and rating transition. For example, "Default + Tax" means the model considers credit risk, personal taxes, and their interactions jointly. The two right-hand-side columns are for results when all three factors are considered. The implied liquidity premium is reported in the relative term as a percentage of the total observed spreads. Default rate and rating transition data used for model calibration and model implementation are from S&P Global Credit Portal (2011) covering 1981 through 2010. Following Rossi (2012), we present two horizons, short- and medium-term horizons representing average of yield spreads for 1 month to 4 years, and 4 years to 10 years, respectively.

(A) CG Model, T = short-term

Rating	Observed Spreads (bps)	Default + Tax		Default + Tax + Transition	
		Spreads (bps)	Liquidity premium (%)	Spreads (bps)	Liquidity premium (%)
AA	67	43	36	51	24
A	103	58	44	72	30
BBB	158	84	47	122	23
BB	343	317	8	289	16
B	429	450	5	352	18

(B) CG Model, T = medium-term

Rating	Observed Spreads (bps)	Def + Tax		Def + Tax + Transition	
		Spreads (bps)	Liquidity premium (%)	Spreads (bps)	Liquidity premium (%)
AA	80	61	24	82	-3
A	111	77	31	111	0
BBB	235	98	58	168	29
BB	412	301	27	289	30
B	558	590	-6	397	29

(C) LT Model, T = short-term

Rating	Observed Spreads (bps)	Def + Tax		Def + Tax + Transition	
		Spreads (bps)	Liquidity premium (%)	Spreads (bps)	Liquidity premium (%)
AA	67	64	4	70	4
A	103	89	14	102	1
BBB	158	124	22	154	3
BB	343	330	4	299	13
B	429	416	3	344	20

(D) LT Model, T = medium-term

Rating	Observed Spreads (bps)	Def + Tax		Def + Tax + Transition	
		Spreads (bps)	Liquidity premium (%)	Spreads (bps)	Liquidity premium (%)
AA	80	83	-4	90	-13
A	111	94	15	115	-4
BBB	235	114	51	175	26
BB	412	282	32	284	31
B	558	568	-2	421	25

Similar to our main findings using an earlier dataset (1973-1993), the structural models with personal taxes and default risk imply a U-shaped unexplained portion of the observed spreads. In other words, the models imply the greatest (percentage) liquidity premium for BBB rating. For example, in Panel C for the LT model, without rating transition, liquidity premium accounts for about 4% of the total yield spreads for AA, BB and B bonds, while it becomes 22% for BBB bond. All four cases share this characteristic. As we argue earlier, this pattern of model-implied liquidity premium is hard to explain, given the fact that investment-grade bonds are far more liquid than junk bonds. However, if we further incorporate rating transition, the U-shaped pattern of the liquidity premium vanishes. Out of the four cases we investigate, three show a clear increase of liquidity premium as bond rating declines. The only case that does not show this trend is the CG model for the short-term bonds. However, it still makes a significant relative improvement versus the very counter-intuitive pattern found when ignoring rating transitions. These findings once again show that incorporating rating transition in the structural framework indeed makes a desirable and essential improvement.

The recent budget crisis in the U.S. resulted in an unprecedented downgrade of the government debt. This certainly casts a system-wide impact on the debt market and beyond, including the corporate and international bond markets. Bond yields would be affected across the board. However, as long as we still use the yield on the government bond as the baseline even if we no longer consider it risk-free rate, corporate yield spreads would not be affected as much. In other words, we expect our model to perform reasonably well under the shadow of the U.S. budget crisis, which looms over the foreseeable future.

5e. Model Limitations: Financial Crisis and International Market

The approach we propose has its limitations. For example, in one case shown in Panel D of Table 3, after incorporating rating transition, the LT model implies -13% and -4% liquidity premium. for AA and A ratings, respectively. Certainly, a negative value makes no sense. Instead, it means that our model has overshot its yield prediction for these two ratings.

The financial crisis has brought system-wide turmoil, which may reveal a limitation in our approach. During a crisis as serious as the subprime loan crisis, market moves violently and the volatility would be poorly captured by the common (geometric) Brownian motion. This is the well-known fat-tail distribution of risk. Brownian motion, relatively easier and more tractable, tends to describe routine, small and independent risks better. When the whole market is experiencing a crisis, psychology and irrationality can no longer be assumed away and market-wide herding effect becomes more pronounced. Some of these effects may be better captured by more complicated mechanisms, such as Levy process.

International environment poses another limitation. Specifically, different countries may have quite different tax practices. Our approach is strictly based on the U.S. tax environment. How to adjust the tax treatment in the model for different countries remains to be a significant challenge. For example, China is removing taxes on corporate bonds. Australia uses the imputation system where personal tax is linked to the amount of corporate tax a specific company pays. In other words, personal tax depends on how much corporate tax the issuing corporation pays, which is quite different from the US's tax system. In addition to the tax complication, political risk can be come a major concern when we move from the domestic to international environment. Furthermore, currency exchange rate risk can be a daily concern for firms operating globally. Once again, these risks are real but not captured by our approach. Some of them, perhaps the exchange rate risk, may be relatively easier to model within the structural framework. Others are considerably more difficult to reliably address and therefore may require different methodology.

6. Conclusions

The spreads generated by structural models for defaultable and taxable bonds reveal a strong rating-related pattern that cannot be characterized by liquidity. We argue existing structural models suffer from a perilous weakness. Quite simply, they implicitly only allow two states of nature: default and no default without rating changes, whereas the probability of bonds being upgraded/downgraded is ignored in the current structural models. We design a framework that combines the structural models with rating transition risk to account for this missing factor, the overall model performance is considerably improved: it explains more of the observed spreads on investment-grade ratings not only in basis points but also in the relative percentage terms, and on the other hand mitigates the overshooting problem exhibited by some structural models for junk bonds. Most importantly, we are able to remove the unreasonable rating-specific pattern in the implied liquidity premium. By capturing the risk of rating migration, significantly more sensible results are obtained. That is, the unexplained portion of the total spreads is consistent with the nature of liquidity risk. This paper's noteworthy findings are a moderate step forward in exploring the credit spread puzzle.

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